Agenda

Review: Quizzes and Homeworks

Review: The Chain Rule

Project: Changing Units

Review of Quizzes

Quiz 4 Q3

Compute the derivative of $f(x) = 6\cos x^3$.

$-18\sin x * \cos^2 x$ ✗

Compute the derivative of $f(x) = 6\cos x^3$.

$-18\sin x * \cos^2 x$ ✗

$y = 6(\cos x)^3$

$= 18(\cos x)^2 \cdot (\sin x)$

$y = 6\cos(x^3)$

$y' = -6\sin(x^3) \cdot 3x^2$

$= -18x^2 \sin(x^3)$

Use the position function $s(t) = \dfrac{74t}{\sqrt{t^2 + 6}}$ to find the velocity at time $t = 1$.

(Assume units of meters and seconds.)

$v(1) =$ ☐ m/s

Use the position function $s(t) = \dfrac{74t}{\sqrt{t^2+6}}$ to find the velocity at time $t = 1$.

(Assume units of meters and seconds.)

$v(1) =$ ☐ m/s

$$s' = \frac{gf' - fg'}{g^2}$$

$$s' = \frac{\sqrt{t^2+6}\,(74) - 74t\,\frac{d}{dt}(t^2+6)^{1/2}}{t^2+6}$$

$$s' = \frac{74\sqrt{t^2+6} - 74t\,\frac{1}{2}(t^2+6)^{-1/2}\cdot(2t)}{t^2+6}$$

$$s'(1) = \frac{74\sqrt{7} - \frac{74}{\sqrt{7}}}{7}\left(\frac{\sqrt{7}}{\sqrt{7}}\right) = 23.97$$

$$\frac{74\cdot 7 - 74}{7\sqrt{7}} = \frac{444}{7\sqrt{7}} = \frac{444\sqrt{7}}{49}$$

Differentiate $y = (x^7 + x - 3)^5$.

$$\frac{dy}{dx} = \square\,(\square)^{\square}\,(\square)$$

Differentiate $y = (x^7 + x - 3)^5$.

$$\frac{dy}{dx} = \boxed{5}\left(\boxed{x^7+x-3}\right)^{\boxed{4}}\left(\boxed{7x^6+1}\right)$$

$\frac{d}{dx} x^1$

$1 x^0$

1

$5(x^7+x-3)^4 \cdot \frac{d}{dx}(x^7+x^1-3)$

$7x^6+1$

Compute the derivative of $f(x) = x\sqrt{x} + \frac{7}{x^3}$.

- A. $f'(x) = \frac{2}{3}x^{1/2} - 21x^{-4}$
- B. $f'(x) = \frac{3}{2}x^{1/2} - 21x^{-3}$
- C. $f'(x) = \frac{3}{2}x^{1/2} - 21x^{-4}$
- D. $f'(x) = \frac{3}{2}x^{1/2} + 21x^{-4}$

Compute the derivative of $f(x) = x\sqrt{x} + \frac{7}{x^3}$.

$x^1 \cdot x^{1/2} + 7x^{-3}$

- A. $f'(x) = \frac{2}{3}x^{1/2} - 21x^{-4}$
- B. $f'(x) = \frac{3}{2}x^{1/2} - 21x^{-3}$
- C. $f'(x) = \frac{3}{2}x^{1/2} - 21x^{-4}$
- D. $f'(x) = \frac{3}{2}x^{1/2} + 21x^{-4}$

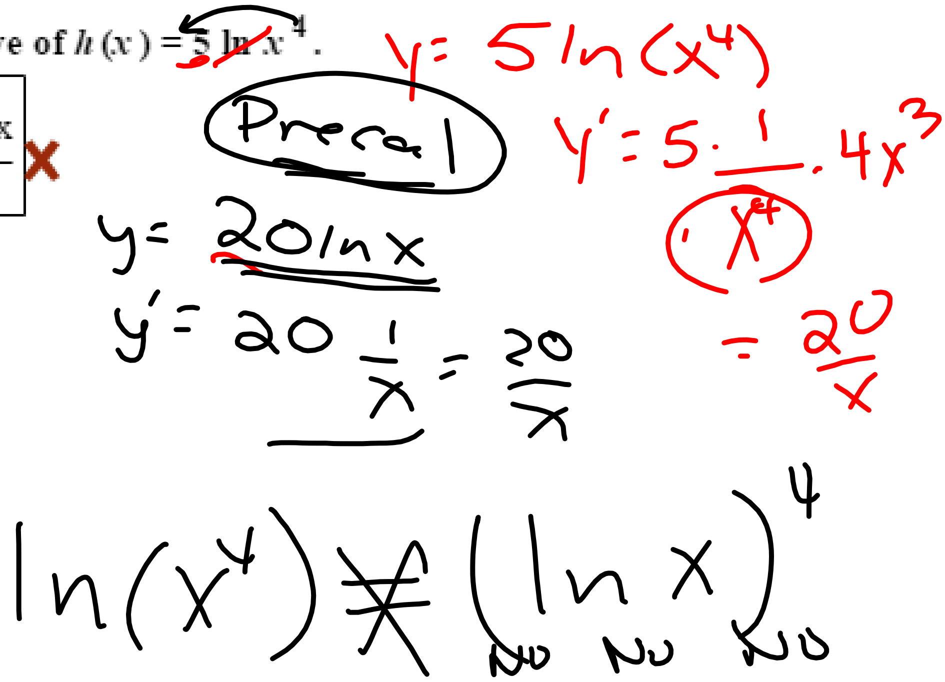
$y = x^{3/2} + 7x^{-3}$

$y = \frac{3}{2}x^{1/2} - 21x^{-4}$

Find the derivative of $h(x) = 5 \ln x^4$.

