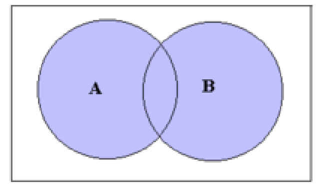
Section K
The Addition Rule and the Rule of Complements

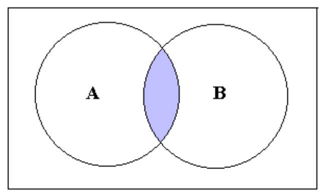
In the previous section, the probabilities found were just for one event, now we will look at how to find probabilities for two or more events together, in other words compound events.
(Round all answers to two or three decimal places.)

A **compound event** is an event that is formed by combining 2 or more events.

P(A or B) = P(A occurs or B occurs or both occur) – inclusive "or"



P(A and B) = P(both A and B occur)



Contingency Table – a table showing the distribution of one variable in rows and another in columns.

Examples:
1) The following table shows the results of a survey for the income level and an individual's favorite form of entertainment.

Income	Favorite Form of Entertainment			Total
	Television	Movies	Theatre (live)	
Under $25,000	35	20	5	60
Between $25,000 and $50,000	25	18	7	50
Over $50,000	12	14	14	40
Total	72	52	26	150

A person is selected at random from this group, calculate the following probabilities:

a) Find the probability that a randomly chosen individual's favorite form of entertainment is going to the movies. P(Movies) = $\frac{52}{150} = \mathbf{0.35}$

b) P(Income is under $25,000) = $\frac{60}{150} = \mathbf{0.40}$

c) P(Income is over $50,000 **or** favorite form of entertainment is going to the Theatre) =$\frac{52}{150} = \mathbf{0.35}$

d) P(Income between $25,000 and $50,000 **and** going to the movies) = $\frac{18}{150} = \mathbf{0.12}$

e) P(Income is over $25,000) = $\frac{90}{150} = \mathbf{0.60}$

2) The following table shows the results of a survey dealing with age and gambling.

Age	Gambling			Total
	Frequently	Occasionally	Never	
Under 20	12	18	20	50
21 – 30	10	17	23	50
31 – 45	28	15	7	50
Over 45	10	10	30	50
Total	60	60	80	200

A person is selected at random from this group, calculate the following probabilities:

a) P(The person gambles occasionally) = $\frac{\mathbf{60}}{\mathbf{200}} = \mathbf{0.30}$

b) P(The person is aged between 21 and 30 or never gambles) = $\frac{\mathbf{107}}{\mathbf{200}} = \mathbf{0.54}$

c) P(The person is over 45 and gambles frequency) **=** $\frac{\mathbf{10}}{\mathbf{200}} = \mathbf{0.05}$

d) P(The person is over 31) = $\frac{\mathbf{100}}{\mathbf{200}} = \mathbf{0.50}$

e) P(The person gambles frequency or occasionally) = $\frac{\mathbf{120}}{\mathbf{200}} = \mathbf{0.60}$

f) P(The person is not under 20) = $\frac{\mathbf{150}}{\mathbf{200}} = \mathbf{0.75}$

In the above examples, since you are given a contingency table you do not need to use formulas to find probabilities, but you are not always given a contingency table so formulas are needed to find certain probabilities.

<u>The General Addition Rule</u>

For any two events A and B, P(A or B) = P(A) + P(B) – P(A and B)

For any two events A and B, P(A and B) = P(A) + P(B) – P(A or B)

Examples:

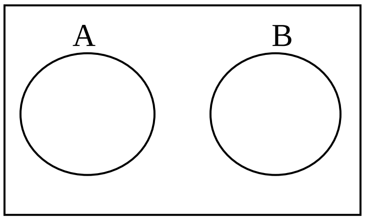
3) If P(A) = 0.35, P(B) = 0.8 and P(A and B) = 0.25. Find P(A or B).

P(A or B) = 0.35 + 0.8 – 0.25 = 0.90

4) If P(A) = 0.58, P(B) = 0.43 and P(A or B) = 0.85. Find P(A and B).

P(A and B) = 0.58 + 0.43 – 0.85 = 0.16

Two events are **<u>mutually exclusive</u>** if it is impossible for both events to occur. P(A and B) = 0



If A and B are **mutually exclusive events**, then P(A or B) = P(A) + P(B)

Examples:

5) If P(A) = 0.4, P(B) = 0.3, and A and B are mutually exclusive. Find P(A or B).

P(A or B) = 0.4 + 0.3 = 0.7

6) If P(A) = 0.7, P(B) = 0.2, and P(A or B) = 0.9. Are A and B mutually exclusive?

