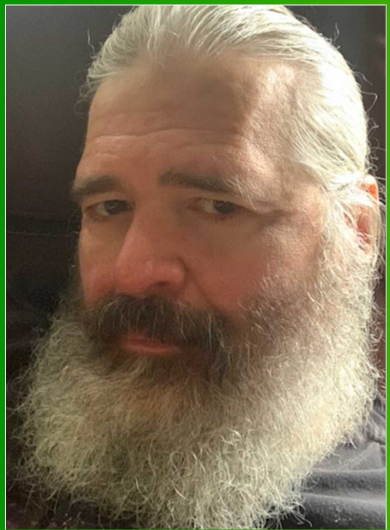
MAT 151 Calculus 1

Prof. Porter



Agenda

Homework Questions

Lecture: Theorems

Group Work: Derivatives

151d6

Homework Questions

*Take time to practice. Derivatives get easier the more you do them

And we will be doing them a lot!

Find an equation of the tangent line to $y = x^2 - 6x$ at $x = -7$.

$y =$ ☐

NOTE: Connect will have yo do this problem using the definition of derivatives.

You can use ANY method to find the solution

1. Definition (too time conuming for a test)
2. Calculator or Desmos (be sure to outline instructions used)
3. Quick and Dirty Rules (fastest for these)

Find an equation of the tangent line to $y = x^2 - 6x$ at $x = -7$.

$y =$ [ ]

$y(-7) = (-7)^2 - 6(-7)$
$= 49 + 42$
$= 91$

Point $(-7, f(-7))$
$(-7, 91)$

Slope $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 6(x+h) - x^2 + 6x}{h} =$$

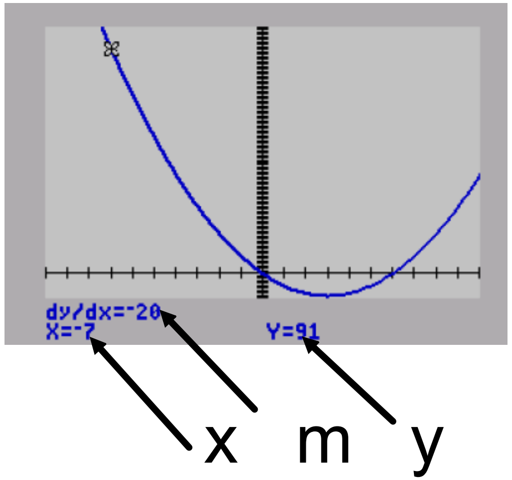
$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 6x - 6h - x^2 + 6x}{h} = \lim_{h \to 0} \frac{2xh + h^2 - 6h}{h} =$$

$$= \lim_{h \to 0} \frac{(2x + h - 6)h}{h} = \lim_{h \to 0} 2x + h - 6 = 2x - 6$$

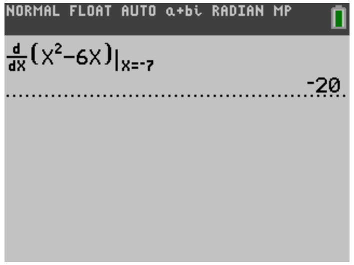
Evaluate for slope $= f'(-7) = 2(-7) - 6 = -20$

