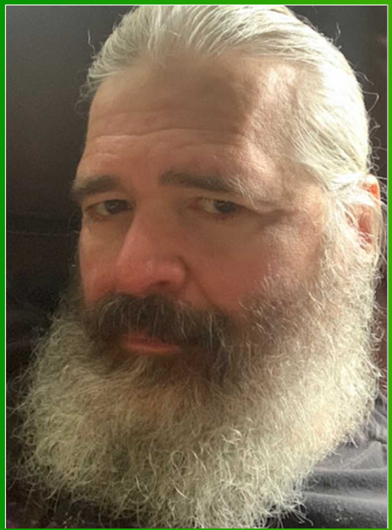
MAT 151 Calculus 1

Prof. Porter



Agenda:

Review of Derivative Rules

Cumulative Review

Lecture: Postion Velocity Acceleration

Group Work

151d8

Review of Differentiation

What is Math?

What is Calculus?

What are the two rates of change?

What is the meaning of velocity?

Velocity is

the Derivative is

the instantaneous rate of change is

the slope of the tangent line

What are the derivatives

250π

$7x$

$7x + 250$

x^{100}

$(2x+1)x^{300}$

$$\frac{(2x+1)}{x^{200}}$$

The "Quick and Dirty" Rules

1. (constant)' = 0   derivative of a constant is zero

2. $(af)' = af'$ constants come out of derivative operations

3. $(f + g)' = f' + g'$ derivative of sum is sum of derivatives

4. $(x^n)' = nx^{n-1}$ <u>Power Rule</u>

5. $(fg) = fg' + gf'$ <u>Product Rule</u>

6. $(f/g) = (gf' - fg')/g^2$ <u>Quotient Rule</u>

What are the derivatives of:

e^x

$\ln(x)$

$\sin(x)$

$\cos(x)$

$\tan(x)$

$\sinh(x)$

$\cosh(x)$

Here's a bunch!

Star ★ means

you must know them

$\left(\frac{1}{x^n}\right)' = -\frac{n}{x^{n+1}}$	$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$
$(\sqrt[m]{x})' = \frac{1}{m\sqrt[m]{x^{m-1}}}$	$(a^x)' = a^x \ln a$
★ $(e^x)' = e^x$	$(\log_a x)' = \frac{1}{x \ln a}$
★ $(\ln x)' = \frac{1}{x}$	$(\sin x)' = \cos x$ ★
★ $(\cos x)' = -\sin x$	$(\tan x)' = \frac{1}{\cos^2 x} = \sec^2 x$ ★
$(\cot x)' = -\frac{1}{\sin^2 x} = -\csc^2 x$	$(\sec x)' = \tan x \sec x$
$(\csc x)' = -\cot x \csc x$	$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$	$(\arctan x)' = \frac{1}{1+x^2}$
$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$	$(\operatorname{arcsec} x)' = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$(\operatorname{arccsc} x)' = -\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$(\sinh x)' = \cosh x$ ★
★ $(\cosh x)' = \sinh x$	$(\tanh x)' = \operatorname{sech}^2 x$
$(\coth x)' = -\operatorname{csch}^2 x$	$(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$
$(\operatorname{csch} x)' = -\operatorname{csch} x \coth x$	$(\operatorname{arcsinh} x)' = \frac{1}{\sqrt{x^2+1}}$
$(\operatorname{arccosh} x)' = \frac{1}{\sqrt{x^2-1}}$	$(\operatorname{arctanh} x)' = \frac{1}{1-x^2}$

What are the derivatives

$(x^2+7x+3)^{25}$

e^{2x+1}

$\ln(2x+5)$

$\sin(x^2)$

$\cos^2(x)$

$\sqrt{(\cosh(x))}$

The derivatives are:

$(x^2+7x+3)^{25} = 25\ (x^2 + 7x + 3)^{24}\ (2x + 7)$

$e^{2x+1} \quad = \quad e^{2x+1}\ (2)$

$\ln(2x+5) = 1/(2x+5) * 2$

$\sin(x^2) = \cos(x^2)\ (2x)$

$\cos^2(x) = 2\cos(x)(-\sin(x))$

$\surd(\cosh(x)) = (1/2)\ (\cosh(x))^{-1/2}\ (\sinh(x))$

Find an equation of the tangent line to $y = x^2 - 9x$ at $x = -8$.

