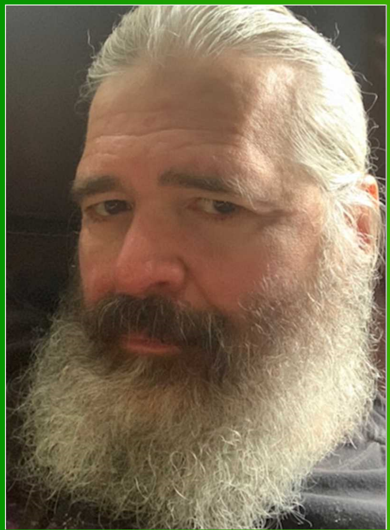
MAT 151 Calculus 1

Prof. Porter



Agenda

Homework Review

Lecture: Derivative Rules

- Transcendentals
- Composite Functions

Group Work: Derivatives

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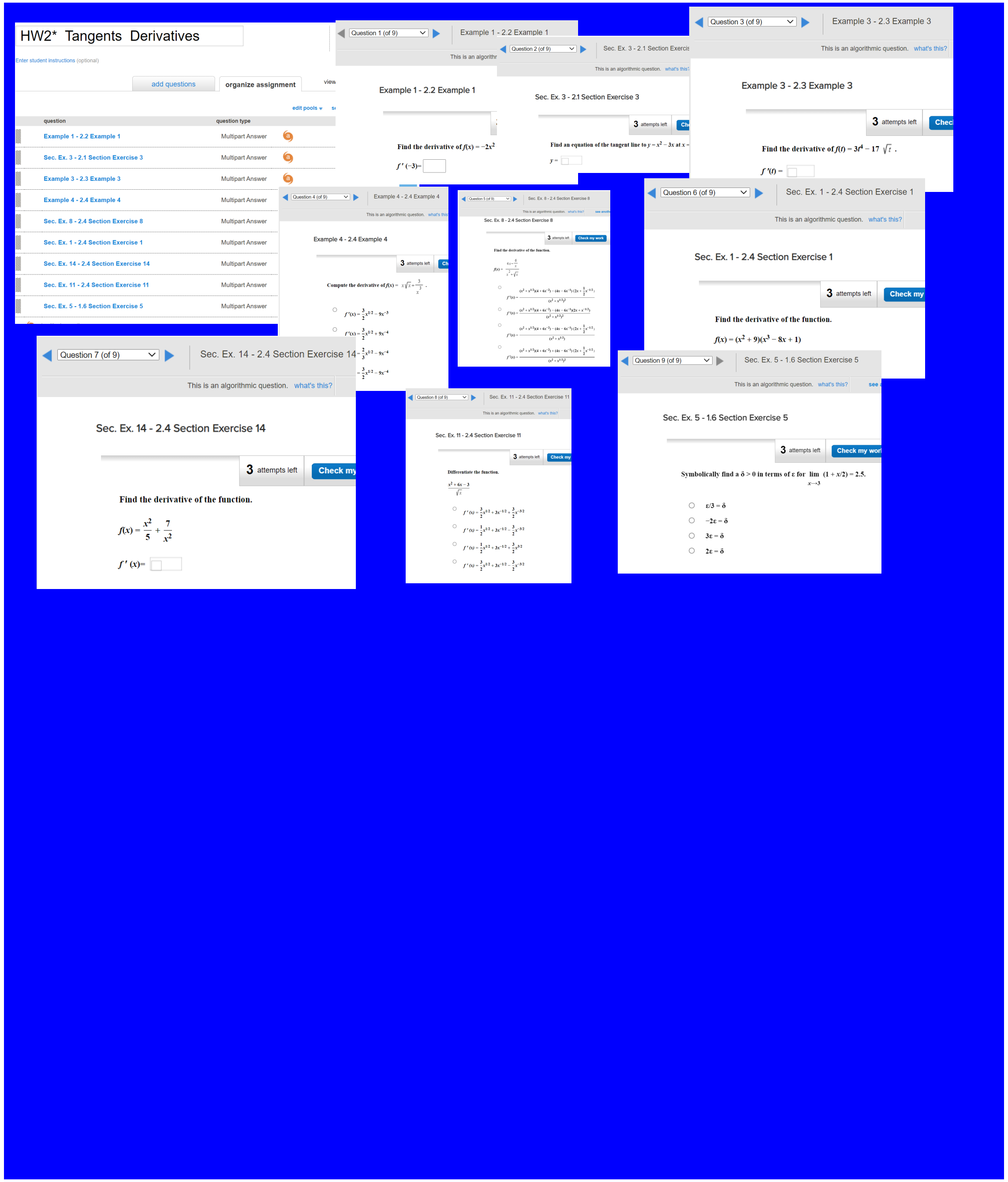

