Section S
<u>Confidence Intervals</u>

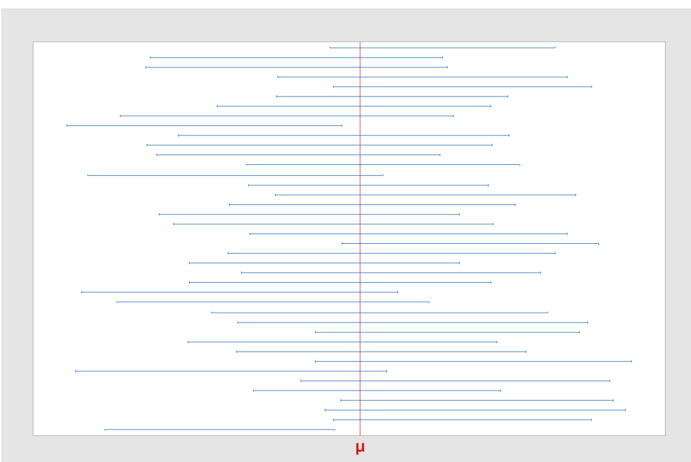
This section starts the study of inferential statistics, specifically methods for estimating the mean of a population. As one might suspect, the statistic used to estimate a population mean, μ, is a sample mean, $\bar{x}$, and because of sampling error, we cannot expect $\bar{x}$ to be exactly equal to μ. Therefore, it is important to provide information about the accuracy of the estimate and this leads to the discussion of confidence intervals. There are two methods for obtaining confidence intervals for the mean of a population; the first method is used when the population standard deviation is known and the second method is used when the population standard deviation is unknown. In addition, confidence intervals can be used to determine whether a claim is valid or invalid.

A **point estimate** of a parameter is a value of a statistic that is used to estimate the value of an unknown parameter.

For example: $\bar{x}$ estimates μ $\hat{p}$ estimates p s estimates σ

A **confidence interval** is an interval of values used to estimate the value of an unknown parameter. A confidence interval is constructed using the point estimate of the unknown parameter along with a percentage, called the **confidence level (CL)** (0% - 100%), that specifies how confident we are that the parameter lies in the interval.

The confidence level represents the expected proportion of intervals that will contain the parameter if a large number of different samples are obtained.



For example in the graph to the left, finding 95% confidence intervals for 40 samples would result in 38 of the 40 intervals containing the population mean and 2 of the intervals would not contain the population mean.

Sampling error will occur with estimation, since it is extremely unlikely for the sample mean to equal the population mean exactly.

The **<u>margin of error</u>**, **m**, is the maximum error when estimating the population parameter of interest using confidence intervals.

Note: The margin of error is equal to half the length of the confidence interval.

Note: The length of a confidence interval for a population mean, μ, and hence the precision with which $\bar{x}$ estimates μ, is determined by the margin of error, m. For a fixed confidence level, increasing the sample size improves the precision, and vice versa.

Case I. Confidence Interval for One Population Mean, μ, when σ is Known.

Assumptions:
1) Simple random sample
2) Normal population or large sample (n > 30)
3) The population standard deviation, σ, is known

Critical z-values (α = 1 – Confidence Level)

$z_{\alpha/2}$ = 1.645 if 90% confidence interval $z_{\alpha/2}$ = 2.326 if 98% confidence interval

$z_{\alpha/2}$ = 1.960 if 95% confidence interval $z_{\alpha/2}$ = 2.576 if 99% confidence interval

$$\bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Margin of error : $m = z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

Confidence Interval $\left(\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right),\ \ \bar{x} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right)$

The **sample size** required for a confidence interval for μ with a specified margin of error, m, is given by the formula

$$n = \left(\frac{z_{\alpha/2} * \sigma}{m}\right)^2$$

Round up to the nearest whole number

Examples:

1) a) Construct a 98% confidence interval for the average time spent waiting for a prescription to be filled at Placebo's Pharmacy, if a random sample of 74 customers showed a mean waiting time of 13.6 minutes and based on past results, the population standard deviation is 4.5 minutes. Interpret your answer.

CL ________

$\bar{x}$ = ________

n = ________

σ = ________

$z_{\alpha/2}$ = _________

b) A claim is made that the population mean wait time is 20 minutes. Using the results from part (a) of this problem, would this claim be valid or invalid?

2) a) A quality-control engineer in a bakery goods plant needs to estimate the mean weight of bags of potato chips that are packed by a machine. He knows from experience the population standard deviation is 0.1oz for this machine and weights are normally distributed. A random sample of 12 bags has a mean weight of 16.01oz. Construct a 95% confidence interval for the population mean. Interpret your answer.

CL ________

$\bar{x}$ = ________

n = ________

σ = ________

$z_{\alpha/2}$ = _________

b) A claim is made that the population mean weight is 16 ounces. Using the results from part (a) of this problem, would this claim be valid or invalid?

3) a) Construct a 99% confidence for the mean salary of teachers in Mercer County if a random sample of 100 teachers had a mean salary of $47,000. Assume the population standard deviation is $1000. Interpret your answer.

CL ________

$\bar{x}$ = ________

n = ________

σ = ________

$z_{\alpha/2}$ = _________

b) A claim is made that the population mean salary is $46,950. Using the results from part (a) of this problem, would this claim be valid or invalid?

4) a) Find a 90% confidence interval for the mean number of hours a child plays a video game per day if a random sample of 43 children has a mean of 3.25 hours and a population standard deviation of 1.75 hours.

CL _______

$\bar{x}$ = _______

n = _______

σ = _______

$z_{\alpha/2}$ = _______

b) A claim is made that the population mean number of hours a child plays a video game per day is 2.5 hours. Using the results from part (a) of this problem, would this claim be valid or invalid?

5) Union officials are concerned about reports of inferior wages being paid to employees of a company under its jurisdiction. How large a sample is needed to obtain a 99% confidence interval for the population mean hourly wage with width equal to $1.00? Assume the population standard deviation is $4.00.

CL _______

σ = _______

$z_{\alpha/2}$ = _______

m = _______

6) An automobile insurance firm wants to find the average amount per claim for auto body repairs. Its summary records combine amounts for body repair with all other amounts, so a sample of individual claims must be taken. A 98% confidence interval within $25 of the population mean is wanted. Assume the population standard deviation is $400. How large a sample is needed?

CL_______

σ = _______

$z_{\alpha/2}$ = _______

m = _______

7) The admission officer wants to determine the average SAT score of students being admitted to XYZ University. A 95% confidence interval within 100 points of the population mean is wanted. How large a sample is needed assuming the population standard deviation is 200?

CL_______

σ = _______

$z_{\alpha/2}$ = _______

m = _______