Section U
Confidence Intervals for a Population Proportion

Confidence intervals can also be constructed to estimate the population proportion, p. A population proportion is the proportion of a population that has a particular characteristic and the sample proportion, $\hat{p}$, estimates the population proportion, p. To construct a confidence interval for a population proportion the z-critical value is used. There is only one method to construct a confidence interval for a population proportion.

Confidence Intervals for the Population Proportion, p.

Assumptions:
1) Simple random sample.
2) The population is at least 20 times as large as the sample.
3) The items in the population are divided into two categories.
4) The sample must contain at least 10 individuals in each category.

Critical z-values (α = 1 – Confidence Level)

$z_{\alpha/2}$ = 1.645 if 90% confidence interval
$z_{\alpha/2}$ = 2.326 if 98% confidence interval
$z_{\alpha/2}$ = 1.960 if 95% confidence interval
$z_{\alpha/2}$ = 2.576 if 99% confidence interval

$$\hat{p} = \frac{x}{n} \qquad \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where n is the sample size and
x is the number of data values with the particular characteristic of interest in the sample

Confidence Interval $\left(\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$

Finding the **sample size** needed to estimate the population proportion, p.

If $\hat{p}$ is known, then $n = \hat{p}(1-\hat{p})\left(\frac{z_{\alpha/2}}{m}\right)^2$ Round up to the nearest whole number

If $\hat{p}$ is unknown, then $n = 0.25\left(\frac{z_{\alpha/2}}{m}\right)^2$ Round up to the nearest whole number

Examples:

1) Suppose that in a sample of 2200 households with one or more television sets, 471 watch a particular network's show at a given time. Construct a 90% confidence interval for the population proportion of households watching this show. Interpret your answer.

CL = **90%**

x = **471**

n = **2200**

$z_{\alpha/2}$ = **1.645**

$$\hat{\mathbf{p}} = \frac{\mathbf{471}}{\mathbf{2200}} = \mathbf{0.21}$$

$$\mathbf{0.21 \pm 1.645\sqrt{\frac{0.21(1-0.21)}{2200}}}$$

(0.20, 0.22)

I am 90% confident the population proportion is between 0.20 and 0.22.

b) A claim is made that the population proportion of households who watch a particular network's show at a given time is 0.25. Using the results from part (a) of this problem, would this claim be valid or invalid?

Invalid

2) As part of a market research study, in a sample of 125, 84 individuals are aware of a certain product. Construct a 98% confidence interval for the proportion of individuals in the population who are aware of the product. Interpret your answer.

CL = **98%**

x = **84**

n = **125**

$z_{\alpha/2}$ = **2.326**

$$\hat{\mathbf{p}} = \frac{\mathbf{84}}{\mathbf{125}} = \mathbf{0.67}$$

$$\mathbf{0.67 \pm 2.326\sqrt{\frac{0.67(1-0.67)}{125}}}$$

(0.57, 0.77)

I am 98% confident the population proportion is between 0.57 and 0.77.

b) A claim is made that the population proportion of individuals who are aware of a certain product is 0.65. Using the results from part (a) of this problem, would this claim be valid or invalid?

Valid

3) In a random sample of 400 registered voters, 120 indicated they plan to vote for Candidate A. Construct a 95% confidence interval for the proportion of all the registered voters who will vote for Candidate A. Interpret this result.

CL = **95%**

x = **120**

n = **400**

$z_{\alpha/2}$ = **1.96**

$$\hat{\mathbf{p}} = \frac{\mathbf{120}}{\mathbf{400}} = \mathbf{0.30}$$

$$\mathbf{0.30 \pm 1.96\sqrt{\frac{0.30(1-0.30)}{400}}}$$

(0.26, 0.34)

I am 95% confident the population proportion is between 0.26 and 0.34.

b) A claim is made that the population proportion of voters who plan to vote for Candidate A is 0.33. Using the results from part (a) of this problem, would this claim be valid or invalid?

Valid

4) A manufacturer claims that 98% of the equipment that is supplied to a factory conformed to specifications. An examination of 700 pieces of equipment reveals that 53 are faulty. Construct a 90% confidence interval for the proportion of the equipment that conforms to specification. Interpret your result.

CL = **90%**

x = **700 – 53 = 647**

n = **700**

$z_{\alpha/2}$ = **1.645**

$$\hat{\mathbf{p}} = \frac{\mathbf{647}}{\mathbf{700}} = \mathbf{0.92}$$

$$\mathbf{0.92 \pm 1.645\sqrt{\frac{0.92(1-0.92)}{700}}}$$

(0.90, 0.94)

I am 90% confident the population proportion is between 0.90 and 0.94.

b) Based on the results found in part (a) of this problem, would the manufacturers claim that 98% of the equipment that is supplied to a factory conformed to specifications, be valid or invalid?

Invalid

5) Rosetta Stone wanted to determine the population proportion of students who drop out of a statistics course. Rosetta finds that of the 160 students, 32 drop out. Construct a 99% confidence interval for population proportion of students who drop out of statistics. Interpret your answer.

CL = **99%**

x = **32**

n = **160**

$z_{\alpha/2}$ = **2.576**

$$\hat{p} = \frac{32}{160} = 0.20$$

$$0.20 \pm 2.576\sqrt{\frac{0.20(1-0.20)}{160}}$$

(0.12, 0.28)

I am 99% confident the population proportion is between 0.12 and 0.28.

b) A claim is made that the population proportion of students who drop out of s statistics course is 0.25. Using the results from part (a) of this problem, would this claim be valid or invalid?

Valid

6) A manufacturer of boxes of candy is concerned about the proportion of imperfect boxes — those containing cracked, broken or otherwise unappetizing candies. How large a sample is needed to get a 95% confidence interval for the population proportion of imperfect boxes with a width no greater than 0.02?

CL = 95%

$\mathbf{z_{\alpha/2}}$ **= 1.96**

m = 0.02/2 = 0.01

$$n = 0.25\left(\frac{1.96}{0.01}\right)^2 = 9604 \longrightarrow$$ **n = 9604**

7) Milly Meter, a theater manager, wants to know what percentage of elementary-school children in West Windsor have seen *Aladdin*. What size sample does she need to be 99% confident that she is within 3% of the population proportion?

CL = 99%

$\mathbf{z_{\alpha/2}}$ **= 2.576**

m = 0.03

$$n = 0.25\left(\frac{2.576}{0.03}\right)^2 = 1843.27 \longrightarrow$$ **n = 1844**

Round up to the nearest whole number