Section N
Binomial Probability Distribution

One of the most important discrete random variables is the binomial random variable. Since this is a specific type of discrete random variable, we will discuss what makes a random variable binomial, learn how to use the binomial formula to calculate probabilities, learn how to use the binomial probability table to find probabilities, as well as learn how to calculate the mean and standard deviation for a binomial random variable.

What makes a random variable binomial?

If you are conducting an experiment in which the random variable X represents the number of successes in a sequence of trials and the following 4 conditions hold:

1. A fixed number of trials are conducted.
2. There are two possible outcomes for each trial. One is labeled "success" and the other is labeled "failure."
3. The probability of success is the same on each trial.
4. The trials are independent. This means that the outcome of one trial does not affect the outcomes of the other trials.

then **X is a binomial random variable**.

Notation: n = number of independent trials of the experiment
p = probability of success for each trial, hence 1 – p = the probability of failure
X denotes the number of successes in n independent trials of the experiment. So $0 \leq \mathrm{X} \leq \mathrm{n}$.

Formula for Binomial Probabilities

For a binomial random variable X that represents the number of successes in n trials with success probability p, the probability of obtaining x successes is

$$\mathbf{P(X = x) = {}_{n}C_{x}p^{x}(1 - p)^{n-x}}$$ where x = 0,1,2,3,…,n

Note: ${}_{\mathrm{n}}\mathrm{C}_{\mathrm{x}} = \frac{n!}{x!(n-x)!}$, this function can be found on most calculators so you really don't need to know this formula

If X is a binomial random variable, then it has a binomial distribution with parameters n and p.

Examples:

1) Determine the indicated probability for a binomial experiment with the given number of trials *n* and the given probability of success, *p*. Round answers to four decimal places.

a) n = 6, p = 0.45, Find P(4). $\mathbf{P(X=4) = {}_{6}C_{4}(0.45)^{4}(1-0.45)^{6-4} = 0.1861}$

b) n = 24, p = 0.6, Find P(20). $\mathbf{P(X=20) = {}_{24}C_{20}(0.6)^{20}(1-0.6)^{24-20} = 0.0099}$

c) n = 50, p = 0.76, Find P(35). $\mathbf{P(X=35) = {}_{50}C_{35}(0.76)^{35}(1-0.76)^{50-35} = 0.0766}$

d) n = 37, p = 0.34, Find P(19). $\mathbf{P(X=19) = {}_{37}C_{19}(0.34)^{19}(1-0.34)^{37-19} = 0.0125}$

e) n = 100, p = 0.82, Find P(72). $\mathbf{P(X=72) = {}_{100}C_{72}(0.82)^{72}(1-0.82)^{100-72} = 0.0044}$

Since the binomial distribution is an important distribution in statistics, tables have been created for many of the different values of n and p and therefore very often you do not need to use the formula to calculate binomial probabilities.

Table A can be used to find many binomial probabilities. This table is found the section N folder.

Examples:
Use Table A to find probabilities for the following problems:

2) A student takes a multiple-choice test that has 15 questions. Each question has four choices. The student forgot about the test and decides to guesses randomly on the questions.

a) n = **15** p = $\mathbf{\frac{1}{4} = 0.25}$ (Since equation has four choices and of which only one is correct.)

b) Find P(X = 4). P(X = 4) = **0.225** (Found using Table A, with n = 15 and p = 0.25)

c) Find P(X is more than 4).
P(X > 4) = **P(5) + P(6) + P(7) + …+P(15) = 0.165 + 0.092 + 0.039 + 0.013 + 0.003 + 0.001 + 0 + …+ 0 = 0.313**

3) A bent coin has a probability of landing on heads equal to 0.40. This coin is tossed 5 times.

a) n = **5** p = **0.40**

b) What is the probability of getting at least 3 heads?

$\mathbf{P(X \geq 3) = P(3) + P(4) + P(5) = 0.230 + 0.077 + 0.010 = 0.317}$

c) What is the probability of getting at most 3 heads?