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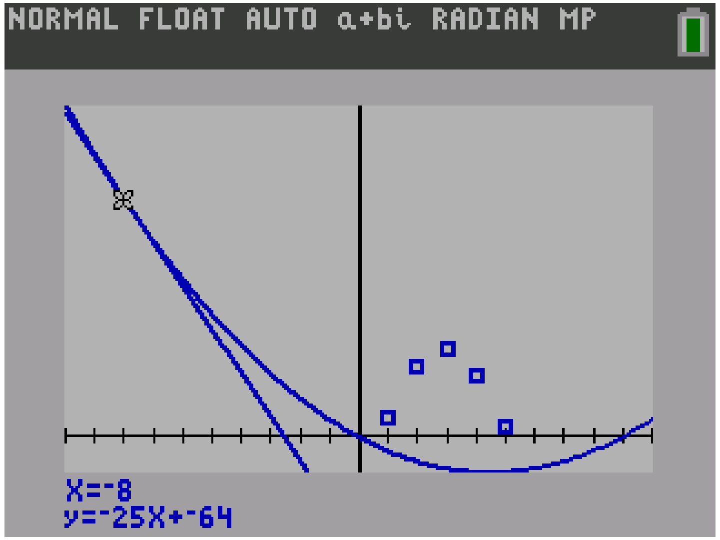

X	Y1	Y2
⁻8	136	⁻25

Point: (-8, 136) f(-8)=136

slope: m = f'(-8) = 2(-8)-9 = -25

line: y - 136 = -25(x - -8)

Determine values of a and b that make the given function continuous.

$$f(x) = \begin{cases} \dfrac{33\sin x}{x} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ b\cos x & \text{if } x > 0 \end{cases}$$

a = ☐ and b = ☐

Determine values of a and b that make the given function continuous.

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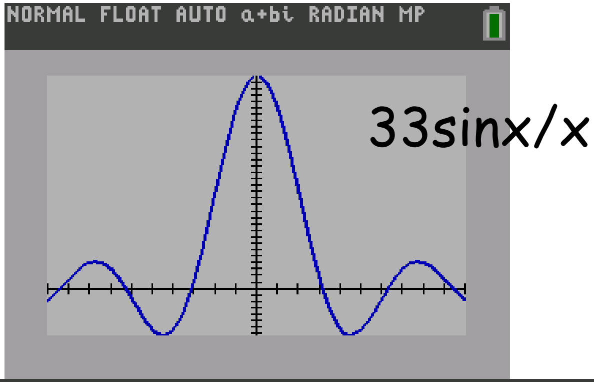
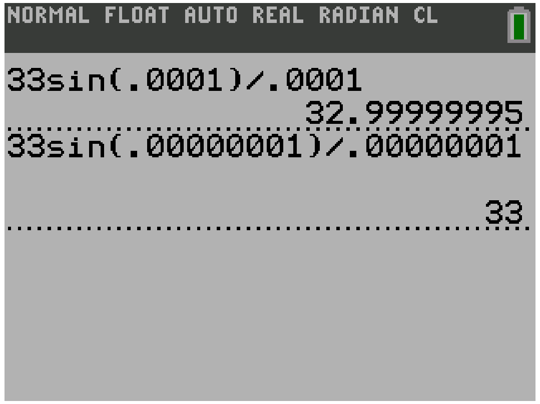


$a =$ ☐ and $b =$ ☐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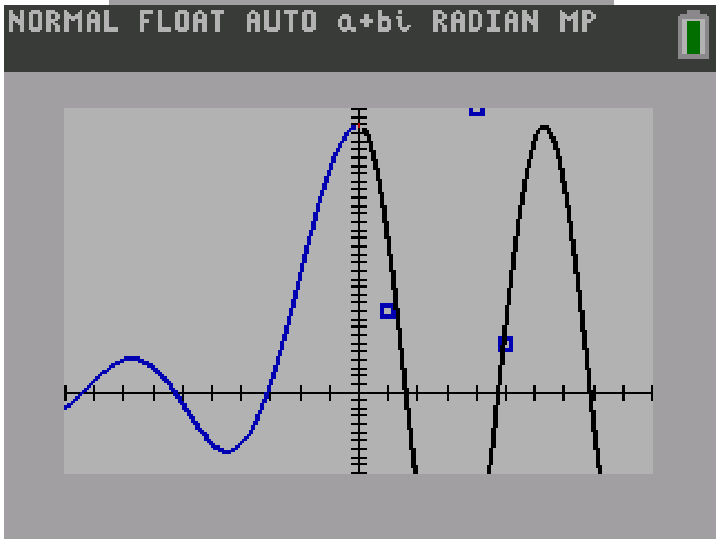
33sin(.0001)/.0001