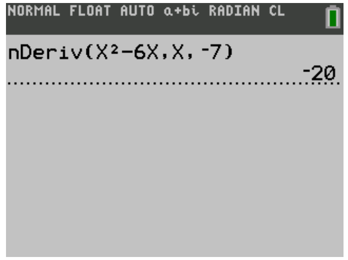
$y - 91 = -20(x + 7)$
$y = -20x - 49$

Calculator: 2nd Trace = Calc

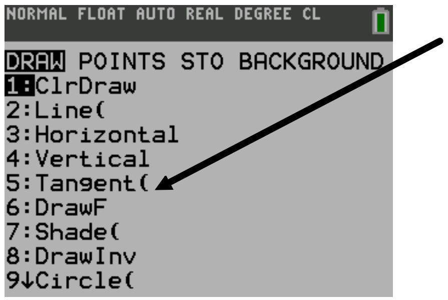


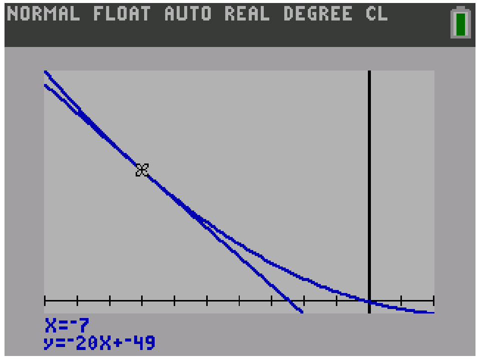
Calc 6 -7

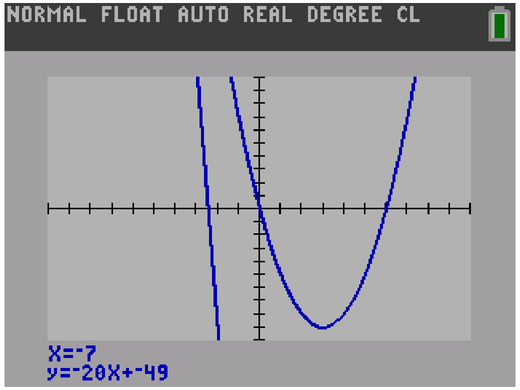




draw 5:







Lecture: Theorems

What is Math?

What is precalculus?

What is Calculus?

What are the rates of change?

How many points do you need?

Why did we invent limits?

What else is the velocity?

What is Math? Language

What is Calculus? Study of Change

What are the rates of change?

- Average and Instantaneous

How many points do you need? 2/1

Why did we invent limits?

-Takes two points to one

What else is the velocity?

slope of the tangent line

is the derivative

is the instantaneous rate of change

is the velocity

is dy/dx

Remember...Quick and Dirty Derivatives

1. (constant)' = 0   derivative of a constant is zero

2. (af) ' = af' constants come out of derivative operations

3. (f + g)' = f' + g' derivative of sum is sum of derivatives

4. $(x^n)' = nx^{n-1}$ Power Rule

5. (fg) = fg' + gf' Product Rule

6. (f/g) = (gf' - fg')/g^2 Quotient Rule

7. (f(g))' = f'(g(x)) g(x) Chain Rule

The Transcendentals:

Star ★ means

you must know them

$\left(\frac{1}{x^n}\right)' = -\frac{n}{x^{n+1}}$	$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$
$(\sqrt[m]{x})' = \frac{1}{m\sqrt[m]{x^{m-1}}}$	$(a^x)' = a^x \ln a$
★ $(e^x)' = e^x$	$(\log_a x)' = \frac{1}{x \ln a}$
★ $(\ln x)' = \frac{1}{x}$	$(\sin x)' = \cos x$ ★
★ $(\cos x)' = -\sin x$	$(\tan x)' = \frac{1}{\cos^2 x} = \sec^2 x$ ★
$(\cot x)' = -\frac{1}{\sin^2 x} = -\csc^2 x$	$(\sec x)' = \tan x \sec x$
$(\csc x)' = -\cot x \csc x$	$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$	$(\arctan x)' = \frac{1}{1+x^2}$
$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$	$(\operatorname{arcsec} x)' = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$(\operatorname{arccsc} x)' = -\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$(\sinh x)' = \cosh x$ ★
★ $(\cosh x)' = \sinh x$	$(\tanh x)' = \operatorname{sech}^2 x$
$(\coth x)' = -\operatorname{csch}^2 x$	$(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$
$(\operatorname{csch} x)' = -\operatorname{csch} x \coth x$	$(\operatorname{arcsinh} x)' = \frac{1}{\sqrt{x^2+1}}$
$(\operatorname{arccosh} x)' = \frac{1}{\sqrt{x^2-1}}$	$(\operatorname{arctanh} x)' = \frac{1}{1-x^2}$

Theorems:

Squeeze Theoerm (limits)

Intermediate Value Theorem (Continuity)

