Section O
The Standard Normal Curve

Now we will discuss the most important distribution in statistics, the normal distribution. The normal distribution is a very common continuous distribution as it occurs often in both theory and practice. Many different variables have populations that form a normal distribution or one that is approximately normally distribution. The normal distribution is also used to make statistical inferences.

A probability density curve represents the probability distribution of a continuous variable.

1. The area under the entire curve is equal to 1.
2. The area under the curve between two values *a* and *b* has two interpretations:
    a. It is the proportion of the population whose values are between *a* and *b*.
    b. It is the probability that a randomly selected individual will have a value between *a* and *b*.

A continuous random variable is **normally distributed**, or has a **normal probability distribution**, if the relative frequency histogram of the random variable has the shape of a normal curve (bell-shaped curve).

Properties of the normal probability distribution

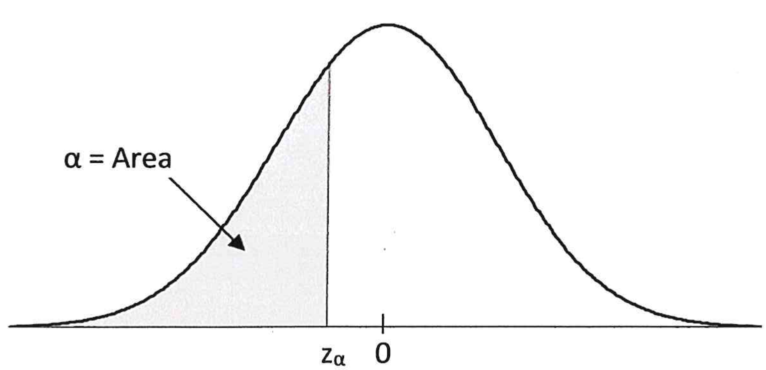
1. Symmetric about its mean, μ.
2. The mean = median = mode
3. The area under the curve equals 1.
4. The area to the right of the mean equals ½ and the area to the left of the mean equal ½.
5. Asymptotic to the x-axis.
6. Empirical rule applies. (68%, 95%, 99.7%)

A normal distribution can have any mean and any positive standard deviation, therefore the mean and standard deviation are the parameters associated with a normal distribution.

The most basic normal distribution is the **standard normal distribution** which has a mean of 0 and standard deviation 1. We use z-scores to represent the values on the x-axis.

The standard normal distribution for this course will be defined by a table. **Table B** is used to find areas under the standard normal curve and it provides the area to the left of a z-score. Finding an area under the standard normal curve is the same as finding a probability; the area under the curve is a probability.

Notation: z_α is the z-score with an area of α to its left



Steps for finding probabilities using Table B:

1) If the probability you are looking for is less than z, then find the area on the table and that is the probability.

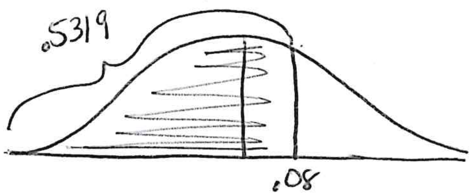
2) If the probability you are looking for is greater than z, then find the area on the table and subtract it from 1 to find the probability.

3) If the probability you are looking for is between two z-scores, then find the area on the table for both z-scores and subtract the smaller area from the larger area to find the probability.

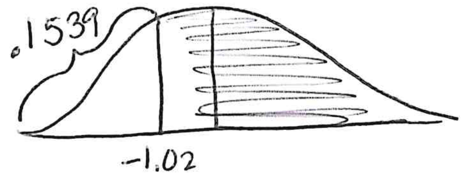
Examples:

Find the following probabilities:

