1. Roughly plot data and regression. Label Axis.

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| Regression <br> used: |  |
| :--- | :--- |
| First x (a) |  |
| Last x (b) |  |

Find the average rate of change between the first and last x -values using regression
\{Y1(b)-Y1(a) $\} /\{b-a\}$

Average Rate of Change
2. Roughly split the graph into two regions and perform different regressions on each side.

Plot data and regressions. Label Axis.

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| left regression split at a Y1=vars 5: >> 1: RegEq /( $x \leq a)$ <br> right regression Y2=vars 5: >>1: $\operatorname{RegEq} /(x \geq a)$ | Left <br> Regression used: |  |
| :---: | :---: | :---: |
|  | Right <br> Regression used: |  |
|  | Location of split (a) |  |
| Find Y1(a) <br>  Y2(a) | $\lim _{x \rightarrow a^{-}} r(x)$ |  |
|  | $\lim _{x \rightarrow a^{+}} r(x)$ |  |

3. Roughly split the graph into two regions and perform different regressions on each side.

Plot data and regressions. Label Axis.

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| left regression split at a | Left |  |
| :--- | :--- | :--- |
|  | Y1 $=$ vars 5: $\gg 1: \operatorname{RegEq} /(\mathrm{x} \leq \mathrm{a})$ | Regression |
| used: |  |  |$\quad$.

4. For a continuous regression: Given $\varepsilon=$ small number Find $\delta>0$ that satisfies

Roughly adjust the regressions so the graph is continuous.
Plot data and graph the regressions. Label Axis.


| $\begin{aligned} & \text { Y1 (x)=regression (y2=split regression) } \\ & \text { Y3 }=\mathrm{L}-\varepsilon \\ & \text { Y4 }=\mathrm{L}+\varepsilon \\ & \text { Calc 5:intersect y1 and y3 }=\mathrm{x} 1 \\ & \text { Calc 5:intersect y1(2) and y4 }=x 2 \\ & \delta=\operatorname{maximum}(\|a-\mathrm{x} 1\|,\|\mathrm{a}-\mathrm{x} 2\|) \end{aligned}$ | $\lim _{x \rightarrow a} r(x)=\mathrm{L}$ |  |
| :---: | :---: | :---: |
|  | Given $\varepsilon=$ |  |
|  | Find $\delta=$ |  |

5. Roughly plot data and regression. Draw the secant and tangent lines at $x=$ a Label Axis.


## Pick x values in order

| $\mathrm{X} 1=$ |  |
| :---: | :--- |
| $\mathrm{X} 2=$ |  |
| $\mathrm{X} 3=$ |  |
| $\mathrm{a}=$ |  |
| $\mathrm{X} 4=$ |  |
| $\mathrm{X} 5=$ |  |
| $\mathrm{X} 6=$ |  |

Find the average rate of change between the exterior x -values around $\mathrm{x}=\mathrm{a}$ using regression

$$
\begin{array}{|l|l}
\hline\{\mathrm{Y} 1(\mathrm{x} 1)-\mathrm{Y} 1(\mathrm{x} 6)\} /\{\mathrm{x} 1-\mathrm{x} 6\}=\mathrm{m}_{\text {sec }} & \begin{array}{l}
\text { Average } \\
\text { Rate of } \\
\text { Change }
\end{array} \\
\hline
\end{array}
$$

Find the average rate of change between an interior x -values around $\mathrm{x}=$ a using regression

$$
\begin{array}{|l|l|}
\hline\{\mathrm{Y} 1(\mathrm{x} 2)-\mathrm{Y} 1(\mathrm{x} 5)\} /\{\mathrm{x} 2-\mathrm{x} 5\}=\mathrm{m}_{\text {sec }} & \begin{array}{l}
\text { Average } \\
\text { Rate of } \\
\text { Change }
\end{array} \\
\hline
\end{array}
$$

Find the average rate of change between the more interior x -values around $\mathrm{x}=\mathrm{a}$ using regression

$$
\begin{array}{|l|l|}
\hline\{\mathrm{Y} 1(\mathrm{x} 3)-\mathrm{Y} 1(\mathrm{x} 4)\} /\{\mathrm{x} 3-\mathrm{x} 4\}=\mathrm{m}_{\text {sec }} & \begin{array}{l}
\text { Average } \\
\text { Rate of } \\
\text { Change }
\end{array} \\
\hline
\end{array}
$$