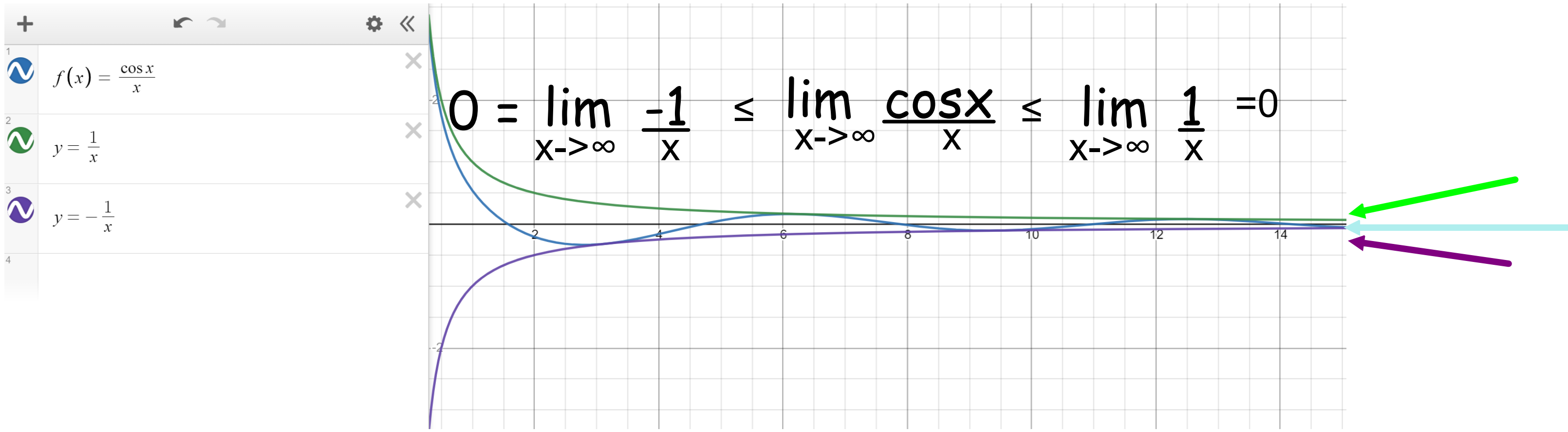
Mean Value Theorem (Continuity)

Fundamental Theorem (later)

Squeeze Theroem

The squeeze (or sandwich) theorem states that **if f(x)≤g(x)≤h(x) for all numbers**, and at some point x=k we have f(k)=h(k), then g(k) must also be equal to them.

We know: $-1 \leq \cos x \leq -1$ so $\frac{-1}{x} \leq \frac{\cos x}{x} \leq \frac{-1}{x}$ ('x' has to be positive)

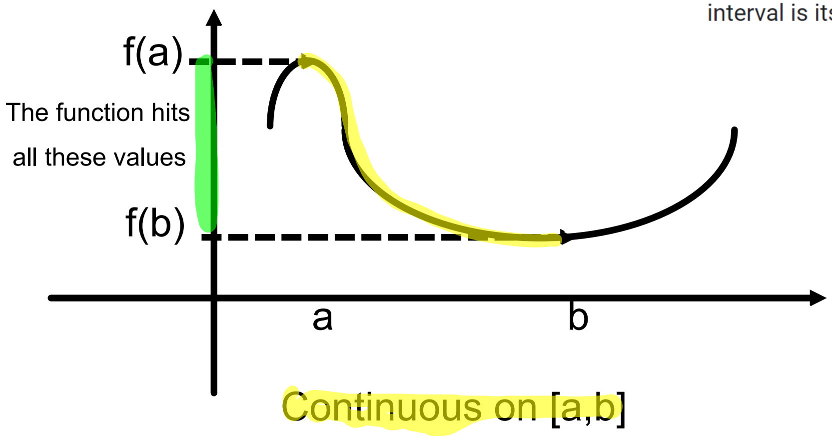


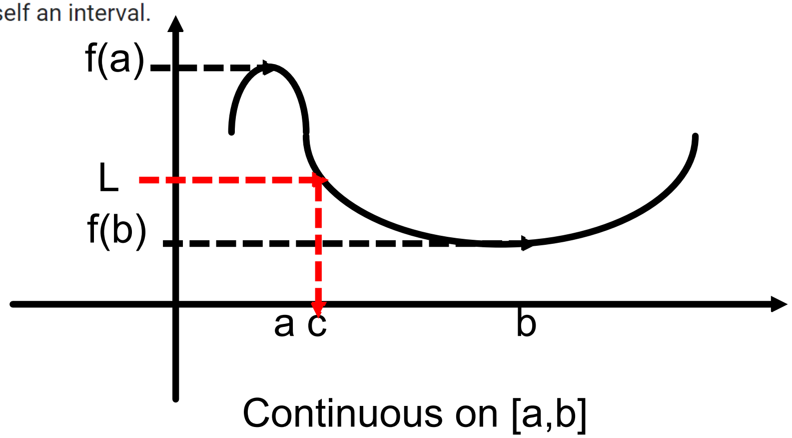
If you have to stay between two fighting friends going out, if they go to the same place, so do you.