Find the derivative of $h(x) = 5 \ln x^4$.

$h'(x) = \dfrac{20\ln^3 x}{x}$ ✘



Your response: ✖

Find the slope of the tangent line at the point $(1, 3)$ **for the ellipse** $6x^2 + 7y^2 = 69$.

$y'(1) =$ 12 ✗

Your response:

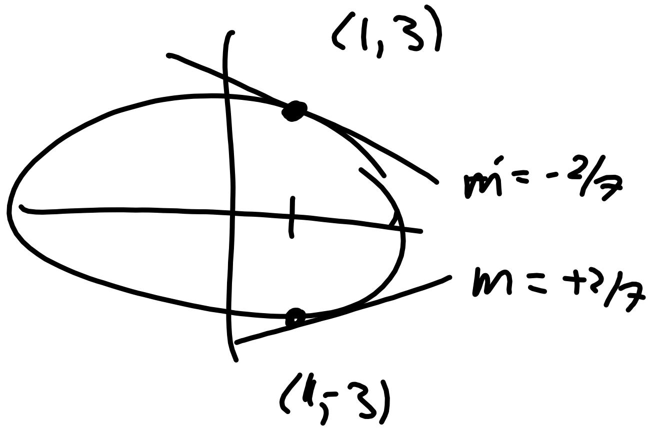
Find the slope of the tangent line at the point (1, 3) for the ellipse $6x^2 + 7y^2 = 69$.

$y'(1) = 12$ ✗

$$12x + 14y\frac{dy}{dx} = 0$$

$$\frac{14y\,\frac{dy}{dx}}{14y} = \frac{0 - 12x}{14y}$$

$$\frac{dy}{dx} = \frac{-12x}{14y}$$



$$-\frac{2x}{14y}$$

Your response:

Find the slope of the tangent line at the point (1,3) for the ellipse $6x^2 + 7y^2 = 69$.

$y'(1) = 12$ ✗

$x = 1$

$6 + 7y^2 = 69$

$7y^2 = 63$

$\frac{7y^2}{7} = \frac{63}{7}$

$y^2 = 9$

$y = \pm 3$

$$\frac{d}{dx}(6x^2 + 7y^2) = \frac{d}{dx}(69)$$

$$12x + 14y \cdot \frac{dy}{dx} = 0$$

(1,3)

$$12(1) + 14(3)\frac{dy}{dx} = 0$$

$12 + 42y' = 0$

$12 + 42y' = 0$

$42y' = -12$

$y' = -12/42$

$$\frac{dy}{dx} = -\frac{12}{42} = -\frac{2}{7}$$

Find the derivative $y\,'(x)$ implicitly.

$\sqrt{xy} - 8y^2 = 22$

Find the derivative $y\,'(x)$ implicitly.

$\sqrt{xy} - 8y^2 = 22$

$$\frac{d}{dx}(xy)^{1/2} + \frac{d}{dx}(-8y^2) = \frac{d}{dx}22$$

$$\frac{1}{2}(xy)^{-1/2} \cdot \frac{d}{dx}(x \cdot y) + -16y\left(\frac{dy}{dx}\right) = 0$$

$$-\frac{1}{2}\frac{1}{\sqrt{xy}} \cdot \left[x\frac{dy}{dx} + y\right] - 16y\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + y + 16y\sqrt{xy}\,\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(x + 16y\sqrt{xy}\right) = -y$$

$$\frac{dy}{dx} = \frac{-y}{x + 16y\sqrt{xy}}$$

Find the derivative $y\,'(x)$ implicitly.

$$\frac{d}{dx}\quad \sqrt{xy} - 8y^2 = 22$$

$$\frac{d}{dx}\sqrt{xy} - \frac{d}{dx}8y^2 = \frac{d}{dx}22$$

$$\frac{d}{dx}(xy)^{1/2} - 16y^1 \cdot \frac{d}{dx}y = 0$$

$$\frac{1}{2}(xy)^{-1/2}\cdot\frac{d}{dx}(xy) - 16y\frac{dy}{dx} = 0$$

Product

$$\frac{d}{dx}(x\cdot y)$$

$$x\frac{d}{dx}y + y\frac{d}{dx}x$$

$$x\frac{dy}{dx} + y\,1$$

$$\frac{1}{2\sqrt{xy}}\left(x\frac{dy}{dx} + y\right) - 16y\frac{dy}{dx} = 0$$

$$2\sqrt{xy}$$

$$\left(x\frac{dy}{dx} + y\right) = 16y\frac{dy}{dx}2\sqrt{xy}$$

$$x\frac{dy}{dx} + y = 32\sqrt{xy}\frac{dy}{dx}$$

$$y = 2\sqrt{xy}\frac{dy}{dx} - x\frac{dy}{dx}$$

$$y = \frac{dy}{dx}\left(2\sqrt{xy} - x\right)$$

$$\frac{y}{2\sqrt{xy} - x} = \frac{dy}{dx}$$

remember chain rule...

