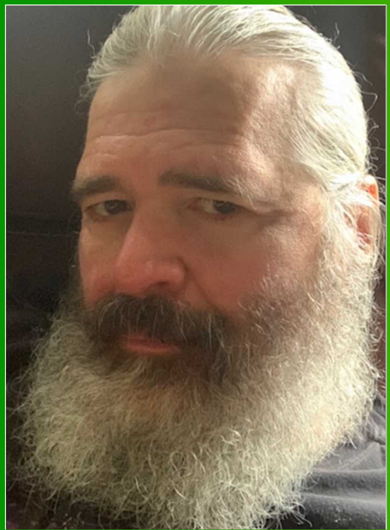
MAT 151 Calculus 1

Prof. Porter



Agenda

Homework Review

Lecture:

- Implicit Differentiation
- Hyperbolic Trig Functions
- Newton's Method

Group Work

151d7

HW3* Chain Rule MVT

11 / 11 questions assigned

Enter student instructions (optional)

add questions | organize assignment | view: list

edit pools ▾ | set all point values...

	question	question type		points
	Example 1 - 2.6 Example 1	Multipart Answer	a	0.90
	Example 2a - 2.6 Example 2a	Multipart Answer	a	0.90
	Example 3a - 2.6 Example 3a	Multipart Answer	a	0.90
	Sec. Ex. 7 - Derivatives	Multipart Answer	a	0.90
	Sec. Ex. 27 - Velocity	Multipart Answer	a	0.90
	Sec. Ex. 5 - 2.7 Section Exercise 5	Multipart Answer	a	0.90
	Example 4b - 2.7 Example 4b	Multipart Answer	a	0.90
	Example 1 - 2.5 Example 1	Multipart Answer	a	0.90
	Sec. Ex. 13a - 2.5 Section Exercise 13a	Multipart Answer	a	0.90
	Sec. Ex. 30 - 2.5 Section Exercise 30	Multipart Answer	a	0.90
	Sec. Ex. 3 - 2.10 Section Exercise 3	Multipart Answer	a	1.00

Question 1 (of 11) — Example 1 - 2.6 Example 1

This is an algorithmic question. what's this?

Example 1 - 2.6 Example 1

3 attempts left | Check my work

Find the derivative of $f(x) = 3x^5\cos(x)$.

Question 2 (of 11) — Example 2a - 2.6 Example 2a

This is an algorithmic question. what's this?

Example 2a - 2.6 Example 2a

3 attempts left | Check my work

Compute the derivative of $f(x) = 2\sin^2(x)$.

$f'(x) =$ ☐

Question 3 (of 11) — Example 3a - 2.6 Example 3a

This is an algorithmic question. what's this?

Example 3a - 2.6 Example 3a

3 attempts left | Check my work

Compute the derivative of $f(x) = 8\cos(x^7)$.

$f'(x) =$ ☐

Question 4 (of 11) — Sec. Ex. 7 - Derivatives

This is an algorithmic question. what's this?

Sec. Ex. 7 - Derivatives

3 attempts left

Find the derivative of $f(x) = \dfrac{\sin(x^8)}{x^8}$.

$f'(x) =$ ☐

Question 5 (of 11) — Sec. Ex. 27 - Velocity

This is an algorithmic question. what's this? see another version

Sec. Ex. 27 - Velocity

3 attempts left | Check my work

Use the position function $s(t) = 2t^2 - 9\sin(3t)$ to find the velocity at time $t = 0$.

Assume units of meters and seconds.

$v(0) =$ ☐ meters/second

Question 6 (of 11) — Sec. Ex. 5 - 2.7 Section Exercise 5

This is an algorithmic question. what's this?

Sec. Ex. 5 - 2.7 Section Exercise 5

3 attempts left | Check my work

Find the derivative of the function $f(x) = 6e^{7x+3}$.

$f'(x) =$ ☐

Question 7 (of 11) — Example 4b - 2.7 Example 4b

This is an algorithmic question. what's this?

Example 4b - 2.7 Example 4b

3 attempts left | Check my work

Find the derivative of $f(x) = 8\ln(x^7)$.

$f'(x) =$ ☐

Question 8 (of 11) — Example 1 - 2.5 Example 1

This is an algorithmic question.

Example 1 - 2.5 Example 1

3

Differentiate $y = (x^5 + x - 6)^6$.

