Section J
Basic Probability Concepts

Before we can begin to discuss inferential statistics, we need to discuss probability. Recall, inferential statistics deals with analyzing a sample from the population to draw conclusions about the population, therefore since the data came from a sample we can never be 100% certain the conclusion is correct. Therefore, probability is an integral part of inferential statistics and needs to be studied before starting the discussion on inferential statistics.

The **theoretical probability** of an event is the proportion of times the event occurs in the long run, as a probability experiment is repeated over and over again.

Law of Large Numbers says that as a probability experiment is repeated again and again, the proportion of times that a given event occurs will approach its probability.

A **sample space** contains all possible outcomes of a probability experiment.

EX:

An **event** is an outcome or a collection of outcomes from a sample space.

A **probability model** for a probability experiment consists of a sample space, along with a probability for each event.

Note: If A denotes an event then the probability of the event A is denoted P(A).

Probability models with equally likely outcomes

If a sample space has n equally likely outcomes, and an event A has k outcomes, then

$$P(A) = \frac{\text{Number of outcomes in A}}{\text{Number of outcomes in the sample space}} = \frac{k}{n}$$

The probability of an event is always between 0 and 1, inclusive.

Important probability characteristics:

1) For any event A, $0 \leq P(A) \leq 1$

2) If A cannot occur, then $P(A) = 0$.

3) If A is certain to occur, then $P(A) = 1$

An **unusual event** is one whose probability is small. Basically, **any probability less than 0.05** would be considered unusual.

Example:

Using a Tree Diagram for Finding a Sample Space

Example: Find the sample space for having three children

Sampling from a Population is a Probability Experiment

Sampling an individual from a population is a probability experiment. The population is the sample space and the members of the population are equally likely outcomes.

Examples:

Cost in thousands of dollars	Frequency
\$75 and over	253
\$65 ≤ \$75	462
\$55 ≤ \$65	571
\$45 ≤ \$55	623
\$35 ≤ \$45	948
\$25 ≤\$35	1324
\$15 ≤ \$25	1235
\$5 ≤ \$15	652
Under \$5	247
Total	6315

1) The following table shows the results of a survey of college freshman, asking how much they pay out-of-pocket, as a student, per year for college.

a) P(student pays between \$55,000 and \$65,000) =

b) P(student pays \$65,000 or over) =

c) P(student pays under \$25,000) =

d) P(student pays under \$5,000) =

e) P(student pays \$5,000 or more) =

2) In a survey of 400 likely voters in a certain city, 215 said that they planned to vote to reelect the incumbent governor.

a) What is the probability that a surveyed voter plans to vote to reelect the incumbent governor?

b) What is the probability that a surveyed voter plans to vote for a new governor?

3) During a recent softball season, a softball pitcher threw 125 fast balls, 242 rise balls, 228 drops and 236 curve balls.

a) What is the probability the softball pitcher threw a rise ball?

b) What is the probability the softball pitcher threw a fast ball?

c) What is the probability the softball pitcher threw a curve ball or a drop?

Empirical Probability – consists of repeating an experiment a large number of times and using the proportion of times an outcome occurs to approximate the probability of the outcome.

Examples

1) A fair 6-sided die is rolled 800 times. On 140 of those rolls the die comes up 3, on 128 of those rolls the die comes up 4 and on 135 of those rolls the die comes up 1.

a) Find the empirical probability the die will come up 3.

b) Find the empirical probability the die will come up 4.

c) Find the empirical probability the die will come up 1.

d) Find the empirical probability the die will come up any number except 1.

2) Two dice are rolled 450 times, the sum of 7 comes up 80 times, the sum of 6 comes up 65 times and the sum of 12 comes up 12 times.

a) Find the empirical probability the sum of the dice will be 7.

b) Find the empirical probability the sum of the dice will be 6.

c) Find the empirical probability the sum of the dice will be 12.