$$\frac{d}{dx} x^4 = 4x^3$$

$$\frac{d}{dx} y^4 = 4y^3 \cdot \frac{dy}{dx}$$

$$\frac{d}{dt} t^7 = 7t^6$$

$$\frac{d}{dt} x^4 = 4x^3 \cdot \frac{dx}{dt}$$

$$(x^4)' = 4x^3$$

Sec. Ex. 9 - 2.8 Section Exercise 9

Your response: ⊗

Find the derivative $y\,'(x)$ implicitly for the equation

$$\frac{2x+1}{y} = 6x + 5y^2.$$

Sec. Ex. 9 - 2.8 Section Exercise 9

Your response: ✖

Find the derivative $y\,'(x)$ implicitly for the equation

$$y \cdot \frac{2x+1}{y} = 6x + 5y^2. \quad \cdot y$$

$$\frac{d}{dx}(2x+1) = \frac{d}{dx}(6x \cdot y + 5y^3)$$

$$2 = 6x \cdot \frac{dy}{dx} + 6y + 15y^2 \frac{dy}{dx}$$

$$2 - 6y = \frac{dy}{dx}(6x + 15y^2)$$

$$\frac{2-6y}{6x+15y^2} = \frac{dy}{dx}$$

Sec. Ex. 9 - 2.8 Section Exercise 9

Your response:

Find the derivative $y'(x)$ implicitly for the equation

$$\frac{2x+1}{y} = 6x + 5y^2.$$

Multiply by y

$$2x+1 = 6xy + 5y^3$$

$$\frac{d}{dx}(2x+1) = \frac{d}{dx}(6xy + 5y^3)$$

$$\frac{d}{dx}6xy + \frac{d}{dx}5y^3$$

$$2 = 6x\frac{d}{dx}y + y(6) + 15y^2$$

$$2 = 6x\frac{dy}{dx} + 6y + 15y^2\frac{dy}{dx}$$

$$\frac{2-6y}{6x+15y^2} = \frac{(6x+15y^2)\frac{dy}{dx}}{6x+15y^2}$$

Algebra

$f(x) = x^3 + 4x - 1$ has an inverse $g(x)$.

Use $g'(x) = \dfrac{1}{f'(g(x))}$ to find $g'(-1)$.

$g'(-1) =$ [ ]

$f(x) = x^3 + 4x - 1$ has an inverse $g(x)$. → ???

Use $g'(x) = \frac{1}{f'(g(x))}$ to find $g'(-1)$.

$g'(-1) = $ ☐

$f'(x) = 3x^2 + 4$

$f'(0) = 4$

$= \frac{1}{4}$

$g(-1) = ?$

x	$f(x)$
0	-1

$f(0) = -1$

$g^{-1}(-1) = 0$

$f^{-1}(x) = g(x)$

x	$f(x)$
a	b
a	c

$y = x^3 + 4x - 1$ solve for x.

$y + 1 = x^3 + 4x$

$f' = 3x^2+4$ $\qquad f^{-1}(x)$

$f(x) = x^3 + 4x - 1$ has an inverse $g(x)$.

$f(0) = -1$

Use $g'(-1) = \dfrac{1}{f'(g(-1))}$ to find $g'(-1)$.

x	$f(x)$
0	-1

$f^{-1}(-1) = 0$

$g^{-1}(-1)$

$g'(-1) = \dfrac{1}{f'(g(-1))} = \dfrac{1}{f'(0)} = \dfrac{1}{4}$

$f(0) = -1$

$g^{-1}(-1) = 0$

$g'(-1) =$ 4

$f'(0) =$

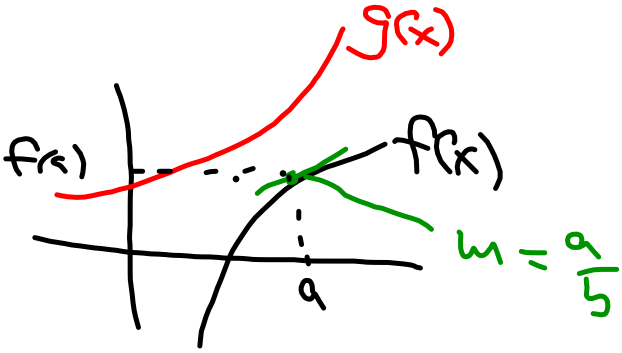
x	$f^{-1}(x)$ $g(x)$
-1	0

$g(-1) = \quad a = 0 \quad f(a) = -1$

$f'(x) = 3x^2+4$ $\qquad f'(0) = 4$

$g'(-1) = \dfrac{1}{f'(g(-1))}$

$= \dfrac{1}{f'(0)} = \dfrac{1}{4}$



$f(0) = -1 \qquad f'(g(x))$

Question

Use the position function $s(t) = 2t^2 - 8\sin 3t$ to find the velocity at time $t = 0$. Assume units of feet and seconds.