HW2* Tangents Derivatives
add questions
organize assignment
question
question type
Example 1 - 2.2 Example 1
Multipart Answer
Sec. Ex. 3 - 2.1 Section Exercise 3
Multipart Answer
Example 3 - 2.3 Example 3
Multipart Answer
Example 4 - 2.4 Example 4
Multipart Answer
Sec. Ex. 8 - 2.4 Section Exercise 8
Multipart Answer
Sec. Ex. 1 - 2.4 Section Exercise 1
Multipart Answer
Sec. Ex. 14 - 2.4 Section Exercise 14
Multipart Answer
Sec. Ex. 11 - 2.4 Section Exercise 11
Multipart Answer
Sec. Ex. 5 - 1.6 Section Exercise 5
Multipart Answer
Question 1 (of 9)
Example 1 - 2.2 Example 1
Find the derivative of f(x) = −2x²
f ′(−3)=
Question 2 (of 9)
Sec. Ex. 3 - 2.1 Section Exercise 3
Find an equation of the tangent line to y = x² − 3x at x =
y =
Question 3 (of 9)
Example 3 - 2.3 Example 3
This is an algorithmic question. what's this?
3 attempts left
Find the derivative of f(t) = 3t⁴ − 17 √t .
f ′(t) =
Example 4 - 2.4 Example 4
Sec. Ex. 8 - 2.4 Section Exercise 8
Question 6 (of 9)
Sec. Ex. 1 - 2.4 Section Exercise 1
This is an algorithmic question. what's this?
3 attempts left
Check my
Find the derivative of the function.
f(x) = (x² + 9)(x³ − 8x + 1)
Question 7 (of 9)
Sec. Ex. 14 - 2.4 Section Exercise 14
This is an algorithmic question. what's this?
3 attempts left
Check my
Find the derivative of the function.
f(x) = x²/5 + 7/x²
f ′(x)=
Sec. Ex. 11 - 2.4 Section Exercise 11
Differentiate the function.
Question 9 (of 9)
Sec. Ex. 5 - 1.6 Section Exercise 5
This is an algorithmic question. what's this?
3 attempts left
Check my work
Symbolically find a δ > 0 in terms of ε for lim (1 + x/2) = 2.5.
ε/3 = δ
−2ε = δ
3ε = δ
2ε = δ

Lecture Review Rules

What is Math?

What is Calculus?

What are the two rates of change?

What are four meanings for velocity?

Velocity is

the Derivative is

the instantaneous rate of change is

the slope of the tangent line

The "Quick and Dirty" Rules

1. (constant)' = 0 derivative of a constant is zero

2. $(af)' = af'$ constants come out of derivative operations

3. $(f + g)' = f' + g'$ derivative of sum is sum of derivatives

4. $(x^n)' = nx^{n-1}$ <u>Power Rule</u>

5. $(fg) = fg' + gf'$ <u>Product Rule</u>

6. $(f/g) = (gf' - fg')/g^2$ <u>Quotient Rule</u>

Try these....

