Section J

Basic Probability Concepts

Before we can begin to discuss inferential statistics, we need to discuss probability. Recall, inferential statistics deals with analyzing a sample from the population to draw conclusions about the population, therefore since the data came from a sample we can never be 100% certain the conclusion is correct. Therefore, probability is an integral part of inferential statistics and needs to be studied before starting the discussion on inferential statistics.

The **theoretical probability** of an event is the proportion of times the event occurs in the long run, as a probability experiment is repeated over and over again.

Law of Large Numbers says that as a probability experiment is repeated again and again, the proportion of times that a given event occurs will approach its probability.

A **sample space** contains all possible outcomes of a probability experiment.

Example:

1) a) Experiment: Roll a 6-sided die; Sample space = X = {1, 2, 3, 4, 5, 6}

b) Experiment: Flip a coin; Sample space = X = {H, T}

c) Experiment: Flip a coin and roll a die;
Sample space = X = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

An **event** is an outcome or a collection of outcomes from a sample space.

Example: **Experiment: Roll a 6-sided die: Sample space = X = {1, 2, 3, 4, 5, 6}**

Event A:{Roll an even #} = {2, 4, 6}
Event B: {Roll a number greater than 5} = {6}
Event C: {Roll a number less than 3} = {1, 2}

A **probability model** for a probability experiment consists of a sample space, along with a probability for each event.

Note: If A denotes an event then the probability of the event A is denoted P(A).

Probability models with equally likely outcomes

If a sample space has n equally likely outcomes, and an event A has k outcomes, then

$$P(A) = \frac{\text{Number of outcomes in A}}{\text{Number of outcomes in the sample space}} = \frac{k}{n}$$

The probability of an event is always between 0 and 1, inclusive.

Important probability characteristics:

1) For any event A, $0 \leq P(A) \leq 1$

2) If A cannot occur, then P(A) = 0.

3) If A is certain to occur, then P(A) = 1

An **unusual event** is one whose probability is small. Basically, **any probability less than 0.05** would be considered unusual.

Example: **Experiment: Roll a 6-sided die: Sample space = X = {1, 2, 3, 4, 5, 6}**

P(roll a 2) = P(2) = 1/6

P(roll a 3 or 4) = P(3 or 4) = 2/6 = 1/3

P(roll an even number) = P(even) = 3/6 = 1/2

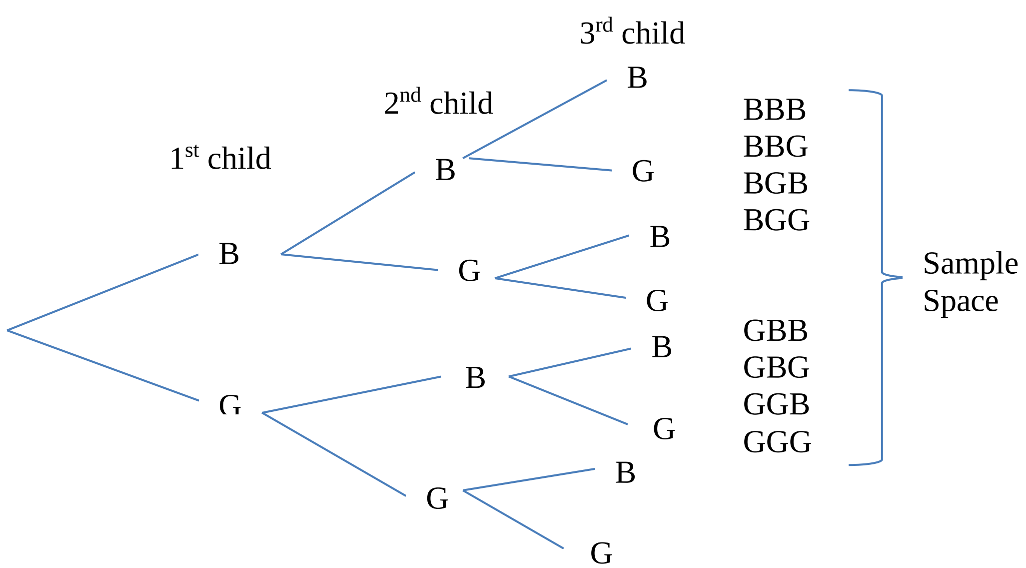
P(roll an odd number) = P(odd) = 3/6 = 1/2

P(roll a 7) = P(7) = 0/6 = 0

P(roll a number greater than 0) = 6/6 = 1

Using a Tree Diagram for Finding a Sample Space

Example: Find the sample space for having three children.