$v(0) =$ ________ feet/second

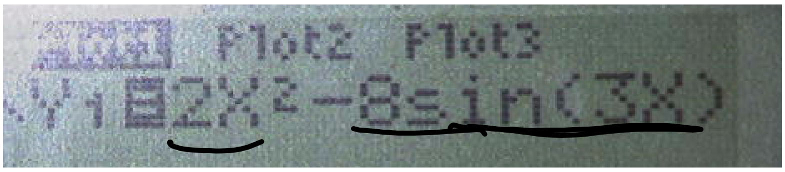
Use the position function $s(t) = 2t^2 - 8\sin(3t)$ to find the velocity at time $t = 0$. Assume units of feet and seconds.

$v(0) =$ ______ feet/second

Chain Rule.

$V(t) = S'(t) = 4t - 8\cos(3t)\cdot\frac{d}{dt}(3t)$

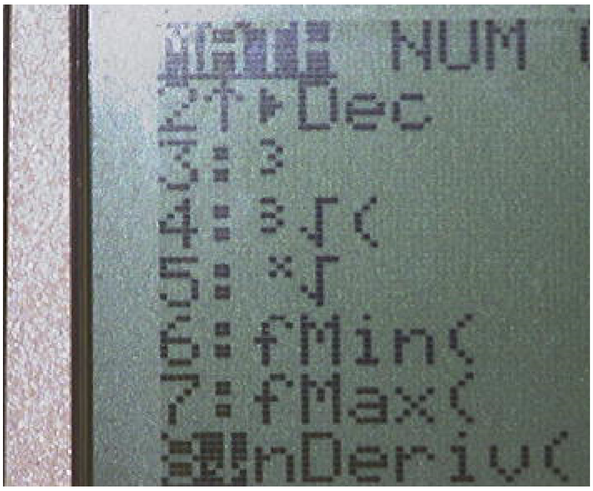
$v(0) = 4(0) - 8\cos(0)3 = -24$

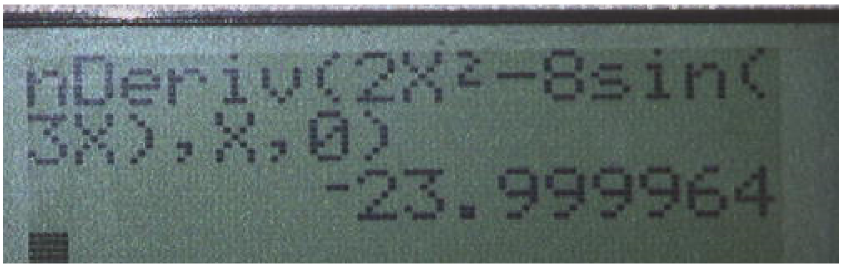


$S(t) =$ Position

$V(t) = S'(t) =$ Velocity

$a(t) = V'(t) = S''(t)$: acceleration





$$\frac{d}{dX}\left(4X^2-4\sin(8X)\right)\Big|_{X=0}$$

Compute the derivative of $f(x) = \dfrac{8}{(5x^3 + 3)^2}$.

Compute the derivative of $f(x) = \frac{8}{(5x^3+3)^2}$.

$8(5x^3+3)^{-2}$

Chain Rule

$y = 8(5x^3+3)^{-2}$

$y' = 8\cdot(-2)(5x^3+3)^{-3}\cdot\frac{d}{dx}(5x^3+3)$

$-16(5x^3+3)^{-3}(15x^2)$

OR $\frac{-240x^2}{(5x^3+3)^3}$

Compute the derivative of $f(x) = \dfrac{10}{\sqrt{4x^3 + 7}}$.

Compute the derivative of $f(x) = \dfrac{10}{\sqrt{4x^3 + 7}}$.

$f(x) = 10(4x^3+7)^{-1/2}$

$f'(x) = -5(4x^3+7)^{-3/2}(12x^2)$

Find the derivative of the function $f(x) = 2x + 7^x$.

Find the derivative of the function $f(x) = 2x + 7^x$.

$$\frac{d}{dx} a^x = a^x \ln a$$

$$f' = 2 + 7^x \cdot \ln 7$$

$y = 7^x$

Find y'

$\ln y = \ln 7^x$

$\ln y = x \ln 7$

$x = \ln y \cdot \frac{1}{\ln 7}$

$\frac{d}{dx} x = \frac{1}{\ln 7} \cdot \frac{d}{dx} \ln y$

$1 = \frac{1}{\ln 7} \frac{1}{y} \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = y \ln 7$

$= 7^x \ln 7$

$$y = 10^x$$

$$\ln y = \ln 10^x$$

$$\frac{d}{dx}\left(\ln y = x \ln 10 \right)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \ln 10$$

$$\frac{dy}{dx} = y \ln 10 = 10^x \ln 10$$

A baseball with mass 0.18 kg and speed 41 m/s is struck by a baseball bat of mass m and speed 37 m/s (in the opposite direction of the ball's motion). After the collision, the ball has initial speed $u(m) = \dfrac{76.4m - 6.63}{m + 0.18}$ m/s. Show that $u'(m) > 0$ and interpret this in baseball terms. Compare $u'(1)$ and $u'(1.4)$. Round your final answer to two decimal places.

$u'(1) \approx$ [ ] and $u'(1.4) \approx$ [ ]. The rate at which this speed is increasing is (select).