$\dfrac{dy}{dx} =$ ☐

Question 9 (of 11) — Sec. Ex. 13a - 2.5 Section Exercise 13a

This is an algorithmic question. what's this? see a

Sec. Ex. 13a - 2.5 Section Exercise 13a

3 attempts left

Compute the derivative of $f(x) = \dfrac{8}{\sqrt{4x^3+3}}$.

$f'(x) =$ ☐

Question 10 (of 11) — Sec. Ex. 30 - 2.5 Section Exercise 30

This is an algorithmic question. what's this? see another version

Sec. Ex. 30 - 2.5 Section Exercise 30

3 attempts left | Check my work

Use the position function $s(t) = \dfrac{84t}{\sqrt{t^2+1}}$ to find the velocity at time $t = 2$. Enter an exact answer, do not use decimal approximation.
(Assume units of meters and seconds.)

$v(2) =$ ☐ m/s

Question 11 (of 11) — Sec. Ex. 3 - 2.10 Section Exercise 3

This is an algorithmic question. what's this? see another version

Sec. Ex. 3 - 2.10 Section Exercise 3

3 attempts left | Check my work

Write your answer in radical form.

Find a value of c satisfying the conclusion of the Mean Value Theorem.

$f(x) = x^3 + 9x^2$, $[0, 4]$

$c =$ ☐

Lecturer: Implicit Differentiation

What is Math?

What is Calculus?

What are the two rates of change?

What is the meaning of velocity?

Velocity is

the Derivative is

the instantaneous rate of change is

the slope of the tangent line

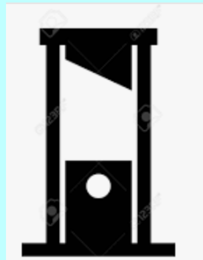
Implicit Differentiation

When my mom was angry, she would say...

"heads are going to roll!'

She was implying a severe punishment

An explicit punishment is to put our head in a guillotine and have it removed



Explicit diferentiation is finding y' when y = f(x)

Implicit differentiation is finding y' when it is in an equation.

To apply implicit differentiation-

1. Take the derivative of both sides of the equation
2. Be careful to use the chain rule when needed
3. Solve for y'. May be more than one correct answer.

remember.... variables don't match

$$\frac{d}{dx}x^2 = 2x$$

$$\frac{d}{dx}y^2 = 2y\,y'$$

$$\frac{d}{dx}t^5 = 5t^4\frac{dt}{dx}$$

Constants: π,e, sometimes a,b,c
Variables: x,y,z, t,θ
Functions: f,g,h

EX:

equation with 'y' in it.

Find the slope of the tangent line at the point (3, 3) for the ellipse $2x^2 + 3y^2 = 45$.

$y'(3) =$ ☐

$$\frac{d}{dx} 2x^2 + 3y^2 = \frac{d}{dx} 45$$

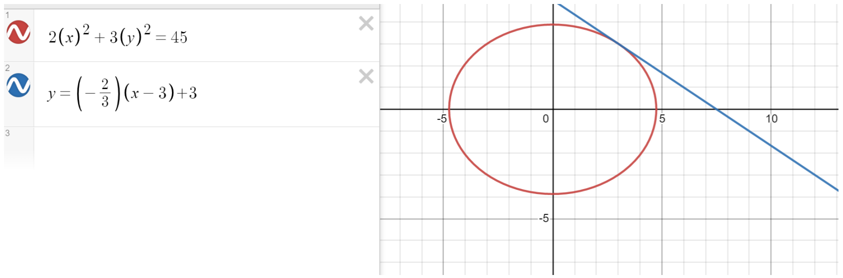
$$4x + 6y\, y' = 0$$

$$6y\, y' = -4x$$

$$y' = -4x / (6y)$$

$$y'(3) = -4(3) / (6(3))$$

$$= -2/3$$



EX:

equation with 'y' in it.

Find the derivative $y'(x)$ implicitly for the equation $\frac{7x+4}{y} = 2x + y^2$.

