Section W
Hypothesis Testing for One Population Proportion

Hypothesis tests for the population proportion can be conducted using the same procedures as we conducted hypothesis tests for one population mean. Unlike hypothesis testing for one population mean there is only one case for hypothesis testing for one sample proportion.

STEPS FOR HYPOTHESIS TESTING FOR ONE POPULATION PROPORTION

Step 1: State Null Hypothesis. $H_o : p = p_o$ (where p_o is a specified value)

Step 2: State Alternative Hypothesis.
1) $H_a : p \neq p_o$
2) $H_a : p > p_o$
3) $H_a : p < p_o$

Step 3: State α. (Usually 0.05, 0.01, or 0.10)

Step 4: Determine the Rejection Region:

two-tailed ($\neq$) : Reject H_o if $z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$

one-tailed (>) : Reject H_o if $z > z_\alpha$

one-tailed (<) : Reject H_o if $z < z_\alpha$

Critical z-values:

	$\alpha = 0.05$		$\alpha = 0.01$		$\alpha = 0.10$	
	(>)	(<)	(>)	(<)	(>)	(<)
1-tailed :	+1.645	−1.645	+2.326	−2.326	+1.282	−1.282
2-tailed (≠):	1.96		2.576		1.645	

Step 5: Calculate the test statistic z: $\hat{p} = \frac{x}{n}$ $\quad z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1-p_o)}{n}}}$

Step 6: Determine if the evaluated statistic is in the critical region or not. Reject or Accept H_o.

Step 7 : State conclusion clearly in words.

P-value approach to hypothesis testing (use with a Minitab Printout):

Step 1: State Null Hypothesis. $H_o : p = p_o$ (where p_o is a specified value)

Step 2: State Alternative Hypothesis. 1) $H_a : p \neq p_o$
2) $H_a : p > p_o$
3) $H_a : p < p_o$

Step 3: State α. (Usually 0.05, 0.01, or 0.10)

Step 4: Determine P-value on Minitab
Step 5: Compare the P-value with α value; If P−value $\leq \alpha$ reject H_o, otherwise accept H_o.
Step 6: State your conclusion in words.

Examples: Hypotheses Testing for One Population Proportion

1) Fargo North, decoder and cryptographer, stated that 40% of the letters in most messages are vowels. Using the first sentence of this exercise as a random sample, test the hypothesis that Fargo has overestimated the percentage of vowels. Use the 0.01 significance level. Number of letters in the first sentence is 78 and the number of vowels is 26.

p = 0.40

x = 26

n = 78

α = 0.01

$$\hat{p} = \frac{26}{78} = 0.33$$

1) H_o: p = 0.40

2) H_a : p < 0.40

3) α = 0.01

4) Reject H_o if z < −2.326

5) $$z = \frac{0.33 - 0.40}{\sqrt{\frac{0.40(1-0.40)}{78}}} = -1.26$$

6) Accept H_o, because −1.26 > −2.326

7) At α = 0.01, the population proportion is equal to 0.40.

2) According to a nationwide poll, 60% of the voters favor a health care bill that would benefit the poor. Senator Ted Gladkowski believes that the percentage of voters who favor the bill is higher in his district. A random sample of 30 voters in his district is taken, and it is found that 24 support the health care bill. Is this sufficient evidence to support Senator Gladkowski's belief at the 0.05 level of significance?

p = 0.60

x = 24

n = 30

α = 0.05

$$\hat{p} = \frac{24}{30} = 0.80$$

1) H_o: p = 0.60

2) H_a : p > 0.60

3) α = 0.05

4) Reject H_o if z > 1.645

5) $$z = \frac{0.80-0.60}{\sqrt{\frac{0.60(1-0.60)}{30}}} = 2.24$$

6) Reject H_o, because 2.24 > 1.645

7) At α = 0.05, the population proportion is greater than 0.60.

3) A certain medicine is said to be 95% effective in giving relief to people with allergic reactions to cats and dogs. Dr. Kay Nyne believes that this claim is incorrect. A random sample of 60 people with such allergies is selected from patients at an allergy clinic. What would you say about the claim at the 0.10 significance level is 58 people got relief?

p = 0.95

x = 58

n = 60

α = 0.10

$$\hat{p} = \frac{58}{60} = 0.97$$

1) **H_o: p = 0.95**

2) **H_a : p ≠ 0.95**

3) **α = 0.10**

4) **Reject H_o if z > 1.645 or z < −1.645**

5) $$z = \frac{0.97 - 0.95}{\sqrt{\frac{0.95(1-0.95)}{60}}} = 0.71$$

6) **Accept H_o, because 0.71 < 1.645**

7) **At α = 0.10, the population proportion is equal to 0.95.**

4) An economics student, Darby Walbert, reads that in his county 70% of the employed earn more than $25,000 per year. He wants to see if the claim is accurate, so he mails out 500 questionnaires to people chosen at random from the phone book. He gets back 100 replies; 85 of them report incomes of more than $25,000. Interpret his results. He plans to test at the 0.01 significance level.

p = 0.70

x = 85

n = 100

α = 0.01

$$\hat{p} = \frac{85}{100} = 0.85$$

1) **H_o: p = 0.70**

2) **H_a : p ≠ 0.70**

3) **α = 0.01**

4) **Reject H_o if z > 2.576 or z < −2.576**

5) $$z = \frac{0.85-0.70}{\sqrt{\frac{0.70(1-0.70)}{100}}} = 3.27$$

6) **Reject H_o, because 3.27 > 2.576**

7) **At α = 0.01, the population proportion is not equal to 0.70.**

5) The usual dropout rate in the freshman class at Wealth College is 50%. A new dean of admissions claims that recent policies have lowered the dropout rate, because in this year's class of 600 freshman only 260 dropped out. Test at the 0.05 significance level. Use the Minitab printout below to test the new dean's claim.

Test and CI for One Proportion of students who drop out

Method

p: event proportion

Descriptive Statistics

N	Event	Sample p	95% Upper Bound for p
600	260	0.433333	0.467633

Test

Null hypothesis	H_0: p = 0.5
Alternative hypothesis	H_1: p < 0.5

P-Value
0.001

1) **H_o: p = 0.5**
2) **H_a : p < 0.5**
3) **α = 0.05**
4) **p-value = 0.001**
5) **0.001 < 0.05, Reject H_o**
6) **At α = 0.05, the population proportion is less than 0.5.**

6) Under certain conditions the probability is 0.20 that a tadpole survives to mature into a frog. Now scientist Ann Juston believes that she has found a way to place vitamins in the frog pond so that more tadpoles will survive. Using this new approach, we take a random sample of 98 tadpoles and find that 31 tadpoles survive. Test at the 0.01 significance level. Using the Minitab, printout below test Ann's claim.

Test and CI for One Proportion - Tadpoles

Method

p: event proportion

Descriptive Statistics

N	Event	Sample p	99% Lower Bound for p
98	25	0.255102	0.159432

Test

Null hypothesis	H_0: p = 0.2
Alternative hypothesis	H_1: p > 0.2

P-Value
0.110

1) **H_o: p = 0.2**
2) **H_a : p > 0.2**
3) **α = 0.01**
4) **p-value = 0.110**
5) **0.110 < 0.01, Accept H_o**
6) **At α = 0.05, the population proportion is equal to 0.2.**