A baseball with mass 0.18 kg and speed 41 m/s is struck by a baseball bat of mass m and speed 37 m/s (in the opposite direction of the ball's motion). After the collision, the ball has initial speed $u(m) = \dfrac{76.4m - 6.63}{m + 0.18}$ m/s. Show that $u'(m) > 0$ and interpret this in baseball terms. Compare $u'(1)$ and $u'(1.4)$. Round your final answer to two decimal places.

Smaller?

$u'(1) \approx$ [$7/1.18^2$] and $u'(1.4) \approx$ [$7/1.58^2$]. The rate at which this speed is increasing is (select) .

decreasing → smaller

$$u' = \frac{(m+.18)(76.4) - (76.4m - 6.63)(1)}{(m+.18)^2}$$

$$= \frac{.18 \cdot 76.4 - 6.63}{(m+.18)^2} = \frac{7}{(m+.18)^2}$$

1 out o

Use the position function $s(t) = \dfrac{74t}{\sqrt{t^2 + 6}}$ to find the velocity at time $t = 1$.

(Assume units of meters and seconds.)

$v(1) =$ ____ m/s

1 out o

Use the position function $s(t) = \dfrac{74t}{\sqrt{t^2+6}}$ to find the velocity at time $t = 1$.

(Assume units of meters and seconds.)

$v(1) =$ ☐ m/s

$\frac{d}{dx}\, 74t \cdot (t^2+6)^{-1/2}$

$V(t) = 74t \cdot (-1/2)(t^2+6)^{-3/2}(2t) + (t^2+6)^{-1/2}(74)$

$74(-1/2)7^{-3/2} \cdot 2 + 7^{-1/2} \cdot 74$

Use the relevant information to compute the derivative of $h(x) = f(g(x))$ at $x = 1$, where $f(1) = -3$, $g(1) = 2$, $f'(2) = -1$, $g'(1) = -3$, and $g'(3) = 5$.

$h'(1) =$ ______

Use the relevant information to compute the derivative of $h(x) = f(g(x))$ at $x = 1$, where $f(1) = -3$, $g(1) = 2$, $f'(2) = -1$, $g'(1) = -3$, and $g'(3) = 5$.

$h'(1) =$ ______

$$h(x) = f(g(x))$$

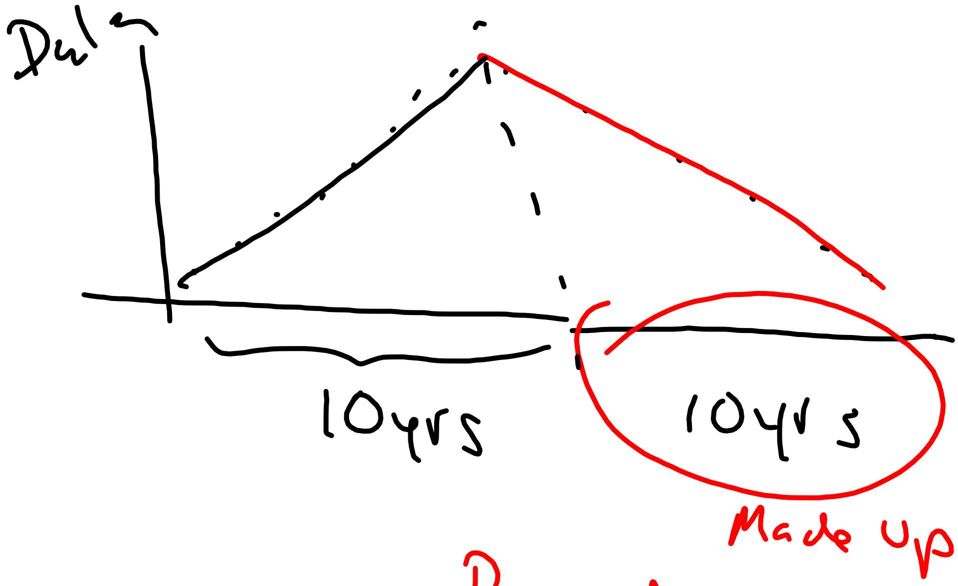
$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$h'(1) = f'(g(1)) \cdot g'(1)$$