32.99999995

33sin(.00000001)/.00000001

33



Question

Suppose that the height of a falling object t seconds after being dropped from a height of 276 feet is given by $f(t) = 276 - 16t^2$ feet. Find the average velocity between times $t = 3$ and $t = 4$.

Your Answer: ______ ft/s

***This velocity is NOT a derivative!

It means ARC.

Question

Suppose that the height of a falling object t seconds after being dropped from a height of 276 feet is given by $f(t) = 276 - 16t^2$ feet. Find the average velocity between times $t = 3$ and $t = 4$.

Your Answer: ____ ft/s

$f(4) = 276 - 16*16 = 20$

$f(3) = 276 - 16*9 = 132$

$f(4) - f(3) = -112$

$$ARC = \frac{f(4) - f(3)}{4 - 3} = -112$$

Determine the limit (answer as appropriate, with a number, ∞, −∞ or does not exist).

$$\lim_{x \to -\infty} \frac{-9x}{\sqrt{16 + x^2}}$$

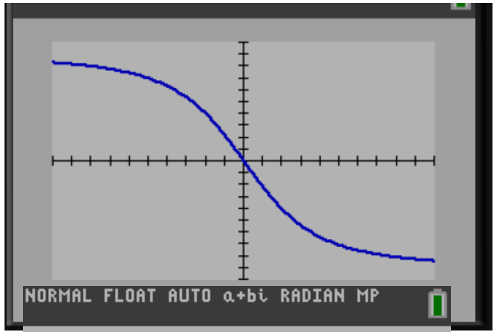
Determine the limit (answer as appropriate, with a number, ∞, −∞ or does not exist).

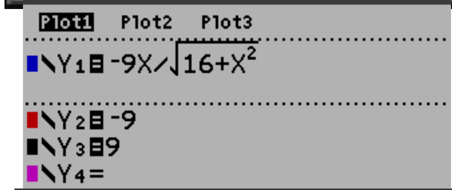
$$\lim_{x \to -\infty} \frac{-9x}{\sqrt{16 + x^2}}$$

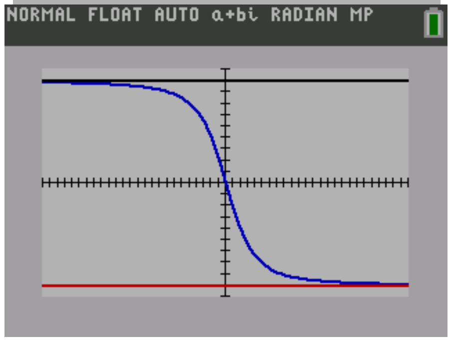
$$= \lim_{x \to -\infty} \frac{-9x / \sqrt{(x^2)}}{\sqrt{(16/x^2 + x^2/x^2)}}$$

$$= \lim_{x \to -\infty} \frac{\overbrace{-9x/|x|}^{-1}}{\sqrt{(\underbrace{16/x^2}_{0} + \underbrace{x^2/x^2}_{1})}}$$

$$= -9\,(-1) = 9$$





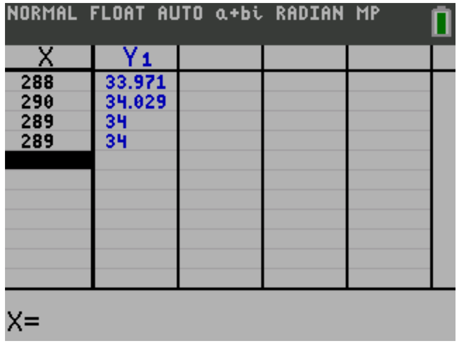


EX: **Evaluate the indicated limit.**

$$\lim_{x \to 289} \frac{x - 289}{\sqrt{x} - 17}$$

Evaluate the indicated limit.

$$\lim_{x \to 289} \frac{x - 289}{\sqrt{x} - 17} \qquad \frac{\sqrt{x} + 17}{\sqrt{x} + 17}$$



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X	Y_1
288	33.971
290	34.029
289	34
289	34

X=

$$= \lim_{x \to 289} \frac{(x-289)(\sqrt{x}+17)}{(x-289)}$$

$$= \lim_{x \to 289} \sqrt{x} + 17 = 17 + 17 = 34$$

Differentiate $y = (x^3 + x - 8)^9$.

