Section L

<u>Conditional Probability and Multiplication Rule</u>

Conditional Probability – a probability that is computed with the knowledge of additional information

The **conditional probability** of an event B, given event A is denoted P(B | A)

P(B | A) is the probability that event B occurs, **given** that event A occurs or has already occurred.

The probability of B given A is given by

$$\boldsymbol{P(B\,|A) = \frac{P(A\ and\ B)}{P(A)}}\text{, where } P(A) \neq 0$$

You can also use: $P(B\,|A) = \frac{Number\ of\ outcomes\ in\ A\ and\ B}{Number\ of\ outcomes\ in\ A}$

This leads to the **General Multiplication Rule:**

$$P(A\ and\ B) = P(A)P(B|A) \quad \text{or } P(A\ and\ B) = P(B)P(A|B)$$

Example:

1) Assume we have an ordinary deck of 52 playing cards.

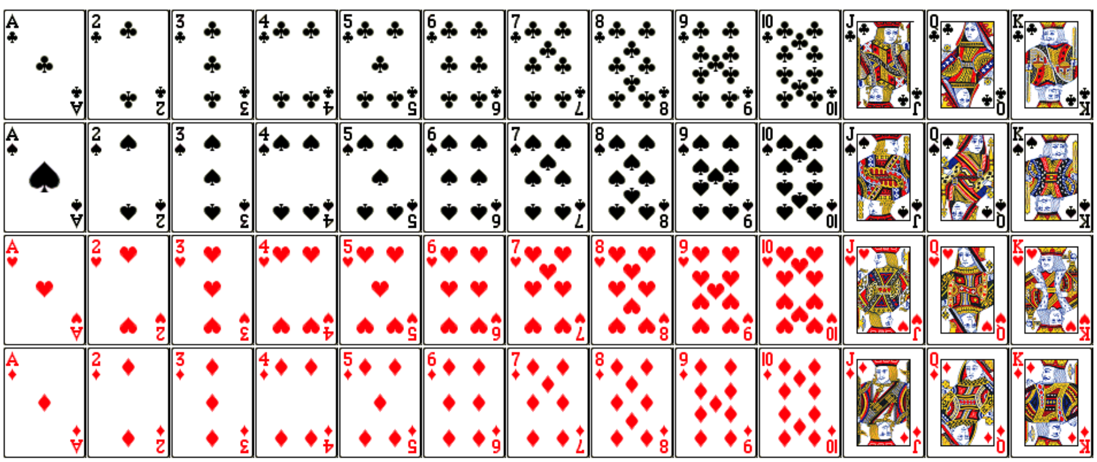
Deck of Cards:

Black: Clubs

Black: Spades

Red: Hearts

Red: Diamonds



One card is chosen at random, find the following probabilities,

a) $P(King|Picture\ card) = \frac{4}{12}$

b) $P(4\ of\ clubs|club) = \frac{1}{13}$

c) $P(2{,}3{,}4\ or\ 5|not\ a\ picture\ card) = \frac{16}{40}$

d) $P(clubs|red\ card) = \frac{0}{26} = 0$

e) $P(4\ |black\ card) = \frac{2}{26}$

2) Let A and B be events with P(A) = 0.35, P(B) = 0.25 and P(A and B) = 0.1. Find $P(A|B)$ and $P(B|A)$.

$P(A|B) = \frac{0.1}{0.25} = 0.4 \quad P(B|A) = \frac{0.1}{0.35} = 0.286$

3) Let A and B be events with P(A) = 0.4, P(B) = 0.6 and $P(B|A) = 0.3$. Find P(A and B).

P(A and B) = (0.4)(0.3) = 0.12

4) The following table displays the 100 Senators of the 115th U.S. Congress on January 3, 2017 viewed by political affiliation and gender.

	Male	Female	Total
Democrat	30	16	46
Republican	47	5	52
Independent	2	0	2
Total	79	21	100

a) P(Senator is a Female) = $\frac{21}{100} = 0.21$

b) P(Democrat) = $\frac{46}{100} = 0.46$

c) P(Female and Democrat) = $\frac{16}{100} = 0.16$

d) $P(Female|Democrat) = \frac{16}{46} = 0.35$

e) $P(Democrat|Female) = \frac{16}{21} = 0.76$

5) At a local business, it was reported that 65 women and 74 men has college degrees. Of the women, 35 have a Master's Degree and of the men 52 have a Master's degree. A person who has a college degree is chosen at random, find the following probabilities:

a) P(female) = $\frac{65}{139} = 0.47$

b) P(Master's Degree) = $\frac{87}{139} = 0.63$

c) P(female and Master's Degree) = $\frac{35}{139} = 0.25$

d) $P(Female|Master's\ Degree) = \frac{35}{87} = 0.40$

e) $P(Master's\ Degree|Female) = \frac{35}{65} = 0.54$

Independence

Two events are **independent** if the occurrence of one does not affect the probability the other event occurs.

In other words, $\boldsymbol{P(B\,|A) = P(B)}$**,** the event A occurring does not affect the probability of event B occurring.

If two events are not independent, then they are called **dependent**.

