

## Section O

### The Standard Normal Curve

Now we will discuss the most important distribution in statistics, the normal distribution. The normal distribution is a very common continuous distribution as it occurs often in both theory and practice. Many different variables have populations that form a normal distribution or one that is approximately normally distribution. The normal distribution is also used to make statistical inferences.

A probability density curve represents the probability distribution of a continuous variable.

1. The area under the entire curve is equal to 1.
2. The area under the curve between two values  $a$  and  $b$  has two interpretations:
  - a. It is the proportion of the population whose values are between  $a$  and  $b$ .
  - b. It is the probability that a randomly selected individual will have a value between  $a$  and  $b$ .

A continuous random variable is **normally distributed**, or has a **normal probability distribution**, if the relative frequency histogram of the random variable has the shape of a normal curve (bell-shaped curve).

#### Properties of the normal probability distribution

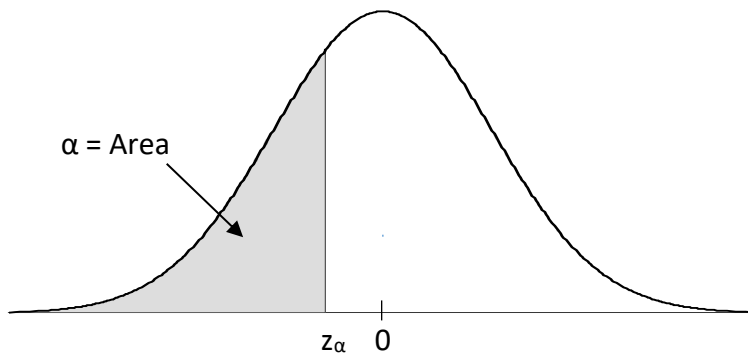
1. Symmetric about its mean,  $\mu$ .
2. The mean = median = mode
3. The area under the curve equals 1.
4. The area to the right of the mean equals  $\frac{1}{2}$  and the area to the left of the mean equal  $\frac{1}{2}$ .
5. Asymptotic to the x-axis.
6. Empirical rule applies. (68%, 95%, 99.7%)

A normal distribution can have any mean and any positive standard deviation, therefore the mean and standard deviation are the parameters associated with a normal distribution.

The most basic normal distribution is the standard normal distribution which has a mean of 0 and standard deviation 1. We use z-scores to represent the values on the x-axis.

The standard normal distribution for this course will be defined by a table. **Table B** is used to find areas under the standard normal curve and it provides the area to the left of a z-score. Finding an area under the standard normal curve is the same as finding a probability; the area under the curve is a probability.

Notation:  $z_\alpha$  is the z-score with an area of  $\alpha$  to its left



Steps for finding probabilities using Table B:

- 1) If the probability you are looking for is less than  $z$ , then find the area on the table and that is the probability.
- 2) If the probability you are looking for is greater than  $z$ , then find the area on the table and subtract it from 1 to find the probability.
- 3) If the probability you are looking for is between two  $z$ -scores, then find the area on the table for both  $z$ -scores and subtract the smaller area from the larger area to find the probability.

**Examples:**

Find the following probabilities:

1)  $P(z < 0.08) =$

2)  $P(z > -1.02) =$

3)  $P(z > 2.36) =$

4)  $P(z < -0.96) =$

5)  $P(0.56 < z < 2.15) =$

6)  $P(-1.03 < z < 1.03) =$

7)  $P(z < 1.54) =$

8)  $P(z > -2.45) =$

9)  $P(-1.34 < z < -0.48) =$

We can also find z-scores corresponding to a given percent.

- 1) Find the z-score in which 99.11 % of the z-scores in a population are below that value.
  
  
  
  
  
  
  
  
  
  
- 2) Find the z-score in which 21.19% of the z-scores in a population are above that value.
  
  
  
  
  
  
  
  
  
  
- 3) Find the z-scores that cut off the middle 60% of the population.