*Fill in each of the boxes

$$\frac{dy}{dx} = \boxed{}\left(\boxed{}\right)^{\boxed{}}\left(\boxed{}\right)$$

Differentiate $y = (x^3 + x - 8)^9$.

$$\frac{dy}{dx} = \boxed{9}\left(\boxed{x^3+x-8}\right)^{\boxed{8}}\left(\boxed{3x^2 + 1}\right)$$

EX: Find $\frac{d}{dt}(\sqrt{113+14t})$.

Compute the derivative of $f(x) = x^3\sqrt{18x+7}$

Find $\frac{d}{dt}(\sqrt{113+14t})$. $= (d/dt)\,(113+14t)^{1/2}$ rewrite exp

power rule

$(1/2)(113+14t)^{-1/2}(14)$ chain rule

$= 7 / \sqrt{(113+14t)}$

Compute the derivative of $f(x) = x^3\sqrt{18x+7}$

$\underline{3x^2}\ \underline{\sqrt{(18x+7)}} + \underline{x^3}\ \underline{(1/2)(18x+7)^{-1/2}\,(18)}$ product

power rule

Product rule: f' g + g' f

chain rule

$= 3x^2\sqrt{18x+7} + 9x^3(18x+7)^{-1/2}$

EX: Compute the derivative of $f(x) = \dfrac{7x}{(x^3+8)^2}$.

Which will you use? Quotient Rule or Rewrite as a Product Rule?

Compute the derivative of $f(x) = \dfrac{7x}{(x^3+8)^2}$.

Quotient Rule

$$\frac{(x^3+8)^2 \cdot 7 - 7x \cdot 6x^2(x^3+8)}{(x^3+8)^4}$$

$\frac{d}{dx}(x^3+8)^2 =$

$2(x^3+8)3x^2$

Power + Chain Rule

Product Rule

$\frac{d}{dx}\, 7x \cdot (x^3+8)^{-2}$

$$\underbrace{7x}_{f} \cdot \underbrace{(-2)(x^3+8)^{-3}(3x^2)}_{g'} + \underbrace{(x^3+8)^{-2}}_{g} \cdot \underbrace{7}_{f'}$$

Power + Chain Rule

Lecture: Postion Velocity Acceleration

Notation: $s(t)$ $v(t)$ $a(t)$ variable 't' usually means time could be 'x' or another.

Already know $s'(t) = v(t)$

Acceleration is how velocity is changing, or $v'(t) = a(t)$

(cars go 0 to 60 in 3 seconds is an example of acceleration)

so $a(t) = (v(t))'$ or $(s'(t))'$ or $s''(t)$ called the second derivative

What are the units?

dy/dx has the 'y' units over the 'x' units.

If we ask how that is changing over the same 'x' units

Then it will be 'y' units/ 'x'units / 'x' units again

So it'll be 'x' units squared in the denominator

Like: 600 miles per hour per hour = 600 m/hr^2

This is 10mph per minute, like my old car

So,

Position = $s(t)$

Velocity = $v(t) = s'(t)$

acceleration = $a(t) = v'(t) = s''(t)$

EX: Use the position function $s(t) = \dfrac{86t}{\sqrt{t^2+3}}$ to find the velocity at time $t = 2$

(Assume units of meters and seconds.)

Ex Use the position function $s(t) = \dfrac{86t}{\sqrt{t^2+3}}$ to find the velocity at time $t = 2$.