EX: $\frac{d}{dx}(x^4 - 3x^2 + 7)$

EX: $\frac{d}{dx}\sqrt{x} + 6$

EX: $\frac{d}{dx} \frac{x^7 + 4x^2 - 9}{x^4}$

Answers

EX: $\frac{d}{dx}(x^4 - 3x^2 + 7) = 4x^3 - 6x$

EX: $\frac{d}{dx}\sqrt{x} + 6 = (1/2)\ x^{-1/2}$

EX: $\frac{d}{dx}\ \frac{x^7 + 4x^2 - 9}{x^4} = \frac{d}{dx}\ x^3 + 4x^{-2} - 9x^{-4} = 3x^2 - 8x^{-3} + 36x^{-5}$

EX: Product

$$y = (\underbrace{x^2+5}_{f})(\underbrace{x^3-9x+3}_{g})$$

$$y' = fg' + gf'$$

$$(x^2+5)(x^3-9x+3)' + (x^3-9x+3)(x^2+5)'$$

$$(x^2+5)(3x^2-9) + (x^3-9x+3)(2x)$$

EX:

$$\frac{d}{dx}\left(x^{50}+2x^{25}+7\right)\left(x^{23}-2x+15\right)$$

$$\left(x^{50}+2x^{25}+7\right)\frac{d}{dx}\left(x^{23}-2x+15\right)+\left(x^{23}-2x+15\right)\frac{d}{dx}\left(x^{50}+2x^{25}+7\right)$$

$$\left(x^{50}+2x^{25}+7\right)\left[23x^{22}-2\right]+\left(x^{23}-2x+15\right)\left[50x^{49}+50x^{24}\right.$$

Quotient Rule.

$$y = \frac{f}{g} \longrightarrow y' = \frac{gf' - fg'}{g^2}$$

EX: Quotient

If $y = \dfrac{x^2 - 7x + 3}{2x + 5}$ Find y'

$$y' = \frac{(2x+5)(2x-7) - (x^2-7x+3)(2)}{(2x+5)^2}$$

EX: $$\frac{d}{dx}\left(\frac{x^7-3x^2+7}{x^4}\right)=$$

$$\frac{x^4\cdot\frac{d}{dx}(x^7-3x^2+7)-(x^7-3x^2+7)\frac{d}{dx}(x^4)}{(x^4)^2}=$$

$$\frac{x^4\cdot(7x^6-6x)-(x^7+3x^2+7)(4x^3)}{x^8}=$$

$$\frac{7x^{10}-6x^5-4x^{10}-12x^5-28x^3}{x^8}=$$

$$\frac{3x^{10}+6x^5-28x^3}{x^8}=$$

$$\boxed{3x^2+\frac{6}{x^3}-\frac{28}{x^5}}$$

EX: $\dfrac{d}{dx}\left(\dfrac{1}{x-5}\right)$

equals zero

$$\frac{(x-5)\cdot\frac{d}{dx}(1) - 1\cdot\frac{d}{dx}(x-5)}{(x-5)^2}$$

$$= \frac{-1}{(x-5)^2}$$

EX:

$$y = \frac{1}{(x+5)^2} = \frac{1}{x^2+10x+25}$$

Quotient Rule:

$$\boxed{\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2}} = \frac{(x^2+10x+25)\frac{d}{dx}(1) - (2x+10)}{(x+5)^4}$$

$$= \frac{-2(x+5)}{(x+5)^4} = \frac{-2}{(x+5)^3}$$

Transcendentals

transcendental function, In mathematics, a function not expressible as a finite combination of the algebraic operations of addition, subtraction, multiplication, division, raising to a **power, and extracting a root**. Examples include the functions log x, sin x, cos x, e^x and any functions containing them.

Here's a bunch!

