Section P

Applications of the Normal Distribution

As one can imagine, not all normal distributions have a mean of zero and a standard deviation of 1, so we need to be able to find probabilities of all different normal distributions. In order to continue to use Table B to find probabilities, we need to standardize a normal random variable that does not have a mean of 0 and a standard deviation of 1.

Standardizing a Normal Random Variable

Suppose that the random variable X is normally distributed with mean μ and standard deviation σ, then the standardized version of x is found by calculating the z-score:

$$z = \frac{x - \mu}{\sigma}$$

Recall, the z-score tells how many standard deviations the original value is above or below the mean.

To find probabilities we need to convert the x-value to a z-value and use Table B to find probabilities.

Converting from z-values to x-values you can solve the above formula as follows: $x = \mu + z\sigma$

Examples:

1) Louis N. Clark discovers that the distribution of heights of students in his class is normally distributed with a mean of 140 cm and a standard deviation of 10 cm. Answer the following questions about the distribution of heights for Louis' class:

a) What proportion of heights are below 148?

b) What proportion of heights lie between 133 and 144?

c) What proportion of heights lie above 138?

d) What proportion of heights are within 1 standard deviation of the mean?

2) The results of a certain blood test performed by nurse Sheri Weine are known to be normally distributed with a mean of 80 and a standard deviation of 4.2. Answer the following :

a) What proportion of results are between 74 and 86?

b) What proportion of results are above 88?

c) What proportion of results are below 71?

d) What proportion of results are above 76?

e) What is the probability that a blood test result picked at random will fall within two standard deviations of the mean?

f) The middle 80% of the distribution is considered to be the healthy range. What two blood test results cut off this middle 80% of the distribution?

3) A distribution of test scores is normally distributed with a mean of 73 and a standard deviation of 8.

a) What test score cuts off the bottom 30% of the distribution?

b) What test score cuts off the top 5% of the distribution?

c) What two test scores cut off the middle 50% of the distribution?

d) What test score is at the 75^{th} percentile?

e) What test score is at the 25^{th} percentile?

f) What test score is at the top 10%?