Intermediate Value Theroem

The IVT states that if a function is continuous on [a, b], and if L is any number between f(a) and f(b), then there must **be a value, x = c**, where a < c < b, such that f(c) = L

In mathematical analysis, the intermediate value theorem states that if **f is a continuous function whose domain contains the interval [a, b]**, then it takes on any given value between f(a) and f(b) at some point within the interval. ... The image of a continuous function over an interval is itself an interval.

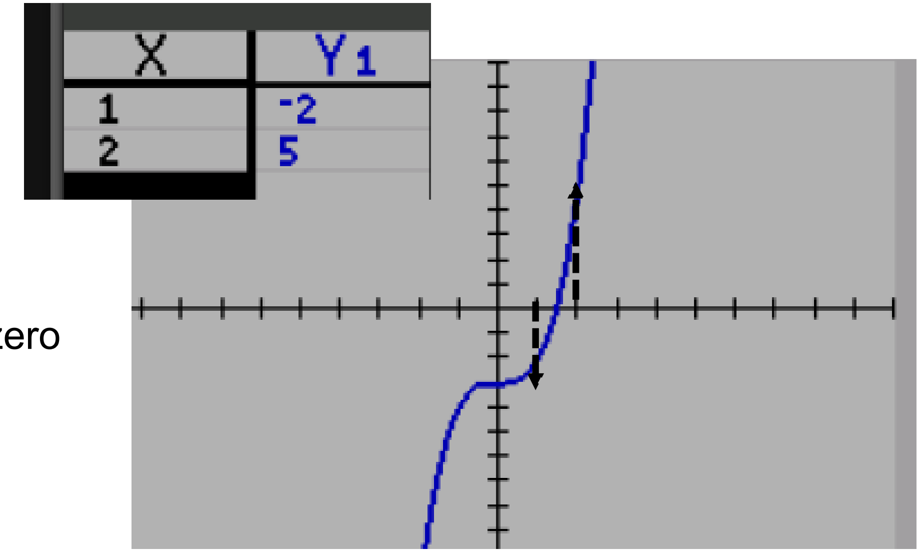




Where IVT used:

Knowing a zero exists:

1. f(x) is Continuous (polynomial)
2. f(1) is -2, f(2) is 5.
3. Zero is between -2 and 5
4. So by the IVT, there must be a zero or x intercept or f(x) =0 on (1,2) or between x=1 and x=2.

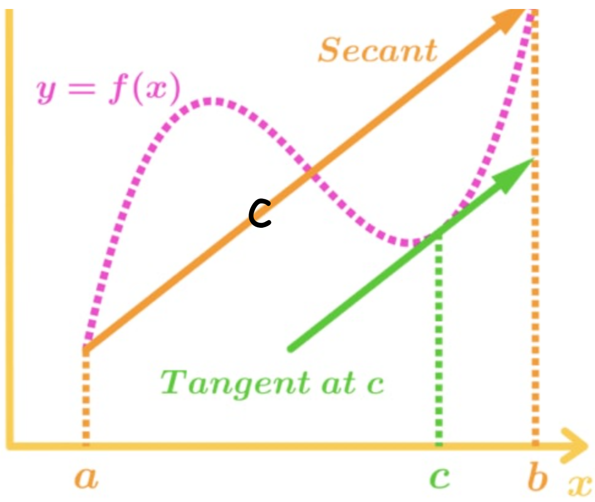


In real life: If the temperature outside drops continuous, and at noon it was 60 degrees and at midnight its -10 degrees, then somewhere between noon and midnight the temp was 32

Mean Value Theorem

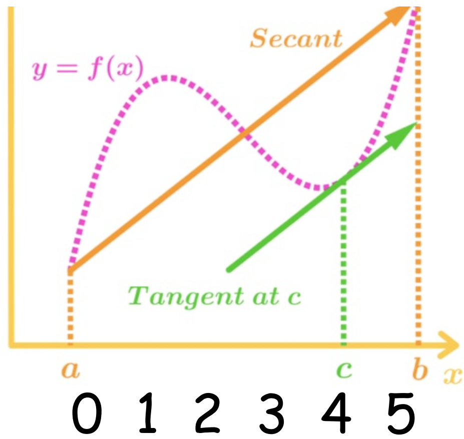
The Mean Value Theorem states that if a function f is continuous on the closed interval [a,b] and differentiable on the open interval (a,b), then there exists a point c in the interval (a,b) such that f'(c) is equal to the function's average rate of change over [a,b

-It's when the average rate of change over an interval [a,b] equals the instantaneous rate of change at 'c'.