Multiply both sides by 'y' to get:

$$7x + 4 = 2xy + y^3$$

Then differentiate implicitly

$$\frac{d}{dx} 7x + 4 = \frac{d}{dx} 2xy + y^3$$

Using product rule and chain rule

$$7 = 2x\, y' + 2y + 3y^2\, y'$$

Now solve for y'

$$7 - 2y = (2x + 3y^2)\, y'$$

$$y' = \frac{7 - 2y}{2x + 3y^2}$$

EX: **Find the derivative $y'(x)$ implicitly.**

$\sqrt{xy} - 9y^2 = 74$

$y'(x) =$ ☐

equation with 'y' in it.

$$\sqrt{(xy)} - 9y^2 = 74$$

$$\frac{d}{dx}\left(\sqrt{(xy)} - 9y^2\right) = \frac{d}{dx}74$$

$$\frac{d}{dx}\left[(XY)^{1/2} - 9Y^2\right] = \frac{d}{dx}74$$

CHAIN RULE

$$(1/2)(XY)^{-1/2}\frac{d}{dx}(XY) - 18Y\,Y' = 0$$

PRODUCT RULE

$$(1/2)(XY)^{-1/2}(XY' + Y) - 18YY' = 0$$

THE REST IS ALGEBRA TO FIND Y'

$$XY' + Y - 36(XY)^{1/2}Y\,Y' = 0$$

$$\boxed{y' = y \,/\, [36(XY)^{1/2}Y - X]}$$

Hyperbolic Trig Functions

Just another transcendental function we didn't discuss in precalculus.

Defined

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}, \quad x \neq 0$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}, \quad x \neq 0$$

Identities

$$\cosh^2 x - \sinh^2 x = 1$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

Derivatives

$$\frac{d}{dx}\sinh x = \cosh x$$

$$\frac{d}{dx}\cosh x = \sinh x$$

$$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$$

$$\frac{d}{dx}\coth x = -\operatorname{csch}^2 x$$

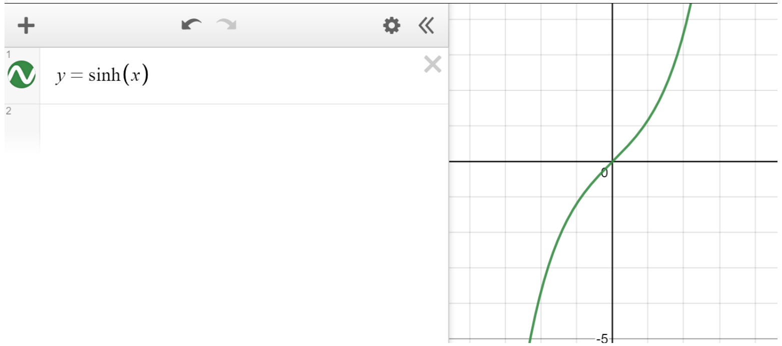
$$\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x$$

hyperbolic sine   also called sinh(x)   said like 'sinch'

Used in conic parameterization

(but not in this class)

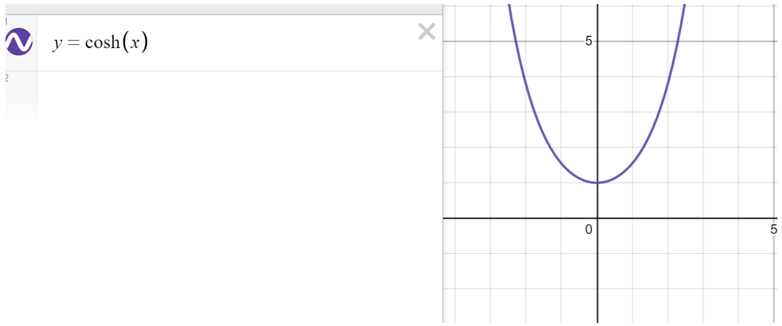


hyperbolic cosine   also called cosh(x)   said like 'kosh'


St. Louis Arch Pictures | Download Fre

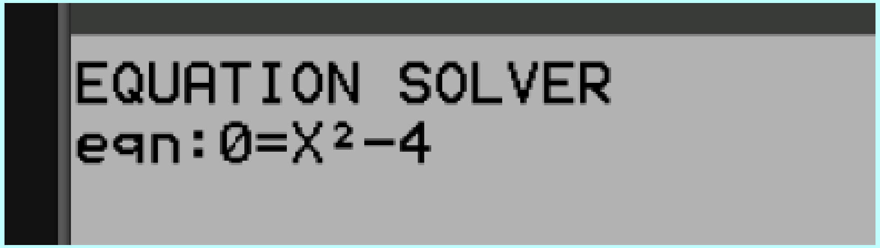
St Loius Arch

is this shape upside down.



