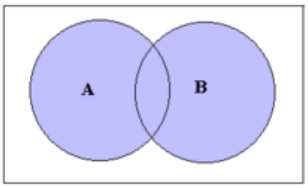
Section K

The Addition Rule and the Rule of Complements

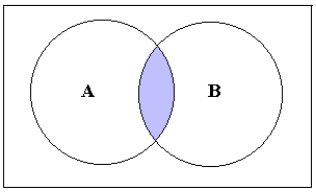
(Round all answers to two or three decimal places.)

A **compound event** is an event that is formed by combining 2 or more events.

P(A or B) = P(A occurs or B occurs or both occur) – inclusive "or"



P(A and B) = P(both A and B occur)



Contingency Table – a table showing the distribution of one variable in rows and another in columns.

Examples:

1) The following table shows the results of a survey for the income level and an individual's favorite form of entertainment.

Income	Favorite Form of Entertainment			Total
	Television	Movies	Theatre (live)	
Under $25,000	35	20	5	60
Between $25,000 and $50,000	25	18	7	50
Over $50,000	12	14	14	40
Total	72	52	26	150

A person is selected at random from this group, calculate the following probabilities:

a) Find the probability that a randomly chosen individual's favorite form of entertainment is going to the movies.

P(Movies) = $\frac{52}{150} = 0.35$

b) P(Income is under $25,000) = $\frac{60}{150} = 0.40$

c) P(Income is over $50,000 **or** favorite form of entertainment is going to the Theatre) = $\frac{52}{150} = 0.35$

d) P(Income between $25,000 and $50,000 **and** going to the movies) = $\frac{18}{150} = 0.12$

e) P(Income is over $25,000) = $\frac{90}{150} = 0.60$

2) The following table shows the results of a survey dealing with age and gambling.

Age	Gambling			
	Frequently	Occasionally	Never	Total
Under 20	12	18	20	50
21 – 30	10	17	23	50
31 – 45	28	15	7	50
Over 45	10	10	30	50
Total	60	60	80	200

A person is selected at random from this group, calculate the following probabilities:

a) P(The person gambles occasionally) = $\frac{60}{200} = 0.30$

b) P(The person is aged between 21 and 30 or never gambles) = $\frac{107}{200} = 0.54$

c) P(The person is over 45 and gambles frequency) = $\frac{10}{200} = 0.05$

d) P(The person is over 31) = $\frac{100}{200} = 0.50$

e) P(The person gambles frequency or occasionally) = $\frac{120}{200} = 0.60$

f) P(The person is not under 20) = $\frac{150}{200} = 0.75$

In the above examples, since you are given a contingency table you do not need to use formulas to find probabilities, but you are not always given a contingency table so formulas are needed to find certain probabilities.

The General Addition Rule

For any two events A and B, P(A or B) = P(A) + P(B) – P(A and B)

For any two events A and B, P(A and B) = P(A) + P(B) – P(A or B)

Examples:

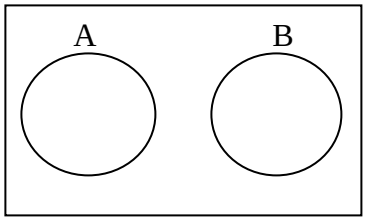
3) If P(A) = 0.35, P(B) = 0.8 and P(A and B) = 0.25. Find P(A or B).

P(A or B) = 0.35 + 0.8 – 0.25 = 0.90

4) If P(A) = 0.65, P(B) = 0.3 and P(A or B) = 0.55. Find P(A and B).

P(A and B) = 0.65 + 0.3 – 0.55 = 0.40

Two events are **mutually exclusive** if it is impossible for both events to occur. P(A and B) = 0



If A and B are **mutually exclusive events**, then P(A or B) = P(A) + P(B)

Examples:

5) If P(A) = 0.4, P(B) = 0.3, and A and B are mutually exclusive. Find P(A or B).

P(A or B) = 0.4 + 0.3 = 0.7

6) If P(A) = 0.7, P(B) = 0.2, and P(A or B) = 0.9. Are A and B mutually exclusive?

Two ways to show mutually exclusive or not:

a) P(A and B) = 0.7 + 0.2 – 0.9 = 0, therefore A and B are mutually exclusive.

or

b) Since P(A) + P(B) = 0.7 + 0.2 = 0.9 = P(A or B), which means P(A and B) = 0 and therefore A and B are mutually exclusive.