Two ways to show mutually exclusive or not:

a) P(A and B) = 0.7 + 0.2 – 0.9 = 0, therefore A and B are mutually exclusive.

or

b) Since P(A) + P(B) = 0.7 + 0.2 = 0.9 = P(A or B), which means P(A and B) = 0 and therefore A and B are mutually exclusive.

7) If P(A) = 0.35, P(B) = 0.45 and P(A or B) = 0.7. Are A and B mutually exclusive?

Two ways to show mutually exclusive or not:

a) P(A and B) = 0.35 + 0.45 – 0.7 = 0.1 ≠ 0, therefore A and B are not mutually exclusive.

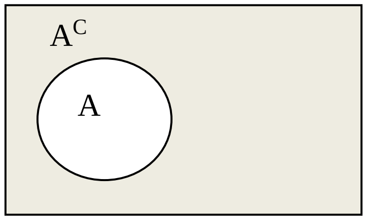
or

b) P(A) + P(B) = 0.35 + 0.45 = 0.8 ≠ 0.7 = P(A or B), therefore A and B are not mutually exclusive.

Complement

If A is any event, the **complement of A** is the event that A does not occur. The complement of A is denoted A^C.

Note: $P(A) + P(A^C) = 1$, so $P(A^C) = 1 - P(A)$



Example:

8) If P(A) = 0.25 and P(B) = 0.45. Find $P(A^C)$ and $P(B^C)$.

$P(A^C) = 1 - 0.25 = 0.75$ and $P(B^C) = 1 - 0.45 = 0.55$

More examples:

9) A survey of type of accommodation a person lives in resulted in the following table:

Type of Accommodation	Frequency
House	468
Condo	279
Apartment	646
Townhouse	343
Total	1736

A person is selected at random. Find the following probabilities:

a) P(the person lives in a Condo) = $\frac{279}{1736} = \mathbf{0.16}$

b) P(the person lives in a House) = $\frac{468}{1736} = \mathbf{0.27}$

c) P(the person lives in an apartment or a townhouse) = $\frac{646+343}{1736} = \frac{989}{1736} = \mathbf{0.57}$

10) Let B be the event that a car brought in for service needs new brakes and let S be the event the car needs new struts. Suppose that P(B) = 0.20, P(S) = 0.15 and P(B and S) = 0.05.

a) Find the probability the car needs brakes or struts or both.

P(B or S) = 0.20 + 0.15 – 0.05 = 0.30

b) Find the probability the car does not need new brakes.

$P(B^C)$ = 1 – 0.20 = 0.80

11) Last semester at Mercer, 250 students enrolled in both MAT125 and ENG101. Of these students 38 earned an A in statistics, 50 earned an A in English and 20 earned an A in both statistics and English.

a) Find the probability a randomly chosen student earned an A in MAT125 or ENG101 or both.

P(M) = $\frac{38}{250} = 0.152$ **P(E) =** $\frac{50}{250} = 0.20$ **P(M and E) =** $\frac{20}{250} = 0.08$

so P(M or E) = 0.152 + 0.20 – 0.08 = 0.272

b) Find the probability a randomly chosen student did not earn an A in MAT125.

$P(M^C)$ = 1 – 0.152 = 0.848

12) In a BIO103: Anatomy and Physiology class there were 40 students. 23 were females and 17 were males.
Three males and six females earned an A in the course. A student is chosen at random from the class.

a) Find the probability the student is a male. **P(male) =** $\frac{17}{40} = 0.425$

b) Find the probability the student earned an A in the course. **P(earned an A) =** $\frac{3+6}{40} = \frac{9}{40} = 0.225$

c) Find the probability the student is male and earned an A. **P(male and A) =** $\frac{3}{40} = 0.075$

d) Find the probability the student is male or earned an A.

P(male or A) = P(male) + P(A) – P(male and A) = 0.425 + 0.225 – 0.075 = 0.575

e) Find the probability the student did not earn an A.

$P(A^C)$ = 1 – 0.225 = 0.775

13) Eight cards are in a box. The cards are numbered one through eight, respectively. The cards numbered 1,2,3,4,5 are blue and the cards numbered 6,7, 8 are red. A single card is drawn from the box at random. Find the following probabilities:

a) P(card is a 2) = $\frac{1}{8}$

b) P(card is a 2 or red) = P(2) + P(red) – P(2 and red) **=** $\frac{1}{8}+\frac{3}{8}-0=\frac{4}{8}$ **or**
you can just count them and get $\frac{4}{8}$

c) P(card is blue and an odd number) = $\frac{3}{8}$ **since there are three cards that are blue and odd**

d) P(card is blue or odd number) = $\frac{5}{8}+\frac{4}{8}-\frac{3}{8}=\frac{6}{8}$ **or you can just count them and get** $\frac{6}{8}$

e) P(card is odd or even) = **1, all the cards are either odd or even.**