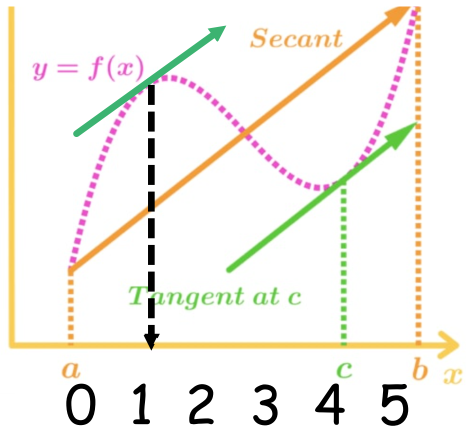
The ARC or slope of the line between 'a' and 'b' is the slope of the orange secant line.

The IRC or slope of the tangent line at 'c' in green



Can you find another value

for x=d where $m_{sec}=m_{tan}(d)$



If you guessed around 1, you were right!

<u>Where is this used:</u> Ever get a ticket for speeding on the turnpike from EZpass? Not anymore, but if you travel 100 miles on the turnpike and EZpass knows you did it in 1 hour, by the MVT, they can say you did 100mph at least once on the trip. The only challenge, it to claim abduction by aliens- that means your trip wasn't continuous!

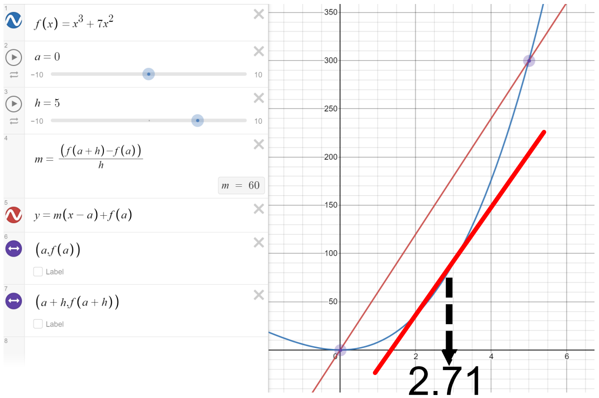
Find a value of c satisfying the conclusion of the Mean Value Theorem.

$f(x) = x^3 + 7x^2$, $[0, 5]$

ARC= $\dfrac{f(5)-f(0)}{5-0} = \dfrac{125 + 7(25)}{5} = 60$

IRC= $f'(x) = 3x^2 + 14x$ (rules used)

MVT: $3c^2 + 14c = 60$



$3c^2 + 14c - 60 = 0$

Quadratic Formula:

$c = \dfrac{-14 \pm \sqrt{(14^2 - 4(3)(-60))}}{2(3)}$

To be on [0,5] must be +

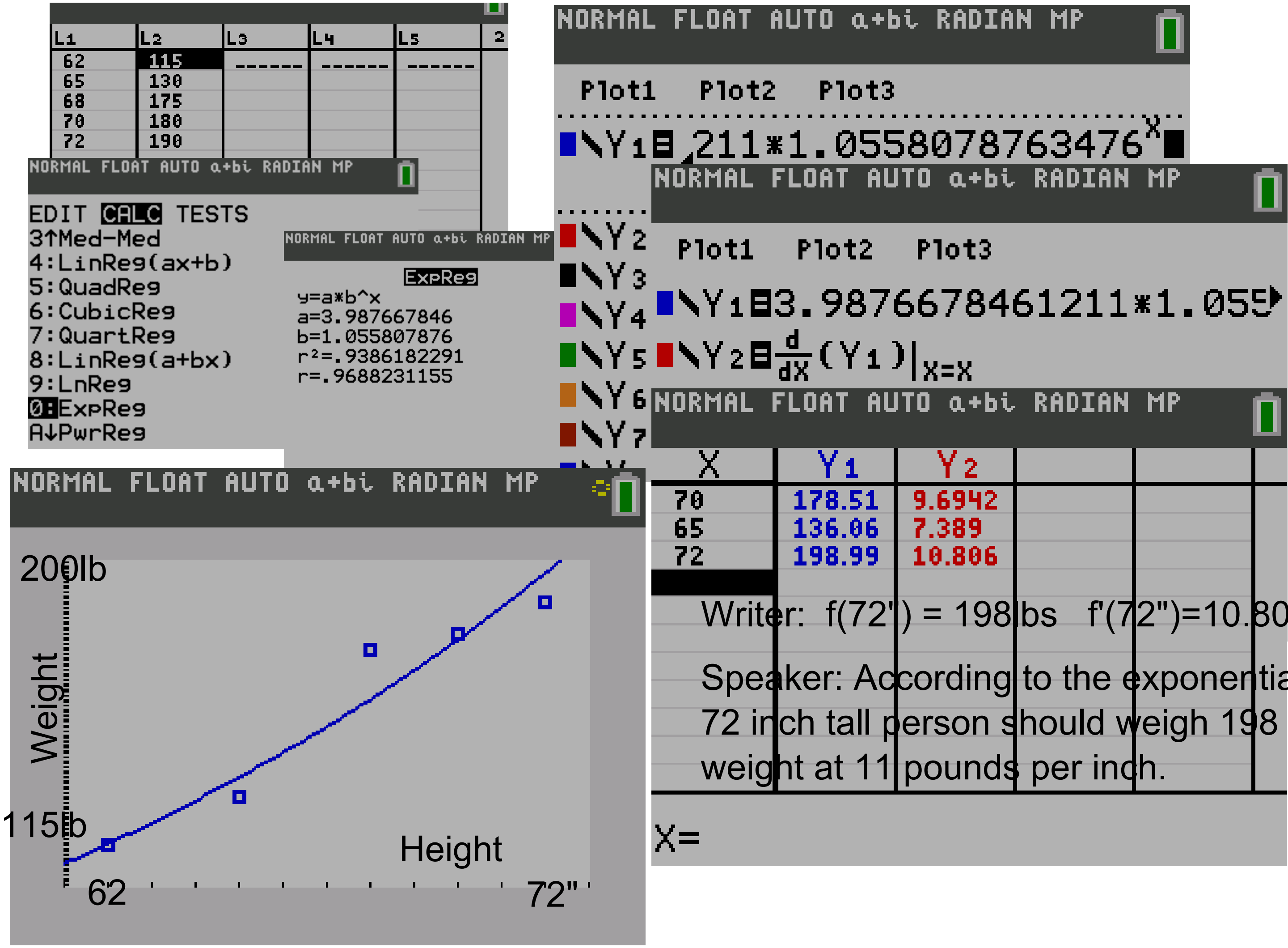
$c = \dfrac{-14 + \sqrt{(916)}}{6} = \dfrac{-7 + \sqrt{(229)}}{3} = 2.71..$

Group Work: Transcendental Derivatives

1. Put Data in
2. Do a transcedental regression

   exponential, logaritmic, sine

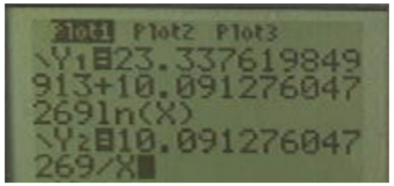
3. Put the derivative in
4. Put in a second derivative
5. Evaluate at a point of interest

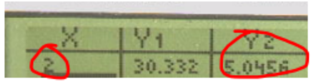


Writer: f(72") = 198lbs f'(72")=10.806lbs/in

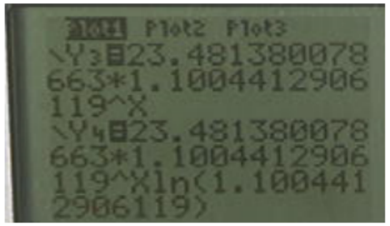
Speaker: According to the exponential regression, a 72 inch tall person should weigh 198 pounds and gain weight at 11 pounds per inch.

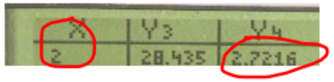
Calculator:



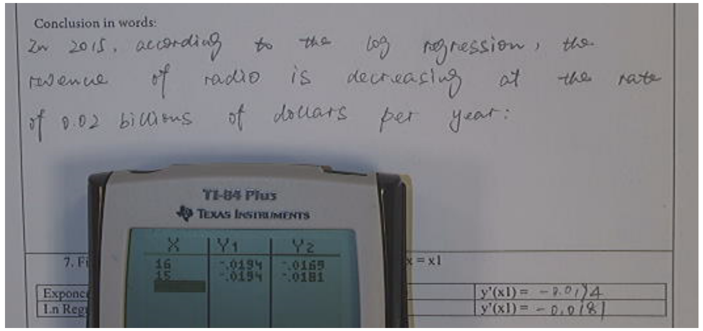


in 2nd year, according to the ln or natural log regression, her revenue is growing at 5.04 billion $ per year.





in 2nd year, according to the exponential regression, her revenue is growing at 2.72 billion $ per year.