NEWTON's METHOD

---used to find the zeros of a function



Need: Function (to be zeroed)

and an initial guess

Newton's Method is an application of derivatives.

Start with $f(x) = 0$ as a goal.

Start with a guess x_0

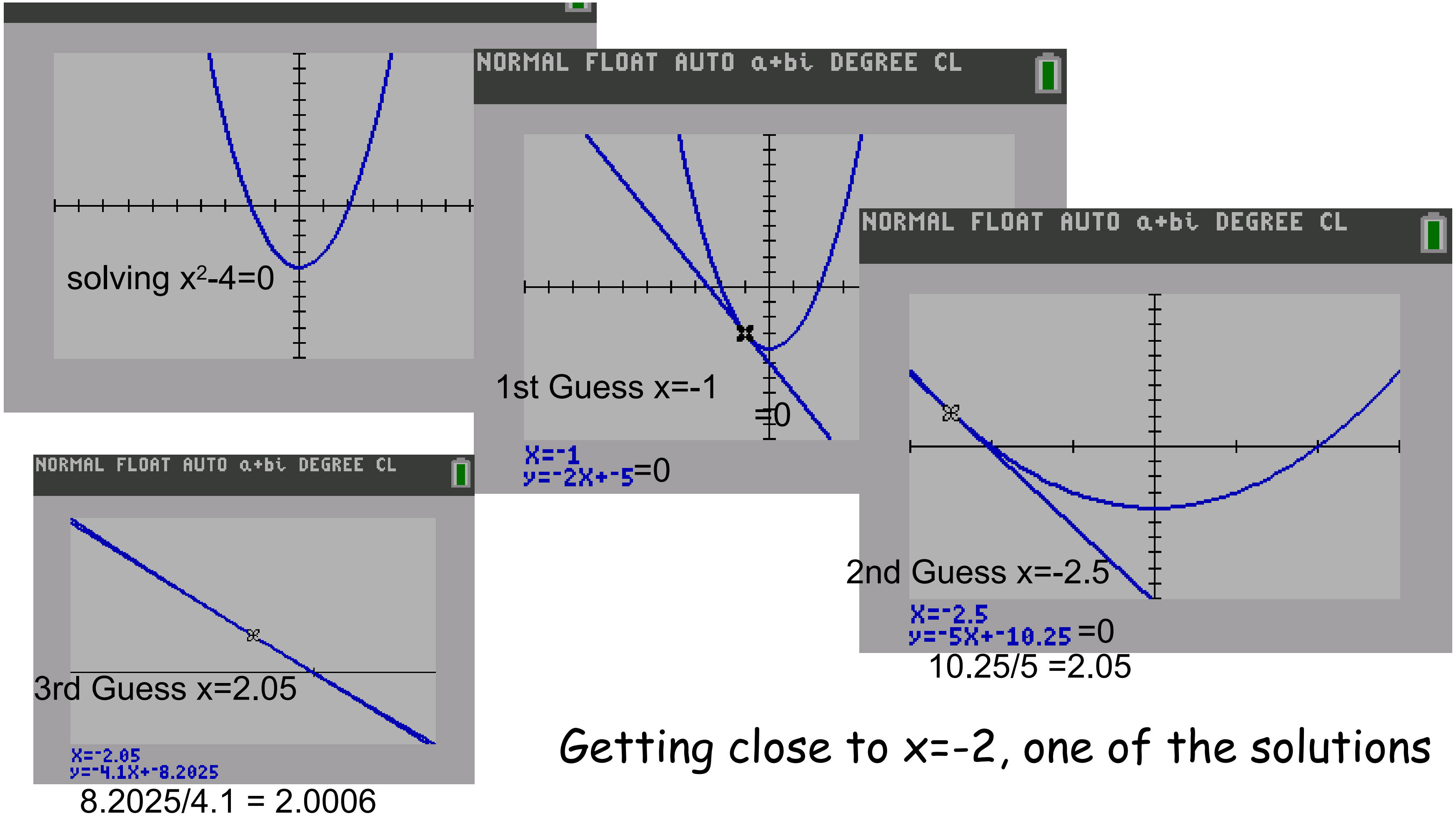
Find the equation of the tangent line $y = f'(x)(x-x_0) + f(x_0)$