$$= f'(2) \cdot (-3)$$

$$= -1 \cdot -3 = 3$$

Group Work



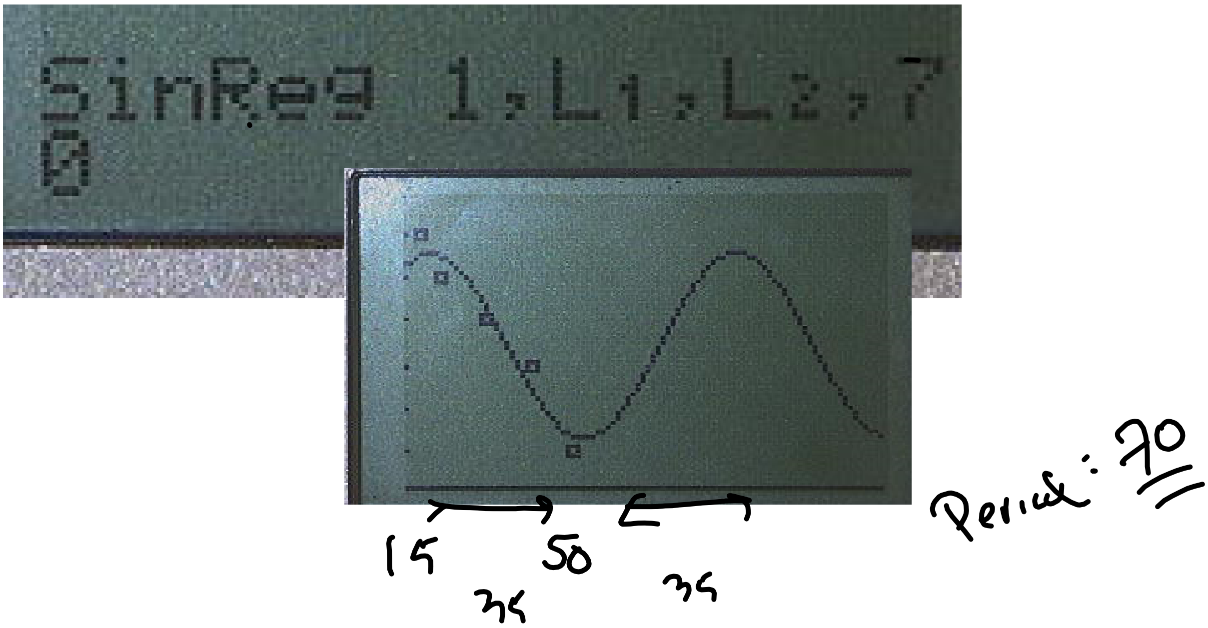
Period = 20

STAT

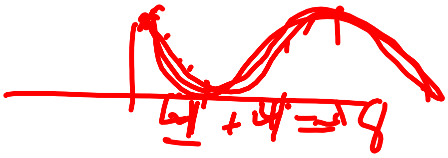
Calc: C SinReg

SinReg 1, L1, L2, 70

Period



Sinreg



$$\frac{d}{dx} \cdot \left(A\sin(Bx+c)+1\right)$$

$$y_1 = A\cos(Bx+c)B$$

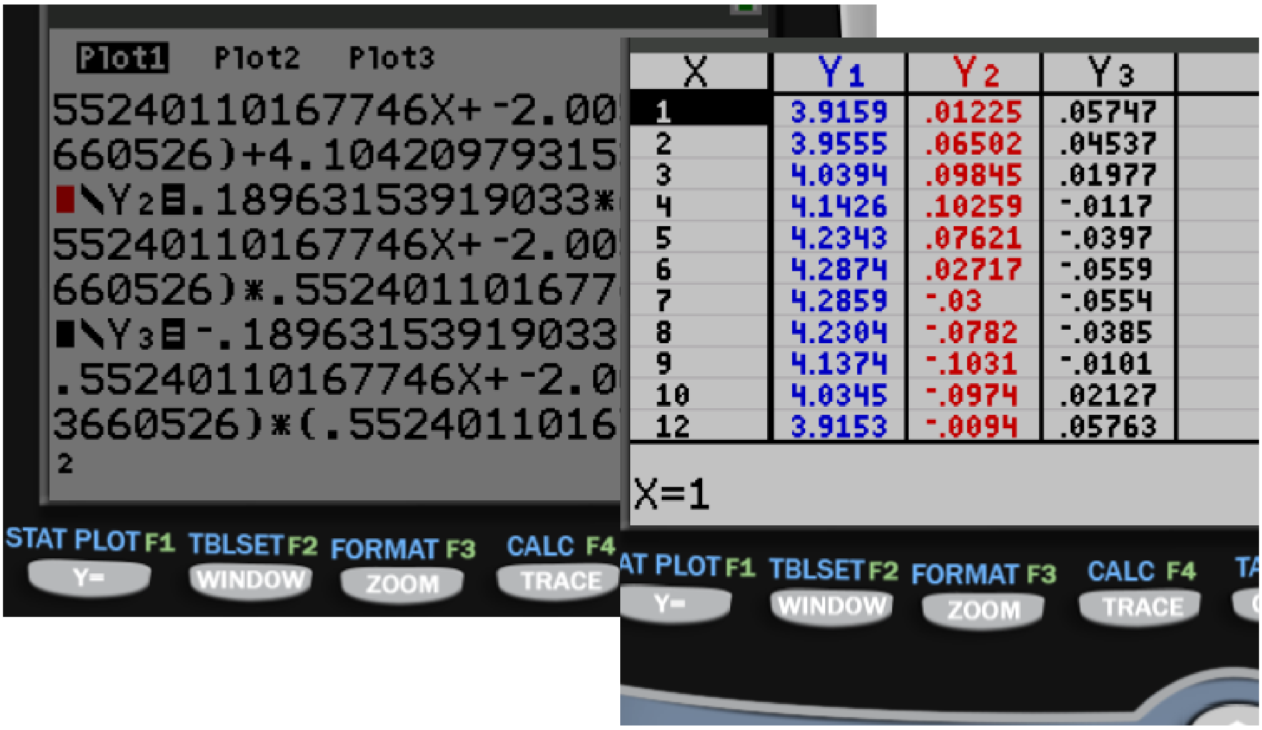
Chain Rule $\frac{d}{dx}(Bx+c)$

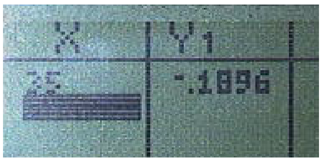
Second derivative

$$\frac{d}{dx}\, A\cos(Bx+c)\cdot B$$

$$-A\sin(Bx+c)B\cdot B$$

$$y_2 = -A\sin(Bx+c)B^2$$


Plot1 Plot2 Plot3
5524011016 7746X+⁻2.00
660526)+4.10420979315
Y2=.18963153919033*
5524011016 7746X+⁻2.00
660526)*.552401101677
Y3=⁻.18963153919033
.5524011016 7746X+⁻2.0
3660526)*(.5524011016
2
STAT PLOT F1 TBLSET F2 FORMAT F3 CALC F4
Y= WINDOW ZOOM TRACE
X Y1 Y2 Y3
1 3.9159 .01225 .05747
2 3.9555 .06502 .04537