Find the instnataneous rate of change at $\mathrm{x}=\mathrm{a}$

| nderiv $(y 1, x, a)$ <br> or calc 6:dydx and $x=a$ | Instant <br> Rate of <br> Change |
| :--- | :--- |

6. Find the derivatives of different regressions using rules at $\mathrm{x}=\mathrm{x} 1$

| Linear Regression $y 1=a x+b$ | $y^{\prime}=a$ | $y^{\prime}(x 1)=$ |
| :--- | :--- | :--- |
| Quadratic Regression $y 2=a x^{2}+b x+c$ | $y^{\prime}=2 a x+b$ | $y^{\prime}(x 1)=$ |
| Cubic Regression $y 3=\mathrm{ax}^{3}+\mathrm{bx}^{2}+c x+d$ | $y^{\prime}=3 a x^{2}+2 b x+c$ | $y^{\prime}(x 1)=$ |
| Quartic Regression $y 4=a x^{4}+\mathrm{bx}^{3}+\mathrm{cx}^{2}+d x+e$ | $y^{\prime}=4 a x^{3}+3 b x^{2}+2 c x+d$ | $y^{\prime}(x 1)=$ |

Compaire to $\mathrm{y} 5=\operatorname{nderv}(\mathrm{y} 4, \mathrm{x}, \mathrm{x})$ at $\mathrm{x}=\mathrm{x} 2, \mathrm{x} 3, \mathrm{x} 4$

| $\mathrm{X} 2=$ | $\mathrm{y} 4^{\prime}(\mathrm{x} 2)=$ |
| :--- | :--- |
| $\mathrm{X} 3=$ | $\mathrm{y} 4^{\prime}(\mathrm{x} 3)=$ |
| $\mathrm{X} 4=$ | $\mathrm{y} 4^{\prime}(\mathrm{x} 4)=$ |

7. Find the derivatives of different regressions using rules at $\mathrm{x}=\mathrm{x}$ 1

| Exponential $\mathrm{y} 6=\mathrm{a}^{*} \mathrm{~b}^{\wedge} \mathrm{x}$ | $\mathrm{y}^{\prime}=\mathrm{a}^{*} \mathrm{~b}^{\wedge} \mathrm{x}^{*} \ln (\mathrm{~b})$ | $\mathrm{y}^{\prime}(\mathrm{x} 1)=$ |
| :--- | :--- | :--- |
| Ln Regression $\mathrm{y} 7=\mathrm{aln} \mathrm{x}+\mathrm{b}$ | $\mathrm{y}^{\prime}=\mathrm{a} / \mathrm{x}$ | $\mathrm{y}^{\prime}(\mathrm{x} 1)=$ |

Compaire to $\mathrm{y} 8=\operatorname{nderv}(\mathrm{y} 6, \mathrm{x}, \mathrm{x})$ at $\mathrm{x}=\mathrm{x} 2, \mathrm{x} 3, \mathrm{x} 4$

| $\mathrm{X} 2=$ | $\mathrm{y} 8^{\prime}(\mathrm{x} 2)=$ |
| :--- | :--- |
| $\mathrm{X} 3=$ | $\mathrm{y} 8^{\prime}(\mathrm{x} 3)=$ |
| $\mathrm{X} 4=$ | $\mathrm{y} 8^{\prime}(\mathrm{x} 4)=$ |

8. .Find the second derivatives of different regressions using rules at $\mathrm{x}=\mathrm{x} 1$

