Forms of Conditional Propositions

Equivalent Propositions

Conditional Forms

- Other translations for $p \rightarrow q$, if p then q
  - All p are q.
  - p only if q.
  - p is sufficient for q.
  - p implies q.
  - q is necessary for p.
  - q, if p.

All squares are rectangles

- The different forms of this implication are listed below.
  - If it is a square then it is a rectangle.
  - Being a square implies it is a rectangle.
  - It is a square only if it is a rectangle.
  - Being a square is sufficient for it to be a rectangle.
  - It is a rectangle, if it is a square.
  - Being a rectangle is necessary for it to be square.

Related Implications

- Converse of $p \rightarrow q$:
  - Switch p and q.
  - $q \rightarrow p$
- Inverse of $p \rightarrow q$:
  - Negate both p and q.
  - $\sim p \rightarrow \sim q$
- Contrapositive of $p \rightarrow q$:
  - Switch and negate both p and q.
  - $\sim q \rightarrow \sim p$

A number is divisible by 2, if it is even.

- Recognize the form: q, if p
  - p: It is even., (The number is even.)
  - q: a number is divisible by 2. (It is divisible by 2.
  - If-then form: If the number is even then it is divisible by 2.
- Converse: If a number is divisible by 2 then it is even.
- Inverse: If the number is not even then it is not divisible by 2.
- Contrapositive: If the number is not divisible by 2 then it is not even.

Propositions That Say the Same Thing

- Propositions that say the same thing are equivalent.
- 2 symbolic propositions are equivalent if and only if under all possible conditions the truth value of one is the same as the truth value of the other.

Check for Equivalency

- Write the given propositions in symbolic form.
- Make a truth table showing all conditions for both symbolic propositions.
- Check to see that the columns for the symbolic propositions are alike.

No in-class assignment problem

$p \rightarrow q$ and $\sim p \vee q$

	p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$
1.	T	T	**T**	F	**T**
2.	T	F	**F**	F	**F**
3.	F	T	**T**	T	**T**
4.	F	F	**T**	T	**T**

- They are equivalent. Therefore, every conditional may be written as a disjunction by negating the if component and changing the $\rightarrow$ to a disjunction sign.

In-class Assignment 12 – 3, 4

Writing Equivalent Propositions from Given Forms

- Given that $\sim q \rightarrow p$ and $q \vee p$ are equivalent forms.
- Write an equivalent proposition to "June is a month of the year or $20 > 17$.
  - Determine in which of the 2 forms the given proposition is.
  - The connective "or" says it is in the second form.
  - Use the second form by using the negation of q and the "if – then" connective.
- Equivalent proposition: If June is not a month of the year then $20 > 17$.

In-class Assignment 12 - 4

De Morgan's Laws – Equivalent Propositions

- De Morgan's Laws
  1. $\sim p \wedge \sim q \leftrightarrow \sim (p \vee q)$
- p: Sunday is not a weekday
  q: May is a month of the year
  - Sunday is a weekday and May is not a month of the year.
  - It is not true that Sunday is not a weekday or May is a month of the year.

In-class Assignment 12 - 5

De Morgan's Laws - continued

2. $\sim p \vee \sim q \leftrightarrow \sim (p \wedge q)$

- p: Sunday is not a weekday
  q: May is a month of the year
  - Sunday is a weekday or May is not a month of the year.
  - It is not true that Sunday is not a weekday and May is a month of the year.

In-class Assignment 12 - 5

Equivalent Statements

- Write each proposition in its symbolic form.
  - Construct a truth table or recognize equivalent forms such as De Morgan's Laws or an implication in its disjunction form.

In-class Assignment 12 – 6, 7