Hence, **if $\boldsymbol{P(A|B) = P(A)}$, then A and B are independent**,

Multiplication Rule for Independent Events

If A and B are independent events, then P(A and B) = P(A)P(B)

Note: You can extend it : P(A and B and C and D and …) = P(A)P(B)P(C)P(D)…

Examples:

6) Let A and B be independent events with P(A) = 0.6 and P(B) = 0.4. Find P(A and B).

P(A and B) = (0.6)(0.4) = 0.24

7) Let A, B, and C be independent events with P(A) = 0.1, P(B) = 0.25 and P(C) = 0.3. Find P(A and B and C).

P(A and B and C) = (0.1)(0.25)(0.3) = 0.0075

8) A fair coin is flipped 5 times. What is the probability that the sequence of tosses is

a) HTHTH? P(HTHTH) = (0.5)(0.5)(0.5)(0.5)(0.5) = 0.03125

b) HHHHH? P(HHHHH) = (0.5)(0.5)(0.5)(0.5)(0.5) = 0.03125

9) A fair die is rolled 3 times, what is the probability all 3 rolls are 1's?

P(1,1,1) = $\frac{1}{6} * \frac{1}{6} * \frac{1}{6} = \frac{1}{216} = 0.005$

Note: Mutually exclusive and independent are different concepts.

Examples:

10) Let A and B be events with P(A) = 0.7, P(B) = 0.8 and P(A and B) = 0.65.

a) Are A and B independent?

$P(A|B) = \frac{0.65}{0.8} = 0.81$ and P(A) = 0.7, so $P(A|B) \neq$ P(A) therefore A and B are not independent.

b) Are A and B mutually exclusive?

Since P(A and B) does not equal zero, A and B are not mutually exclusive.

c) P(A or B) = 0.7 + 0.8 – 0.65 = 0.85

11) Let A and B be events with P(A) = 0.4, P(B) = 0.5 and P(A or B) = 0.9.

a) Find P(A and B).

P(A and B) = 0.4 + 0.5 – 0.9 = 0

b) Are A and B mutually exclusive?

Yes, since P(A and B) = 0

c) Are A and B independent?

$P(A|B) = \frac{0}{0.5} = 0$ and P(A) = 0.4, so $P(A|B) \neq$ P(A) therefore A and B are not independent.

12) Let A and B be events with P(A) = 0.6, P(B) = 0.4 and P(A or B) = 0.76

a) Find P(A and B).

P(A and B) = 0.6 + 0.4 – 0.76 = 0.24

b) Are A and B mutually exclusive?

No, because P(A and B) ≠ 0.

c) Are A and B independent?

$P(A|B) = \frac{0.24}{0.4} = 0.6$ and P(A) = 0.6, so $P(A|B)$ = P(A) therefore A and B are independent.

13) Jimmy needs to read 7 books for his English class *Fahrenheit 451*, *The Great Gatsby*, *The Lord of the Flies*, *Romeo and Juliet*, *To Kill a Mockingbird*, *The Scarlett Letter*, and *Of Mice and Men*. His teacher said the books will be read in random order. What is the probability Jimmy will read *The Great Gatsby* first and *Of Mice and Men* second.

P(*The Great Gatsby*, then *Of Mice and Men)* = $\frac{1}{7} * \frac{1}{6} = \frac{1}{42} = 0.024$

14) The U.S. National Center for Education Statistics publishes information about school enrollment in *Digest of Education Statistics*. The table below provides information for enrollment in public and private schools levels.

	Type		
Level	Public	Private	Total
Elementary	33,903	4,640	38,543
High School	13,537	1,366	14,903
College	11,626	3,263	14,889
Total	59,066	9,269	68,335

a) P(Private school) = $\frac{9269}{68335} = 0.136$

b) P($Private|High\ School$) $= \frac{1366}{14903} = 0.092$

c) Are events Private and High School independent? Explain your answer in terms of probabilities.

No, because P(Private) ≠ P($Private|High\ School$)

or

P(Private and High School) = $\frac{1366}{68335} = 0.02$ and P(Private)*P(High School) = $\frac{9269}{68335} * \frac{14903}{68335}$ = 0.03

since 0.02 ≠ 0.03, Private and High School are not independent.

d) Are events Private and High School mutually exclusive? Why or why not?

No, because P(Private and High School) ≠ 0.

15) An ice chest contains 5 cans of coke, 3 cans of root beer, and 4 cans of sprite. Three cans are selected at random, without replacement. Find the following probabilities:

a) P(All three cans are coke) =$\frac{5}{12} * \frac{4}{11} * \frac{3}{10} = \frac{60}{1320} = 0.045$

b) P(All three cans are sprite) = $\frac{4}{12} * \frac{3}{11} * \frac{2}{10} = \frac{24}{1320} = 0.018$

c) P(The first two cans are root beer and the third is coke) = $\frac{3}{12} * \frac{2}{11} * \frac{5}{10} = \frac{30}{1320} = 0.023$

d) P(None of the cans are coke) = $\frac{7}{12} * \frac{6}{11} * \frac{5}{10} = \frac{210}{1320} = 0.159$

e) P(the first is sprite, the second is coke and the third is root beer) = $\frac{4}{12} * \frac{5}{11} * \frac{3}{10} = \frac{60}{1320} = 0.045$