Sampling from a Population is a Probability Experiment

Sampling an individual from a population is a probability experiment. The population is the sample space and the members of the population are equally likely outcomes.

Examples: 1) The following table shows the results of a survey of college freshman, asking how much they pay out-of-pocket, as a student, per year for college.

Cost in thousands of dollars	Frequency
$75 and over	253
$65 < $75	462
$55 < $65	571
$45 < $55	623
$35 < $45	948
$25 <$35	1324
$15 < $25	1235
$5 < $15	652
Under $5	247
Total	6315

a) P(student pays between $55,000 and $65,000) = $\frac{571}{6315} = \mathbf{0.09}$

b) P(student pays $65,000 or over) = $\frac{462+253}{6315} = \frac{715}{6315} = \mathbf{0.113}$

c) P(student pays under $25,000) = $\frac{1235+652+247}{6315} = \frac{2134}{6315} = \mathbf{0.34}$

d) P(student pays under $5,000) = $\frac{247}{6315} = \mathbf{0.04}$

e) P(student pays $5,000 or more) = $\frac{253+462+571+623+948+1324+1235+652}{6315} = \frac{6068}{6315} = \mathbf{0.96}$

2) In a survey of 400 likely voters in a certain city, 215 said that they planned to vote to reelect the incumbent governor.

a) What is the probability that a surveyed voter plans to vote to reelect the incumbent governor?

$$\frac{215}{400} = \mathbf{0.54}$$

b) What is the probability that a surveyed voter plans to vote for a new governor? $\frac{185}{400} = \mathbf{0.46}$

3) During a recent softball season, a softball pitcher threw 125 fast balls, 242 rise balls, 228 drops and 236 curve balls.

a) What is the probability the softball pitcher threw a rise ball? $\frac{242}{831} = \mathbf{0.29}$

b) What is the probability the softball pitcher threw a fast ball? $\frac{125}{831} = \mathbf{0.15}$

c) What is the probability the softball pitcher threw a curve ball or a drop? $\frac{236+228}{831} = \frac{464}{831} = \mathbf{0.56}$

<u>Empirical Probability</u> – consists of repeating an experiment a large number of times and using the proportion of times an outcome occurs to approximate the probability of the outcome.

Examples

1) A fair 6-sided die is rolled 800 times. On 140 of those rolls the die comes up 3, on 128 of those rolls the die comes up 4 and on 135 of those rolls the die comes up 1.

a) Find the empirical probability the die will come up 3. $\frac{\mathbf{140}}{\mathbf{800}} = \mathbf{0.175}$

b) Find the empirical probability the die will come up 4. $\frac{\mathbf{128}}{\mathbf{800}} = \mathbf{0.160}$

c) Find the empirical probability the die will come up 1. $\frac{\mathbf{135}}{\mathbf{800}} = \mathbf{0.169}$

d) Find the empirical probability the die will come up any number except 1. $\frac{\mathbf{665}}{\mathbf{800}} = \mathbf{0.831}$

2) Two dice are rolled 450 times, the sum of 7 comes up 80 times, the sum of 6 comes up 65 times and the sum of 12 comes up 12 times.

a) Find the empirical probability the sum of the dice will be 7. $\frac{\mathbf{80}}{\mathbf{450}} = \mathbf{0.178}$

b) Find the empirical probability the sum of the dice will be 6. $\frac{\mathbf{65}}{\mathbf{450}} = \mathbf{0.144}$

c) Find the empirical probability the sum of the dice will be 12. $\frac{\mathbf{12}}{\mathbf{450}} = \mathbf{0.027}$