Find the x-intercept (y=0) $0 = f'(x_0)(x-x_0) + f(x_0)$

$x = x_0 - f(x_0)/f'(x_0)$

Let that be your next guess x_1 $x_1 = x_0 - f(x_0)/f'(x_0)$

repeat until numbers stop changing $x_2 = x_1 - f(x_1)/f'(x_1)$



Getting close to x=-2, one of the solutions

$f(x) = x^2 - 4$ $f'(x) = 2X$

First guess 15 sto X

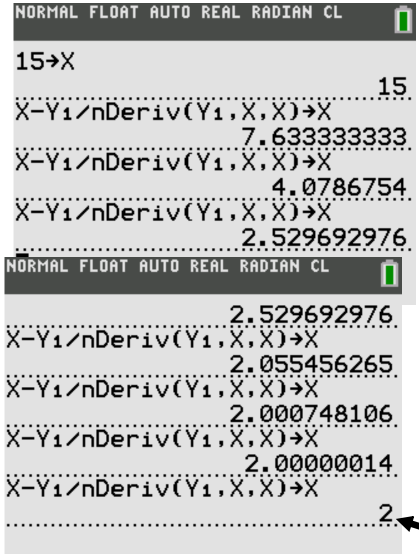
Second guess $15-(15^2-4)/(2*15) = 229/30$

Third Guess $(229/30)-((229/30)^2-4)/(229/15)$

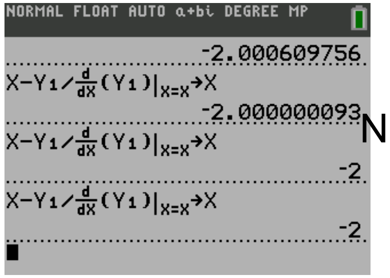
$(48841/900)(15/229)$

$1573230/206100 - 732615/206100$

$840615/206100 = 4.0786...$



Final Answer



Newer OS

Desmos was easier, and reduced fraction.

$f(x) = x^2 - 4$

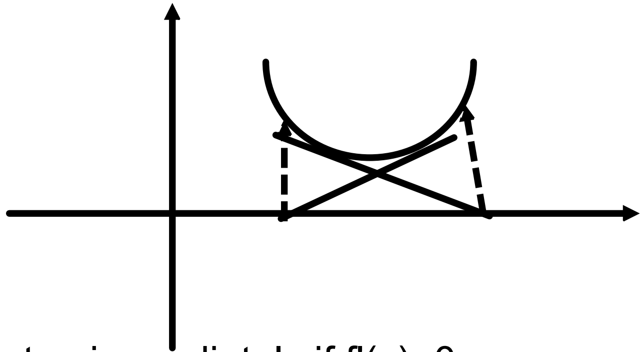
$g(x) = 2x$

$15 - \frac{f(15)}{g(15)} = \frac{229}{30}$

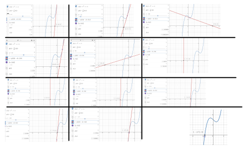
$\frac{229}{30} - \frac{f\left(\frac{229}{30}\right)}{g\left(\frac{229}{30}\right)} = \frac{56041}{13740}$

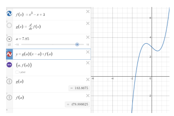
Watch out for a bad guess,

could never converge on a number



or stop immediately if f'(x)=0





Give your final answers as reduced improper fractions.

Use Newton's method with the given x_0 to compute x_1 and x_2 by hand.

$x^3 + 3x^2 - 1 = 0, x_0 = 1$

$x_1 =$ ☐ and $x_2 =$ ☐

$f(x)=x^3+3x^2-1$

$f'(x)=3x^2+6x$

$x_1 = x_0 - f(x_0)/ f'(x_0)$

$x_1 = 1 - f(1)/ f'(1)$

$x_1 = 1 - (1^3+3*1^2-1)/ (3*1^2+6*1)$

$= 1 - 3/9 = 6/9 = \boxed{2/3}$

$x_2 = (2/3)-[(2/3)^3+3(2/3)^2-1]/[3(2/3)^2+6(2/3)]$

$$\left(\frac{2}{3}\right)-\frac{\left(\frac{2}{3}\right)^3+3\left(\frac{2}{3}\right)^2-1}{3\cdot\left(\frac{2}{3}\right)^2+6\left(\frac{2}{3}\right)}$$

$$= \boxed{\frac{79}{144}}$$

Calculator is more difficult to get this part

EX:

Use Newton's method to approximate $\sqrt[3]{18}$.