Star ★ means

you must know them

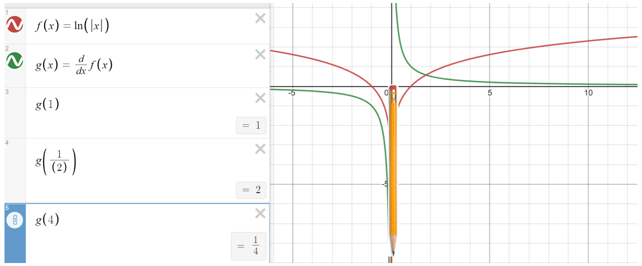
$\left(\frac{1}{x^n}\right)' = -\frac{n}{x^{n+1}}$	$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$
$(\sqrt[m]{x})' = \frac{1}{m\sqrt[m]{x^{m-1}}}$	$(a^x)' = a^x \ln a$
$(e^x)' = e^x$	$(\log_a x)' = \frac{1}{x \ln a}$
$(\ln x)' = \frac{1}{x}$	$(\sin x)' = \cos x$
$(\cos x)' = -\sin x$	$(\tan x)' = \frac{1}{\cos^2 x} = \sec^2 x$
$(\cot x)' = -\frac{1}{\sin^2 x} = -\csc^2 x$	$(\sec x)' = \tan x \sec x$
$(\csc x)' = -\cot x \csc x$	$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$	$(\arctan x)' = \frac{1}{1+x^2}$
$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$	$(\operatorname{arcsec} x)' = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$(\operatorname{arccsc} x)' = -\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$(\sinh x)' = \cosh x$
$(\cosh x)' = \sinh x$	$(\tanh x)' = \operatorname{sech}^2 x$
$(\coth x)' = -\operatorname{csch}^2 x$	$(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$
$(\operatorname{csch} x)' = -\operatorname{csch} x \coth x$	$(\operatorname{arcsinh} x)' = \frac{1}{\sqrt{x^2+1}}$
$(\operatorname{arccosh} x)' = \frac{1}{\sqrt{x^2-1}}$	$(\operatorname{arctanh} x)' = \frac{1}{1-x^2}$

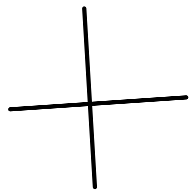
<u>AnotherTranscendental</u>

Also,

$$\frac{d}{dx}\ln(|x|) = 1/x$$

$$x \neq 0$$

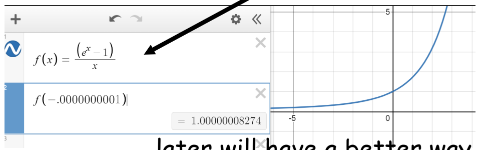




To prove these, various methods are used.

For example: $\frac{d}{dx}e^x = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h} = \lim_{h\to 0}\frac{e^x e^h-e^x}{h} = e^x \lim_{h\to 0}\frac{e^h-1}{h} = e^x$

$\boxed{\frac{d}{dx}e^x = e^x}$



later will have a better way

$\frac{d}{dx}\frac{\sin x}{\cos x} = \frac{(\cos x)(\cos x)-(\sin x)(-\sin x)}{(\cos x)^2} = \frac{1}{(\cos x)^2} = \sec^2 x$

quotient identity

quotient rule here

these are by the trig identities
Pythagorean and reciprocal

$\boxed{\frac{d}{dx}\tan x = \sec^2 x}$

Chain Rule

The derivative of a composite function

$$\frac{d}{dx} f\big(g(x)\big) = f'\big(g(x)\big)g'(x)$$

COMPOSITE FUNCTIONS : (f o g)(x) = f (g (x))

Like an assembly line of functions where the inner function g(x) goes first, and the outer function f(x) goes last

Chain Rule The derivative of a composite function

$$\frac{d}{dx}f\big(g(x)\big) = f'\big(g(x)\big)g'(x)$$

In words, take the derivative of the outside function, leave the inside alone, then multiply by the derivative of the inside

EX: Find y' if $Y=(X^2+3X+8)^{100}$

$\boxed{\text{Inside function}}$ usually in a parenthesis, exponent or denominator

$g(x) = X^2 + 3X + 8$

$\boxed{\text{Outside function}}$ would be done last in order of operations PEMDAS

$h(x) = X^{100}$

$h'(x) = 100X^{99}$ (Power Rule)

to leave inside function alone, must put it into h', so

$h'(g(x))=100(x^2+3x+8)^{99}$

now multiply by $g'(x) = 2x+3$ so $\boxed{y' = 100(X^2+3X+8)^{99}(2X+3)}$

by power and chain rule

EX: **Compute the derivative of $f(x) = \frac{8}{\sqrt{x^3+6}}$**

Or..... $y = 8(x^3 + 6)^{-1/2}$

Inside function is $g(x) = x^3 + 6$ Outside is $f(x) = x^{-1/2}$

$f'(x) = (-1/2)x^{-3/2}$ $g'(x) = 3x^2$

$f'(g(x)) = (-1/2)(x^3 + 6)^{-3/2}$

so

$y' = (-1/2)(x^3 + 6)^{-3/2}3x^2$ or $(-3x^2/2)(x^3 + 6)^{-3/2}$ or $\dfrac{-3x^2}{2\sqrt{(x^3 + 6)^3}}$