3 4.0394 .09845 .01977
4 4.1426 .10259 ⁻.0117
5 4.2343 .07621 ⁻.0397
6 4.2874 .02717 ⁻.0559
7 4.2859 ⁻.03 ⁻.0554
8 4.2304 ⁻.0782 ⁻.0385
9 4.1374 ⁻.1031 ⁻.0101
10 4.0345 ⁻.0974 .02127
12 3.9153 ⁻.0094 .05763
X=1
STAT PLOT F1 TBLSET F2 FORMAT F3 CALC F4
Y= WINDOW ZOOM TRACE

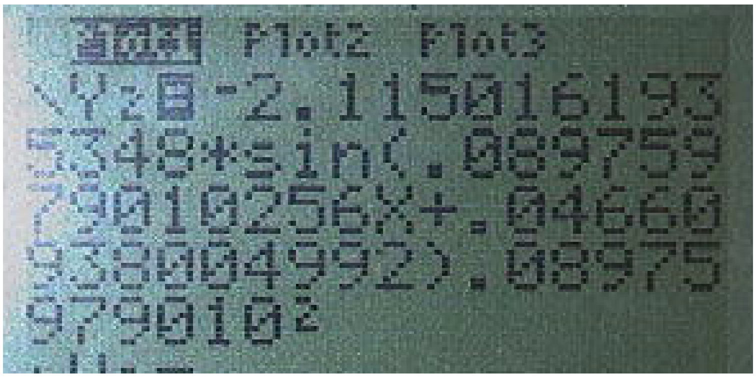


at $35 my sales are dropping by a rate of

0.18 people per dollar

according to the sine regression

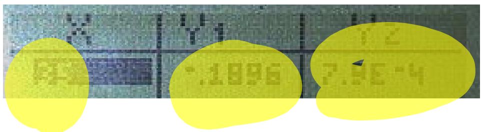
insert (-)



$y = A\sin(Bx+c)$

$y' = A\cos(Bx+c)\cdot B$

$y'' = -A\sin(Bx+c)\cdot B^2$



squared

according to the sine regression with a period of 70...

At $35 my sales are decreasing but accelerating (because y'' is >0)

if y'' is negative the function is decelerating

Acceleration

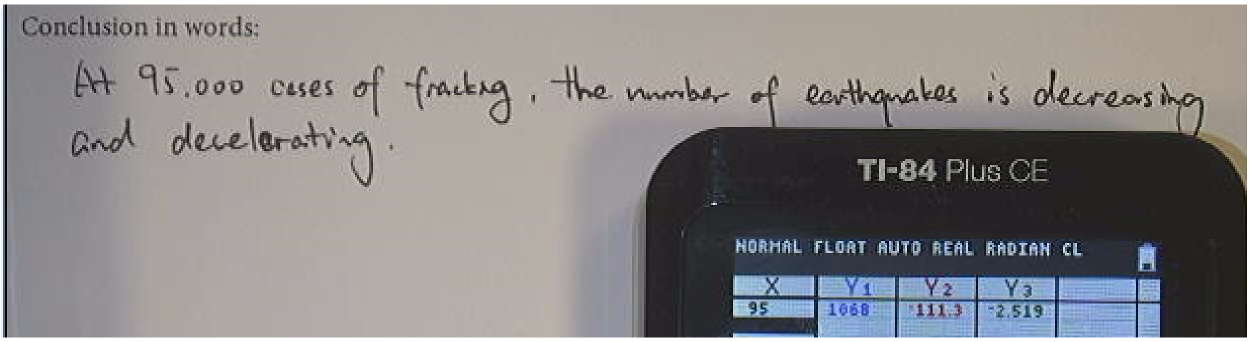
$$s'' \approx \frac{\Delta v}{\Delta t} \qquad \frac{mi/hr}{hr}$$