(Assume units of meters and seconds.)

$$s(t) = 86t \cdot (t^2+3)^{-1/2}$$

Product

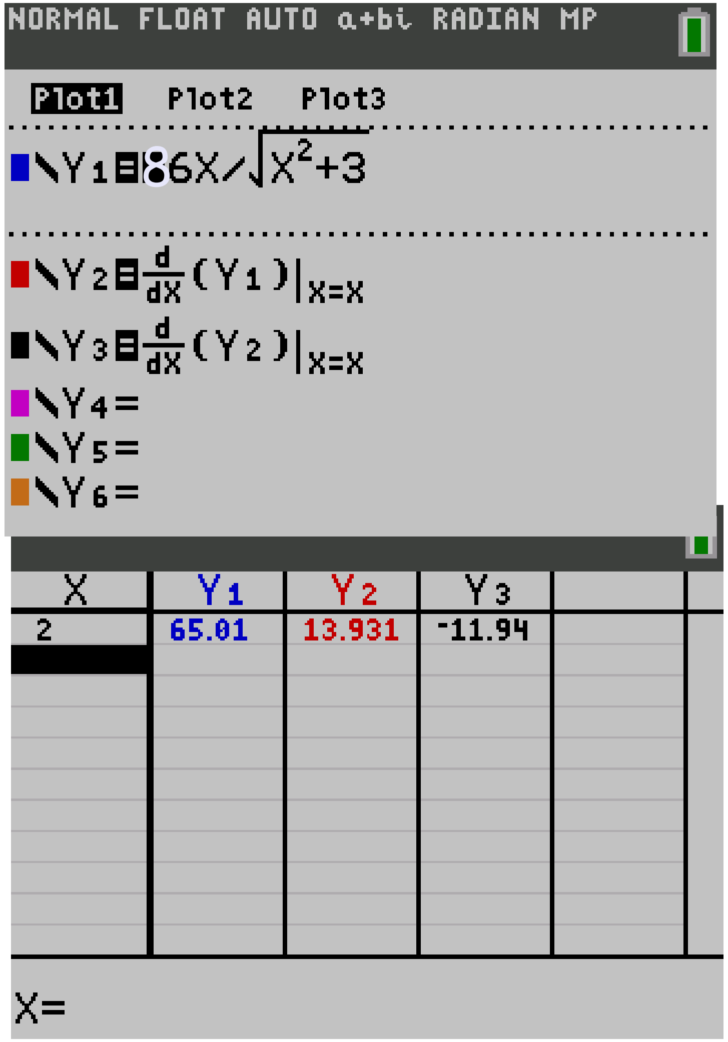
$$s'(t) = 86t \cdot -\frac{1}{2}(t^2+3)^{-3/2} \cdot \frac{d}{dt}(t^2+3) + (t^2+3)^{-1/2} \cdot \frac{d}{dt} 86t$$

$$= 86t \cdot -\frac{1}{2}(t^2+3)^{-3/2} \cdot (2t) + (t^2+3)^{-1/2} \cdot 86$$

$$s'(2) = 86(2)\left(-\frac{1}{2}\right)(4+3)^{-3/2}(2)(2) + (7)^{-1/2} \cdot 86$$

$$= \frac{86}{\sqrt{7}}\left(-\frac{4}{7}+1\right) = \frac{86 \cdot 3}{7\sqrt{7}} = \frac{258}{7\sqrt{7}}$$