Chain Rule- variables don't match

$\frac{d}{dx}x^2 = 2x$

$\frac{d}{dx}y^2 = 2y\,y'$

$\frac{d}{dx}\pi^2 = 0$ (pie is a constant)

$(x^2)' = 2x$ (prime is used for same variable)

Constants: π,e, sometimes a,b,c

Variables: x,y,z, t,θ

Functions: f,g,h

Chain Rule with Transcendentals:

	inside $g(x)$	out $f(x)$	out' $f'(x)$	out'(in) $f'(g(x))$	out'(in) $f'(g(x))g'(x)$
EX: $y = e^{\sqrt{x}}$	$\sqrt{x}$	e^x	e^x	$e^{\sqrt{x}}$	$e^{\sqrt{x}}(1/2)x^{-1/2}$
EX: $y = \ln(x^3)$	x^3	$\ln x$	$1/x$	$1/x^3$	$(1/x^3)(3x^2)$
EX: $y = \sin^2 x$	$\sin x$	x^2	$2x$	$2\sin x$	$2\sin x \cos x$
EX: $y = \cos(x^2)$	x^2	$\cos x$	$\sin x$	$\sin(X^2)$	$\sin(X^2)\ (2x)$
EX: $y = 1/\cos x$	$\cos x$	$1/x$	$-1/x^2$	$-1/\cos^2 x$	$(-1/\cos^2 x)(\sin x)$

EX: $\frac{d}{dx}\frac{5}{x+2} = \frac{d}{dx}5(x+2)^{-1}$

rewrite

$= -5(x+2)^{-2}\cdot(1)$

Chain Rule.

$\frac{d}{dx}\frac{5}{(x+2)^2} = \frac{d}{dx}5(x+2)^{-2}$

$= -10(x+2)^{-3}(1)$

RULE:	EX (with Chain rule):
$\frac{d}{dx} e^{u} = e^{u} \frac{du}{dx}$	$\frac{d}{dx} e^{x^2} = e^{x^2} \cdot 2x$
$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$	$\frac{d}{dx} \ln(x^3+7) = \frac{1}{x^3+7} \cdot 3x^2$
$\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$	$\frac{d}{dx} \sin(e^{x}) = \cos(e^{x}) \cdot e^{x}$

Ex $\frac{d}{dx}\sec x = \frac{d}{dx}\frac{1}{\cos x} = \frac{d}{dx}(\cos x)^{-1} = +\cos^{-2}x \sin x$

$$= \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x} = \tan x \sec x$$

Ex $\frac{d}{dx}\sec(e^x) = \tan(e^x)\sec(e^x)\cdot e^x$

$$f(g(x)) = f'(g(x))\cdot g'(x)$$

tricky

$y = \sin(\pi/3)$ so $y' = 0$

$y = x \sin(\pi/3)$ $y' = \sin(\pi/3)$

$y = \sqrt{(x)} \sin(\pi/3)$ $y' = (1/2)x^{-1/2}\sin(\pi/3)$

because $\sin(\pi/3)$ is a constant!

Groupwork: Second Derivative

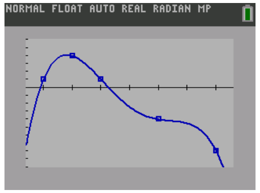
Last time....