For Posting:

Producer: Table of Values , Regression, Units

a	A	B	C

natural log   YEARS, VALUES, ETC.

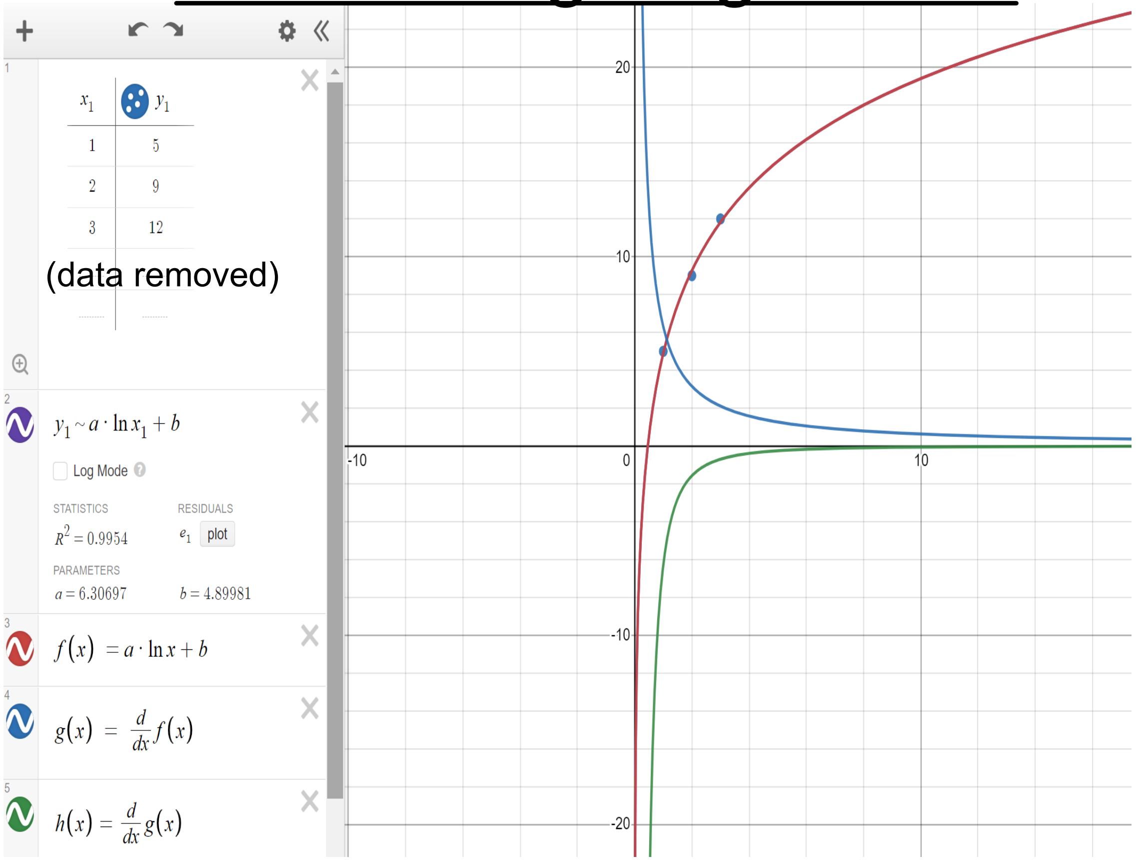
Writer: y(a) = A , y'(a) = B and y'(a) = C, include units!

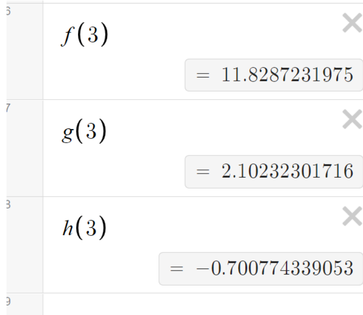
Speaker: According to the natural log regression, when A happens, we end up getting B, and it is accellerating (decellerating if C is negative)