| Linear Regression $\mathrm{y} 1=\mathrm{ax}+\mathrm{b}$ | $\mathrm{y}^{\prime \prime}=0$ | $\mathrm{y}^{\prime \prime}(\mathrm{x} 1)=$ |
| :--- | :--- | :--- |
| Quadratic Regression $\mathrm{y} 2=\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}$ | $\mathrm{y}^{\prime \prime}=2 \mathrm{a}$ | $\mathrm{y}^{\prime \prime}(\mathrm{x} 1)=$ |
| Cubic Regression $\mathrm{y} 3=\mathrm{ax}^{3}+\mathrm{bx}^{2}+\mathrm{cx}+\mathrm{d}$ | $\mathrm{y}^{\prime \prime}=6 \mathrm{ax}+2 \mathrm{~b}$ | $\mathrm{y}^{\prime \prime}(\mathrm{x} 1)=$ |
| Quartic Regression $\mathrm{y} 4=\mathrm{ax}^{4}+\mathrm{bx}^{3}+\mathrm{cx}^{2}+\mathrm{dx}+\mathrm{e}$ | $\mathrm{y}^{\prime \prime}=12 \mathrm{ax}^{2}+6 \mathrm{bx}+2 \mathrm{c}$ | $\mathrm{y}^{\prime \prime}(\mathrm{x} 1)=$ |

Compaire to $\mathrm{y} 5=\operatorname{nderv}(\operatorname{nderiv}(\mathrm{y} 4, \mathrm{x}, \mathrm{x}), \mathrm{x}, \mathrm{x})$ at $\mathrm{x}=\mathrm{x} 2, \mathrm{x} 3, \mathrm{x} 4$

| $\mathrm{X} 2=$ | $\mathrm{y} 4{ }^{\prime} \prime(\mathrm{x} 2)=$ |
| :--- | :--- |
| $\mathrm{X} 3=$ | $\mathrm{y} 4{ }^{\prime \prime}(\mathrm{x} 3)=$ |
| $\mathrm{X} 4=$ | $\mathrm{y} 44^{\prime}(\mathrm{x} 4)=$ |

## 9. Make a transformation of your $x$-values and your $y$-values


10. Find the derivatives of sine regression using rules at $\mathrm{x}=\mathrm{x} 1$

| Sine Regression $y 2=a \sin (b x+c)+d$ | $y^{\prime}=a \cos (b x+c) * b$ | $y^{\prime}(x 1)=$ |
| :--- | :--- | :--- |

Find the second derivatives of sine regression using rules at $\mathrm{x}=\mathrm{x} 1$

| Sine Regression $\mathrm{y} 2=\mathrm{asin}(\mathrm{bx}+\mathrm{c})+\mathrm{d}$ | $\mathrm{y}^{\prime \prime}=-\mathrm{asin}(\mathrm{bx}+\mathrm{c})^{*} \mathrm{~b}^{\wedge} 2$ | $\mathrm{y}^{\prime}(\mathrm{x} 1)=$ |
| :--- | :--- | :--- |

## OPTIONAL

11. Find the derivatives of the invers sine regression usio evils at end

12. Use the mean value theorem on the two end points OF a regression and identify a point on the graph with a similar slope?
Y1=regEq
Y2=nderiv ( $\mathrm{y} 1, \mathrm{x}, \mathrm{x}$ )
Y3="average rate of change"
Calc 5:intersect

| Regression <br> used: |  |
| :--- | :--- |
| Ave Rate of <br> change: |  |
| Point(s) of <br> intersection: |  |

13. Was the zero found by using Newton's Method for by using $\mathrm{x}=0$ or $\mathrm{x}=1$ as an initial guess?

Y1=cubicregression
0 sto x
x-yl/nderv(y1,x,x)stox
iteration $\qquad$
iteration $\qquad$
iteration $\qquad$
zero:
14. Related rates

## OMIT THIS QUESTION

15. Graph the cubic or quartic regression, identify all critical points, concavity, and inflection points.

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| X: |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y' |  |  |  |  |  |  |  |  |  |
| Increasing <br> or <br> Deceasing |  |  |  |  |  |  |  |  |  |
| Y" |  |  |  |  |  |  |  |  |  |
| Concavity? <br> Up or <br> Down |  |  |  |  |  |  |  |  |  |

16. 

Find $y^{\prime}=0$ to identify critical values a1,a2

Critical
Points

Find y'(a1) and y"(a2) to determine max/min

| Y'' at |  |
| :---: | :--- |
| critical |  |
| Points |  |
| Max or |  |
| Min |  |

17. Find $y$ ' $=0$ to identify inflection points Did the student take the second derivative and identify concavity for the zero of the cubic regression? $Y^{\prime \prime}=0$ at $-b /(6 a)$ : $\qquad$

| Inflection |
| :---: |
| Points |