1. Put Data in
2. Do a polynomial regression
3. Put the derivative in
4. Evaluate at a point of interest

add

5. Put in a second derivative

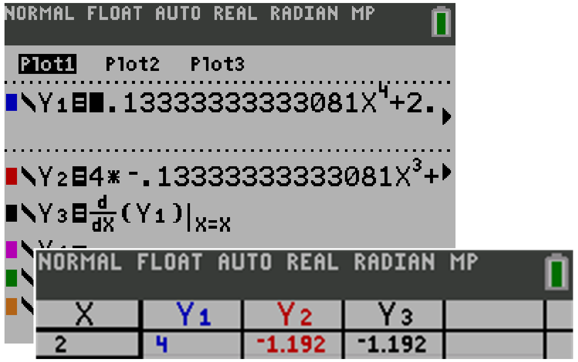
In Calculator



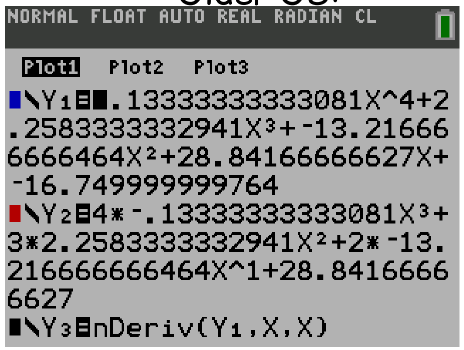
y1=$Ax^4+Bx^3+cx^2+Dx+E$ Quartic Regression

y2=$4*Ax^3+3*Bx^2+2*cx+D$ Derivative

y3 = math 8 nderiv y1 , x , x second derivative



Older OS:

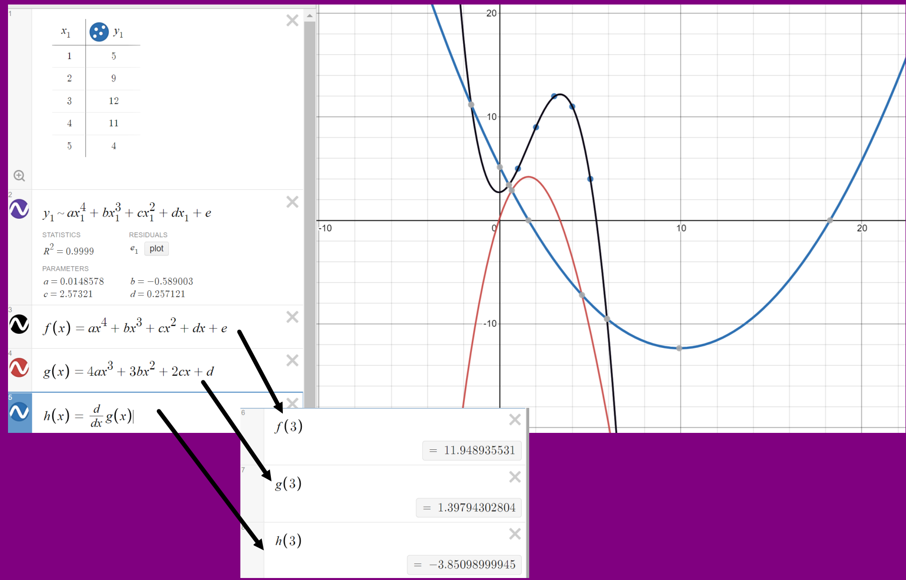


DESMOS

y=

y'=

y''=



x_1	y_1
1	5
2	9
3	12
4	11
5	4

$y_1 \sim ax_1^4 + bx_1^3 + cx_1^2 + dx_1 + e$
STATISTICS
$R^2 = 0.9999$
RESIDUALS
e_1 plot
PARAMETERS
$a = 0.0148578$
$b = -0.589003$
$c = 2.57321$
$d = 0.257121$
$f(x) = ax^4 + bx^3 + cx^2 + dx + e$
$g(x) = 4ax^3 + 3bx^2 + 2cx + d$
$h(x) = \frac{d}{dx} g(x)$
$f(3)$
$= 11.948935531$
$g(3)$
$= 1.39794302804$
$h(3)$
$= -3.85098999945$

For Posting:

Producer: Table of Values , Regression, Units

a	A	B	C

QUARTIC YEARS, VALUES, ETC.

Writer: $y(a) = A$, $y'(a) = B$ and $y'(a) = C$, include units!

Speaker: According to the quartic regression, when A happens, we end up getting B, and it is accellerating (decellerating if C is negative)

CLASS ANNOUNCEMENETS:

Make sure you are Honorlock ready