DESMOS

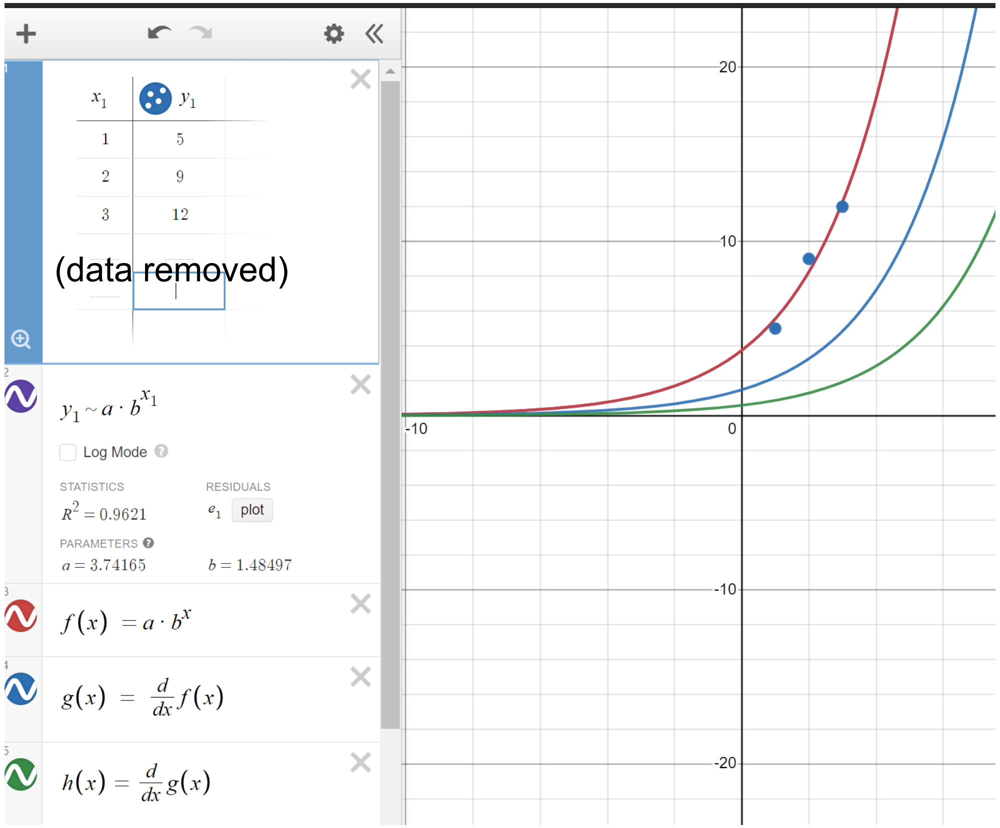
Many more regression available than in the calculator. Don't feel restricted to the following three that are in the calculator.

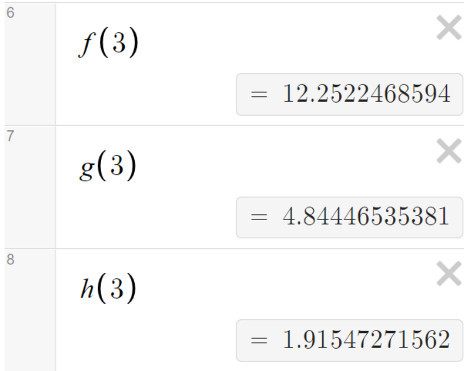
Natural Log Regression



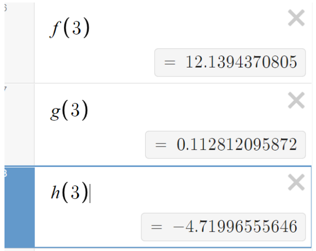


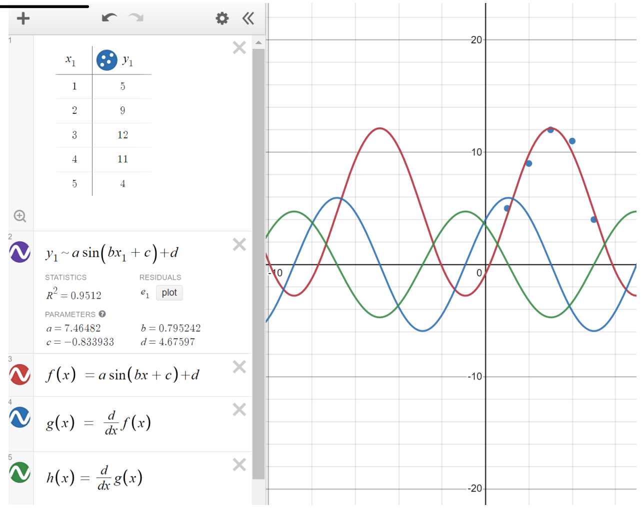
Exponential Regression





Sine Regression





CLASS ANNOUNCEMENETS:

Have a study plan for the first test soon.