7) If P(A) = 0.35, P(B) = 0.45 and P(A or B) = 0.7. Are A and B mutually exclusive?

Two ways to show mutually exclusive or not:

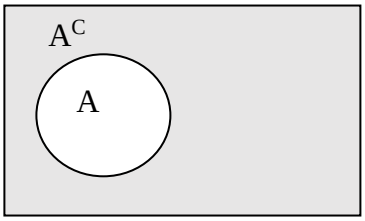
a) P(A and B) = 0.35 + 0.45 – 0.7 = 0.1 $\neq$ 0, therefore A and B are not mutually exclusive.

or

b) P(A) + P(B) = 0.35 + 0.45 = 0.8 $\neq$ 0.7 = P(A or B), therefore A and B are not mutually exclusive.

If A is any event, the **complement of A** is the event that A does not occur. The complement of A is denoted A^C.

Note: $P(A) + P(A^C) = 1$, so $P(A^C) = 1 - P(A)$



Example:

8) If P(A) = 0.25 and P(B) = 0.45. Find $P(A^C)$ and $P(B^C)$.

$P(A^C) = 1 - 0.25 = 0.75$ and $P(B^C) = 1 - 0.45 = 0.55$

More examples:

9) A survey of type of accommodation a person lives in resulted in the following table:

Type of Accommodation	Frequency
House	468
Condo	279
Apartment	646
Townhouse	343
Total	1736

A person is selected at random. Find the following probabilities:

a) P(the person lives in a Condo) = $\frac{279}{1736} = 0.16$ b) P(the person lives in a House) = $\frac{468}{1736} = 0.27$

c) P(the person lives in an apartment or a townhouse) = $\frac{646+343}{1736} = \frac{989}{1736} = 0.57$

10) Let B be the event that a car brought in for service needs new brakes and let S be the event the car needs new struts. Suppose that P(B) = 0.20, P(S) = 0.15 and P(B and S) = 0.05.

a) Find the probability the car needs brakes or struts or both.

P(B or S) = 0.20 + 0.15 – 0.05 = 0.30

b) Find the probability the car does not need new brakes.

$P(B^C) = 1 - 0.20 = 0.80$

11) Last semester at Mercer, 250 students enrolled in both MAT125 and ENG101. Of these students 38 earned an A in statistics, 50 earned an A in English and 20 earned an A in both statistics and English.

a) Find the probability a randomly chosen student earned an A in MAT125 or ENG101 or both.

Let M = the student earned an A in MAT125 and let E = the student earned an A in MAT101

P(M) = $\frac{38}{250} = 0.152$ P(E) = $\frac{50}{250} = 0.20$ P(M and E) = $\frac{20}{250} = 0.08$

so P(M or E) = 0.152 + 0.20 – 0.08 = 0.272

b) Find the probability a randomly chosen student did not earn an A in MAT125.

$P(M^C) = 1 - 0.152 = 0.848$

12) In a BIO103: Anatomy and Physiology class there were 40 students. 23 were females and 17 were males. Three males and six females earned an A in the course. A student is chosen at random from the class.

a) Find the probability the student is a male. P(male) = $\frac{17}{40} = 0.425$

b) Find the probability the student earned an A in the course. P(earned an A) = $\frac{3+6}{40} = \frac{9}{40} = 0.225$

c) Find the probability the student is male and earned an A. P(male and A) = $\frac{3}{40} = 0.075$

d) Find the probability the student is male or earned an A.

P(male or A) = P(male) + P(A) – P(male and A) = 0.425 + 0.225 – 0.075 = 0.575

e) Find the probability the student did not earn an A.

$P(A^C) = 1 - 0.225 = 0.775$

13) Eight cards are in a box. The cards are numbered one through eight, respectively. The cards numbered 1,2,3,4,5 are blue and the cards numbered 6,7, 8 are red. A single card is drawn from the box at random. Find the following probabilities:

a) P(card is a 2) = $\frac{1}{8}$

b) P(card is a 2 or red) = P(2) + P(red) – P(2 and red) = $\frac{1}{8}+\frac{3}{8}-0=\frac{4}{8}$ or
you can just count them and get $\frac{4}{8}$

c) P(card is blue and an odd number) = $\frac{3}{8}$ since there are three cards that are blue and odd

d) P(card is blue or odd number) = $\frac{5}{8}+\frac{4}{8}-\frac{3}{8}=\frac{6}{8}$ or you can just count them and get $\frac{6}{8}$

e) P(card is odd or even) = 1, all the cards are either odd or even.