Check on Calculator



Checked on DESMOS, but Connect may want exact answer

$$f(t) = \frac{86t}{\sqrt{t^2+3}}$$

$$g(t) = \frac{d}{dt} f(t)$$

$$g(2)$$

$= 13.9306905766$

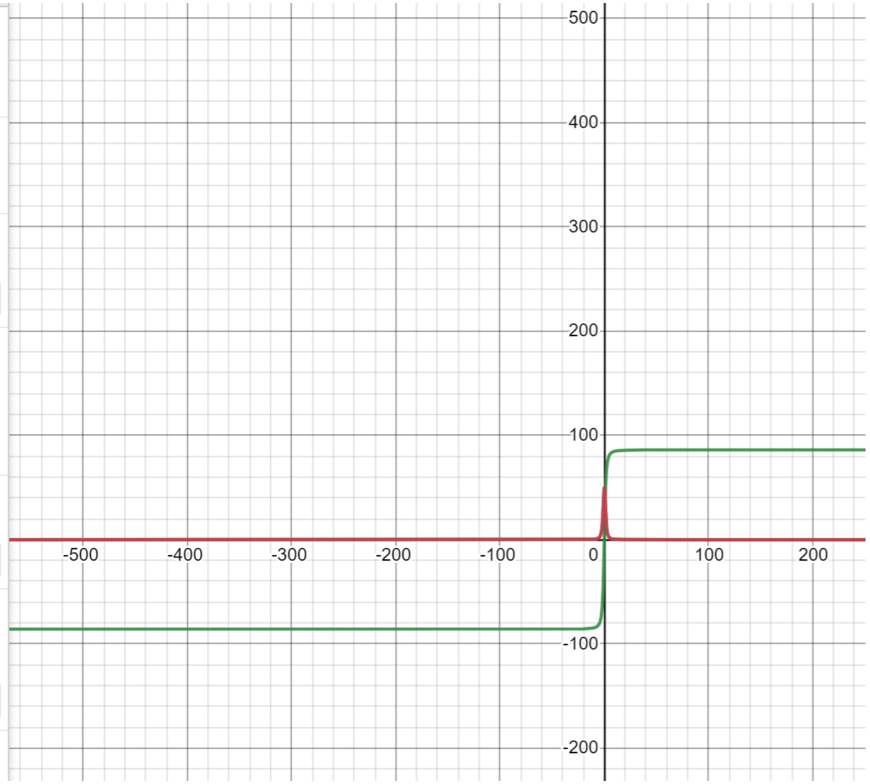
$$h(t) = \frac{\sqrt{t^2+3}\cdot 86 - 86t\left(\frac{1}{(2)}\right)\left(t^2+3\right)^{-\frac{1}{2}}}{t^2+3}$$

$$h(2)$$

$= 13.9306905766$

$$\frac{86}{\sqrt{7}}\left(\frac{3}{7}\right)$$

$= 13.9306905766$



Higher Order Derivatives

Notations:

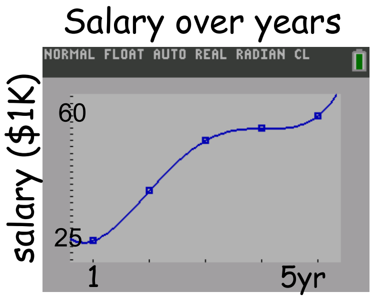
1st Derivative:	y'	$f'(x)$	$Dx[y]$	dy/dx	$d/dx[f(x)]$
2nd Derivative:	y''	$f''(x)$	$D^2x[y]$	d^2y/dx^2	$d^2/dx^2[f(x)]$
3rd Derivative:	y'''	$f'''(x)$	$D^3x[y]$	d^3y/dx^3	$d^3/dx^3[f(x)]$
4th Derivative:	$y^{(4)}$	$f^{(4)}(x)$	$D^4x[y]$	$d^4y/dx4$	$d^4/dx^4[f(x)]$

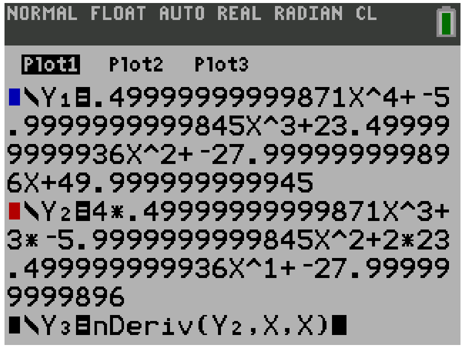
Second derivative measures accelerations

third derivative measures 'jerking motions'

Groupwork

Second derivatives

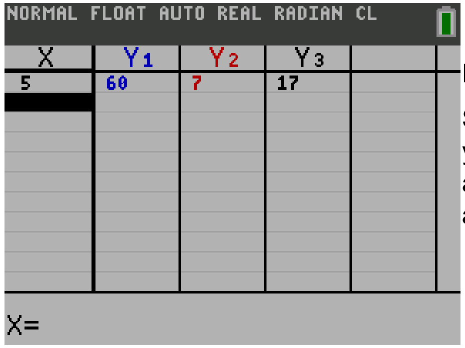




Quartic Regression

Derivative

Second Derivative



Evaluated at 5th year.

So...according to the quartic regression, in the fifth year of work, a salary of 60,000 dollars will be earned and it is increasing by only 7 dollars a year but its accleerating by 17 dollars per year per year.

STOP

Complete Practice Test after

Homework 4 and Quiz 4.