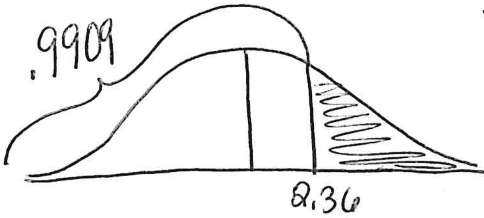
1) P(z < 0.08) = .5319



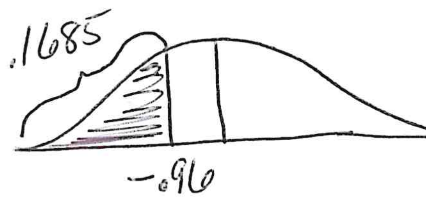
2) P(z > −1.02) = 1 − .1539 = .8461



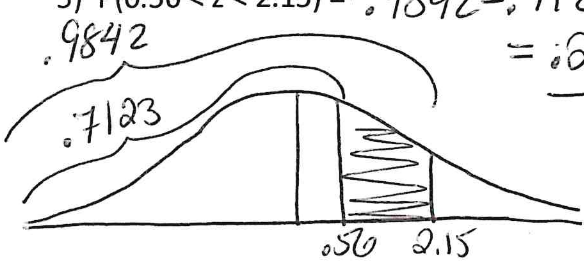
3) P(z > 2.36) = 1 − .9909 = .0091



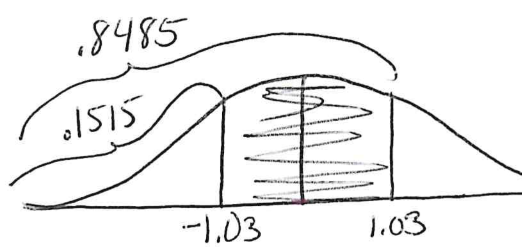
4) P(z < −0.96) = .1685



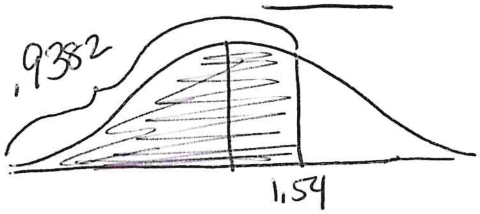
5) P(0.56 < z < 2.15) = .9842 − .7123 = .2719



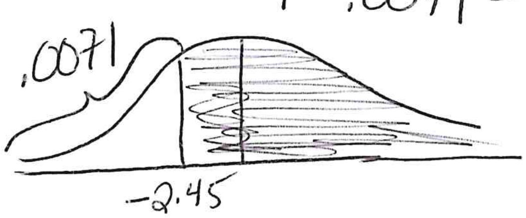
6) P(−1.03 < z < 1.03) = .8485 − .1515 = .6970



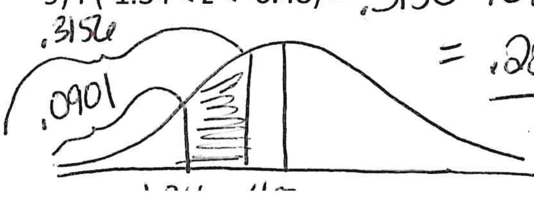
7) P(z < 1.54) = .9382



8) P(z > -2.45) = 1 − .0071 = .9929

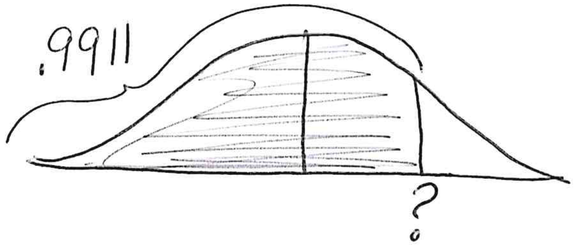


9) P(-1.34 < z < -0.48) = .3156 − .0901 = .2255



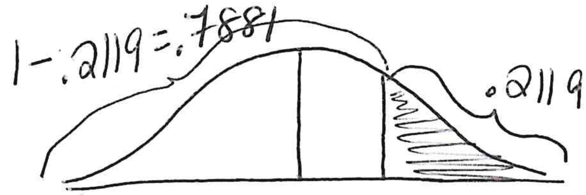
We can also find z-scores corresponding to a given percent.

1) Find the z-score in which 99.11 % of the z-scores in a population are below that value.



$Z = 2.37$

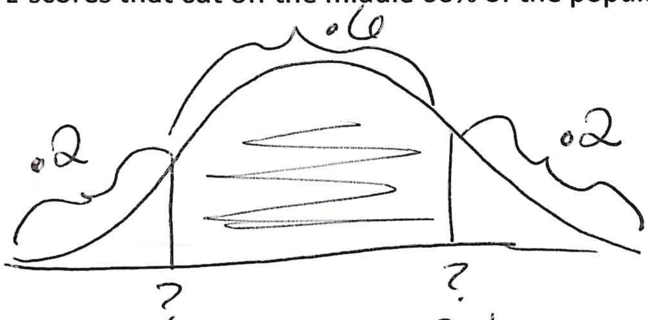
2) Find the z-score in which 21.19% of the z-scores in a population are above that value.



$Z = .80$

3) Find the z-scores that cut off the middle 60% of the population.

$1 - .6 = \frac{.4}{2} = .2$



$-.84$ $.84$