$\sqrt[3]{18} \approx$

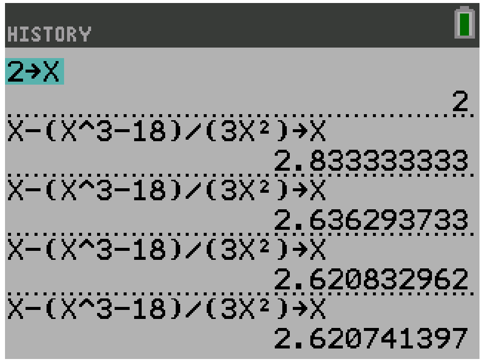
$x = \sqrt[3]{18}$

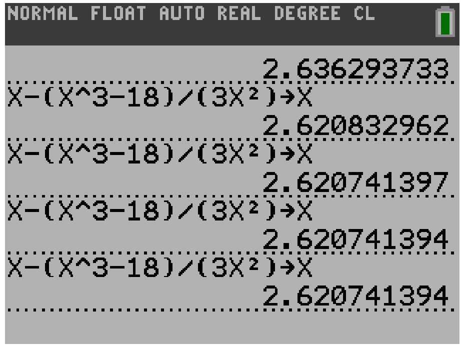
$x^3 = 18$

$x^3 - 18 = 0$

$f(x) = x^3 - 18$

$f'(x) = 3x^2$





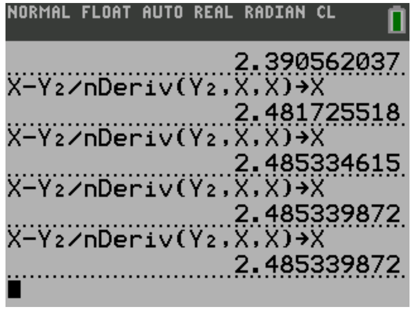
Groupwork: MVT

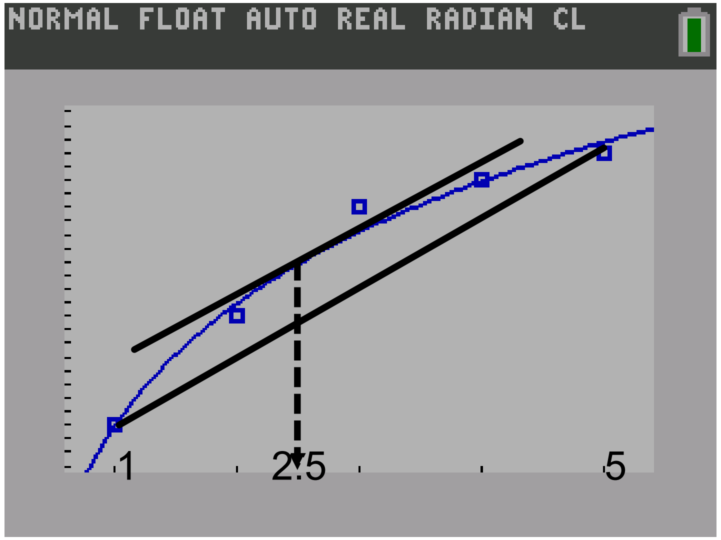
1. Put Data in
2. Do a regression. find ARC (see ARC post)
3. Put the derivative minus ARC in y2
4. Put in a second derivative y3
5. Store a guess between endpoints c_0 sto x
6. Type x - y2/y3 sto x, repeat enter