$\mathbf{P(X \leq 3) = P(0) + P(1) + P(2) + P(3) = 0.078 + 0.259 + 0.346 + 0.230 = 0.913}$

4) Assistant Professor Ratso, a leading experimental psychologist, is in the habit of sending mice through mazes. She predicts that a mouse reaching the end of a T-shaped maze is more likely to turn left than right. She believes that the proportion of mice which turn left is 0.70. If this is true and she sends 9 mice down the maze, what is the probability that

a) n = **9** p = **0.70**

b) exactly 3 will turn left. $\mathbf{P(X = 3) = 0.021}$

c) Less than 5 will turn left. $\mathbf{P(X < 5) = 0.000 + 0.000 + 0.004 + 0.021 + 0.074 = 0.099}$

d) All the mice turn left. $\mathbf{P(X = 9) = 0.040}$

5) According to CTIA, 25% of all U.S. households are wireless-only households (no landline). In a random sample of 14 households, what is the probability that

a) n = **14** p = **0.25**

b) Exactly 5 are wireless-only? $\mathbf{P(5) = 0.147}$

c) Fewer than 3 are wireless-only? $\mathbf{P(X < 3) = P(0) + P(1) + P(2) = 0.018 + 0.083 + 0.180 = 0.281}$

d) At least 3 are wireless-only?

$\mathbf{P(X \geq 3) = P(3) + P(4) + \ldots + P(14) =}$

$\mathbf{= 0.24 + 0.220 + 0.147 + 0.073 + 0.028 + 0.008 + 0.002 + 0 + \ldots + 0 = 0.718}$

e) Between 5 and 7, inclusive are wireless-only?

$\mathbf{P(5 \leq X \leq 7) = P(5) + P(6) + P(7) = 0.147 + 0.073 + 0.028 = 0.248}$

Mean (expected value) and standard deviation of a binomial random variable

Let X be a binomial random variable with n trials and success probability p.

Then the mean of X = $\mathbf{\mu_x = np}$ and standard deviation of X = $\boldsymbol{\sigma_X} = \sqrt{\mathbf{np(1-p)}}$

Note: For a fixed p, as the number of trials n in a binomial experiment increases, the probability distribution of the random variable X becomes bell-shaped. As a rule of thumb, if $np(1 - p) \geq 10$, the probability distribution will be approximately bell-shaped.

Examples:

6) According to CTIA, 25% of all U. S. households are wireless-only households. In a simple random sample of 200 households, determine the mean and standard deviation for the number of wireless-only households.

n = **200** $\mu_X = np = (200)(0.25) = 50$

p = **0.25** $\sigma_X = \sqrt{np(1-p)} = \sqrt{200(0.25)(1-0.25)} = 6.12$

7) According to flightstats.com, American Airlines flights from Dallas to Chicago are on time 80% of the time. Suppose 100 flights are randomly selected. Determine the mean and standard deviation for the number of on time flights.

n = **100** $\mu_X = np = (100)(0.80) = 80$

p = **0.80** $\sigma_X = \sqrt{np(1-p)} = \sqrt{100(0.80)(1-0.80)} = 4$

8) In a recent poll, the Gallup Organization found that 45% of adults Americans believe that the overall state of moral values in the United States is poor. Compute the mean and standard deviation of the number of adults who believe that the overall state of moral values in the United States is poor based on a random sample of 500 adult Americans.

n = **500** $\mu_X = np = (500)(0.45) = 225$

p = **0.45** $\sigma_X = \sqrt{np(1-p)} = \sqrt{500(0.45)(1-0.45)} = 11.12$

9) According to the American Lung Association, 90% of adult smokers started smoking before turning before turning 21 years old. Compute the mean and standard deviation of the number of smokers who started before turning 21 years old in 200 trials of a probability experiment.

n = **200** $\mu_x = np = (200)(0.90) = 180$

p = **0.90** $\sigma_x = \sqrt{np(1-p)} = \sqrt{200(0.90)(1-0.90)} = 4.24$