$$\frac{0mph \quad 60mph}{1/12\ hr}$$

mi/hr^2

720 mphph

$mi/hr/h$

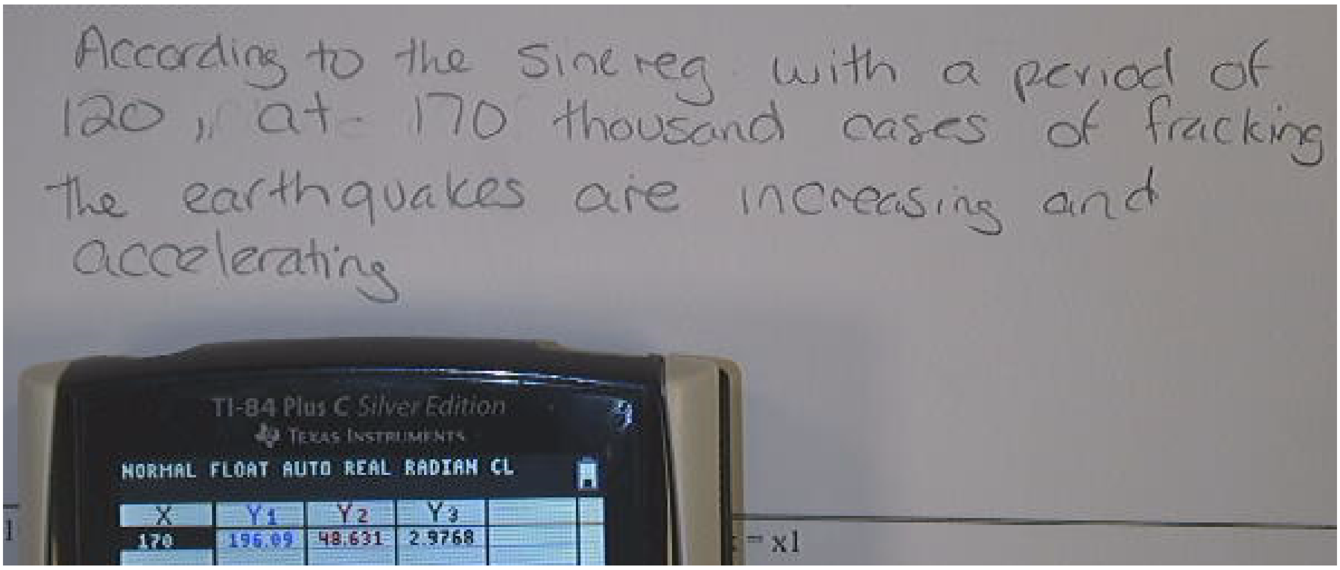


According to the sine regression with a period of 80.

Decreasing at a rate of 111.3 earthquakes per 1000 fracking cases.

Decellerating by 2.5 fracking ~~per cases per case.~~ 1000s

At 170



increasing by 50 earthquakes per 1000 cases of fracking

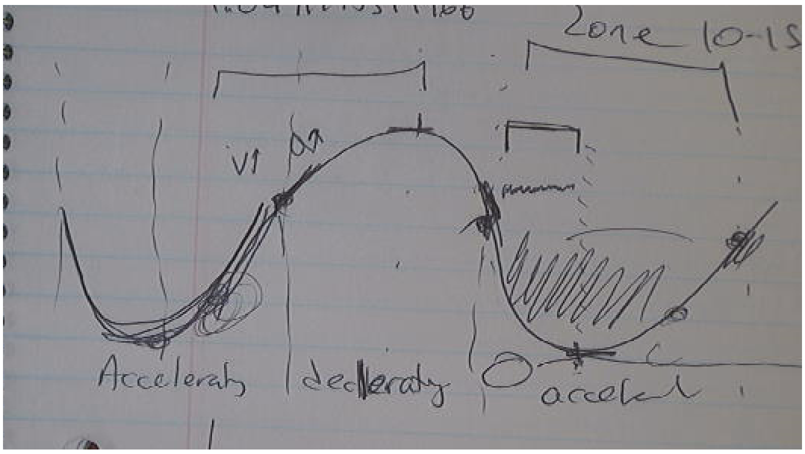
accelerating by 3 earthquakes per 1000 cases per 1000 cases

Conclusion in words:

In the year 2015, according to the sine regression the radio is decreasing at the rate of 0.8 billion dollars per year.

X	Y2	Y3
15	-.7759	.97509

accelerating



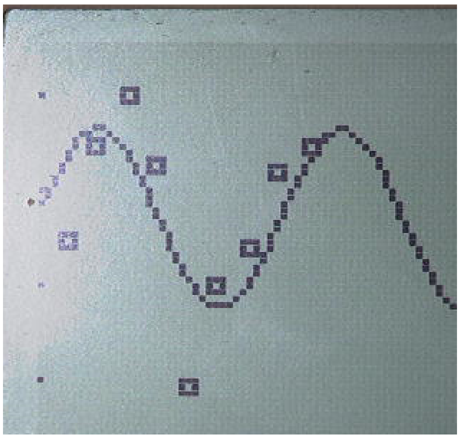
Conclusion in words:

In 2019, the revenue of radio will increase at a rate of 502 million dollars per year, and that rate will increase at a rate of 411 million dollars per $year^2$. This gives assurity, as the rate itself continues to increase.

* According to sine regression.

X	Y2	Y3
19	.50239	[illegible]

opportunity with less competition



Do sin regression (Hard)

Manually find y' y''

$y_1 = A\sin(Bx+c)+D$

$y_2 = A\cos(Bx+c)\cdot B$

$y_3 = -A\sin(Bx+c)\cdot B^2$