Section P
Applications of the Normal Distribution

As one can imagine, not all normal distributions have a mean of zero and a standard deviation of 1, so we need to be able to find probabilities of all different normal distributions. In order to continue to use Table B to find probabilities, we need to standardize a normal random variable that does not have a mean of 0 and a standard deviation of 1.

Standardizing a Normal Random Variable

Suppose that the random variable X is normally distributed with mean μ and standard deviation σ, then the standardized version of x is found by calculating the z-score:

$$z = \frac{x - \mu}{\sigma}$$

Recall, the z-score tells how many standard deviations the original value is above or below the mean.

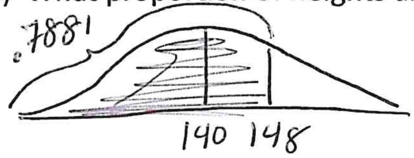
To find probabilities we need to convert the x-value to a z-value and use Table B to find probabilities.

Converting from z-values to x-values you can solve the above formula as follows: $X = \mu + z\sigma$

Examples:

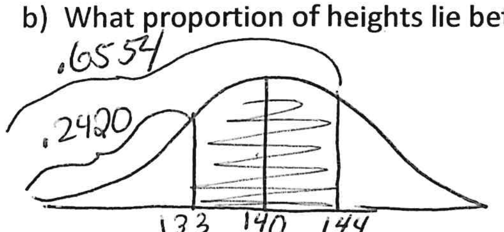
1) Louis N. Clark discovers that the distribution of heights of students in his class is normally distributed with a mean of 140 cm and a standard deviation of 10 cm. Answer the following questions about the distribution of heights for Louis' class:

a) What proportion of heights are below 148? $P(X<148) = .7881$



$Z = \frac{148-140}{10} = .8$

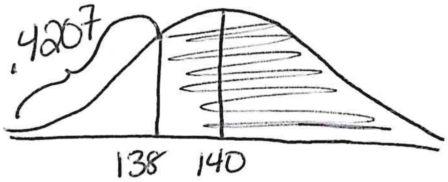
b) What proportion of heights lie between 133 and 144? $P(133<X<144) = .6554 - .2420 = .4134$



$Z = \frac{133-140}{10} = -0.7 \Rightarrow .2420$

$Z = \frac{144-140}{10} = 0.4 \Rightarrow .6554$

c) What proportion of heights lie above 138? $P(X>138) = 1 - .4207 = .5793$

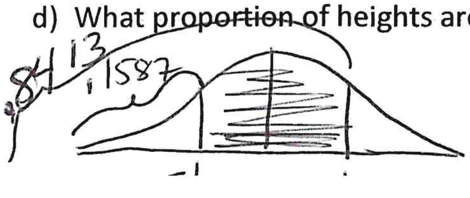


$Z = \frac{138-140}{10} = 0.2 \Rightarrow .4207$

d) What proportion of heights are within 1 standard deviation of the mean? so $Z=-1$ and $Z=1$

$\Downarrow$ .1587 $\Downarrow$ .8413

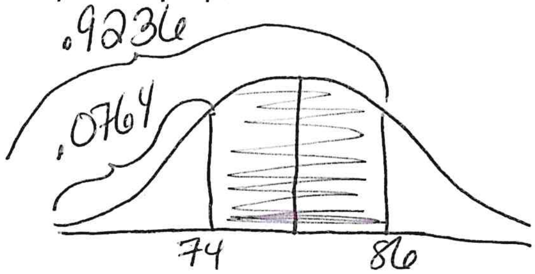
$.8413 - .1587 = .6826$



2) The results of a certain blood test performed by nurse Sheri Weine are known to be normally distributed with a mean of 80 and a standard deviation of 4.2. Answer the following :

a) What proportion of results are between 74 and 86? $P(74<X<86)=.9236-.0764$

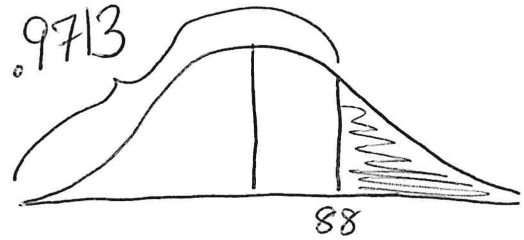
$=\underline{.8472}$



$Z=\frac{74-80}{4.2}=1.43 \Rightarrow .0764$

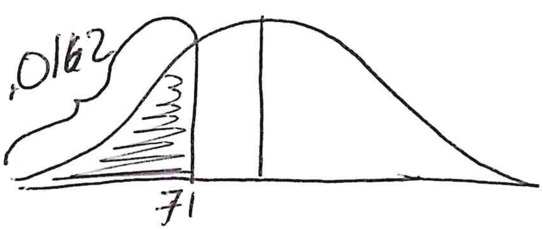
$Z=\frac{86-80}{4.2}=1.43 \Rightarrow .9236$