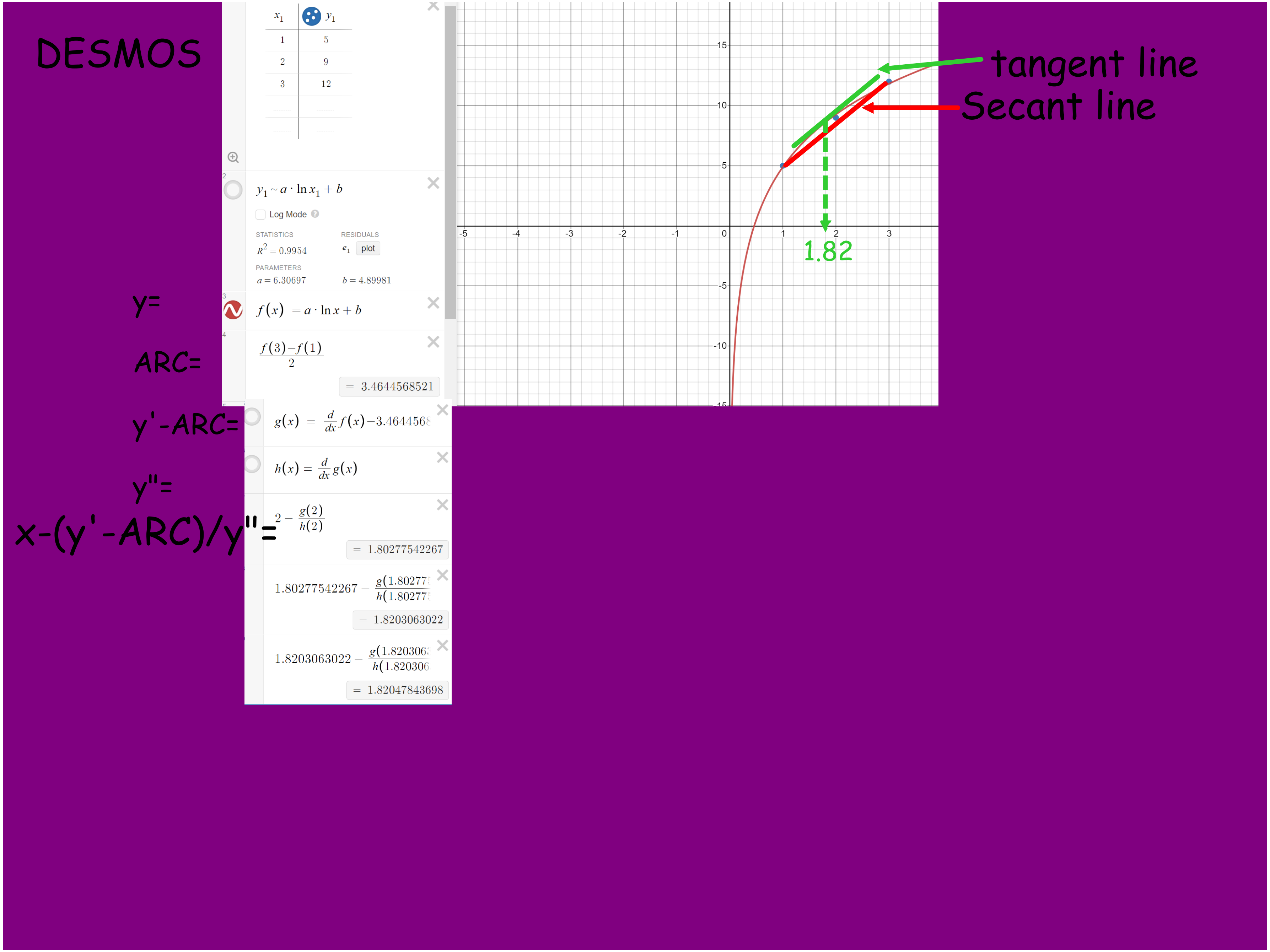
Calculator:

Data ARC y'-ARC=0

Newtons Method






DESMOS
y=
ARC=
y'-ARC=
y"=
x-(y'-ARC)/y"=
x1 | y1
1 | 5
2 | 9
3 | 12
y1 ~ a · ln x1 + b
Log Mode
STATISTICS
R² = 0.9954
RESIDUALS
e1 plot
PARAMETERS
a = 6.30697
b = 4.89981
f(x) = a · ln x + b
(f(3)−f(1))/2
= 3.4644568521
g(x) = d/dx f(x)−3.464456
h(x) = d/dx g(x)
2 − g(2)/h(2)
= 1.80277542267
1.80277542267 − g(1.80277)/h(1.80277)
= 1.8203063022
1.8203063022 − g(1.820306)/h(1.820306)
= 1.82047843698
tangent line
Secant line
1.82

For Posting:

Producer: Function, ARC (w/ end points), Newtons steps, units
QUARTIC YEARS, VALUES, ETC.

Writer: ARC = y'(c) include units!

Speaker: According to the quartic regression, when 'c' happens, we end up getting an instantaneous rate of change equal to the average over the interval between 'a' and 'b'

STOP

CLASS ANNOUNCEMENETS:

Derivatives Test Soon