b) What proportion of results are above 88? $P(X>88)=1-.9713=\underline{.0287}$



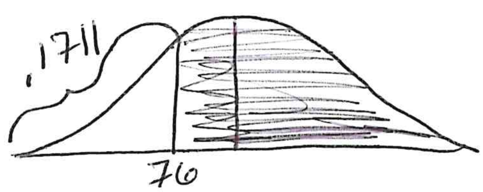
$Z=\frac{88-80}{4.2}=1.90 \Rightarrow .9713$

c) What proportion of results are below 71? $P(X<71)=\underline{.0162}$



$Z=\frac{71-80}{4.2}=-2.14 \Rightarrow .0162$

d) What proportion of results are above 76? $P(X>76)=1-.1711=\underline{.8289}$

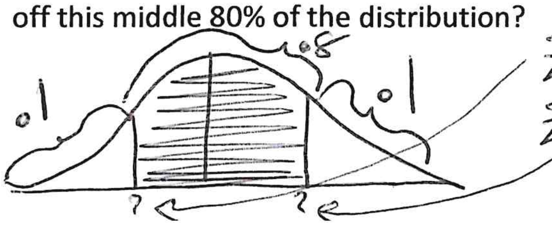


$Z=\frac{76-80}{4.2}=-.95 \Rightarrow .1711$

e) What is the probability that a blood test result picked at random will fall within two standard deviations of the mean? So $Z=-2$ and $Z=2$

$.9772-.0228=\underline{.9544}$

f) The middle 80% of the distribution is considered to be the healthy range. What two blood test results cut off this middle 80% of the distribution?



$Z=-1.28$

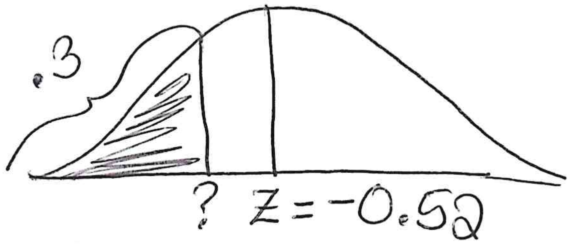
$Z=1.28$

$X=80+(-1.28)(4.2)=74.62$

$X=80+(1.28)(4.2)=85.38$

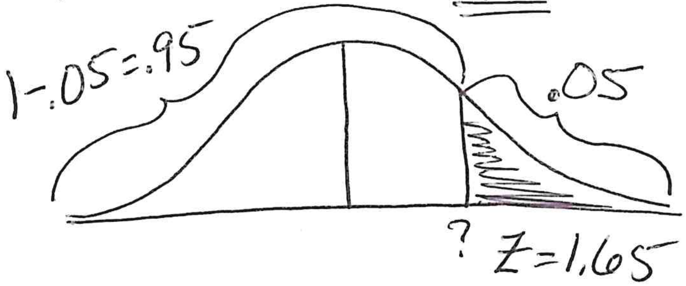
3) A distribution of test scores is normally distributed with a mean of 73 and a standard deviation of 8.

a) What test score cuts off the bottom 30% of the distribution?



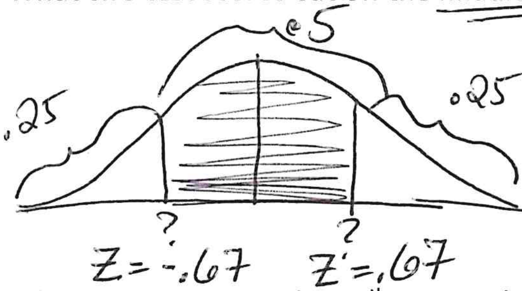
$$X = 73 + (-.52)(8)$$
$$= \underline{68.84}$$

b) What test score cuts off the top 5% of the distribution?



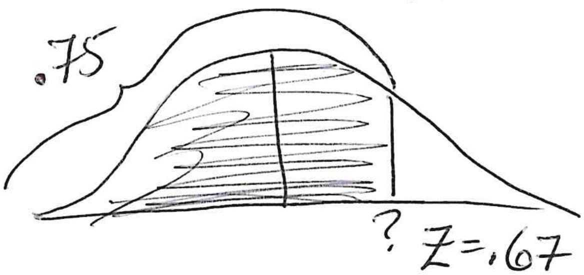
$$X = 73 + 1.65(8)$$
$$= \underline{86.2}$$

c) What two test scores cut off the middle 50% of the distribution?



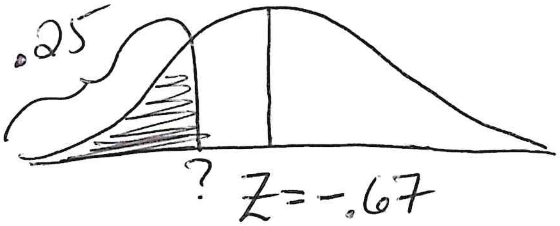
$$X = 73 + (-.67)(8) = \underline{67.64}$$
$$X = 73 + (.67)(8) = \underline{78.36}$$

d) What test score is at the 75th percentile?



$$X = 73 + (.67)(8) = \underline{78.36}$$

e) What test score is at the 25th percentile?



$$X = 73 + (-.67)(8) = \underline{67.64}$$

f) What test score is at the top 10%?

$$X = 73 + 1.28(8)$$
$$= \underline{83.24}$$