

Agenda

Review Quiz A,B - ALEKS Test

Lecture epsilon delta

Groupwork

Review

Quiz A

Question ▼	Answered Correctly
1. Using a graphing calculator to find local extrema of a polynomial function	72%
2. Union and intersection of intervals	69%
3. Finding limits for a piecewise-defined function	69%
4. Finding a limit by using the limit laws: Problem type 3	67%
5. Determining a parameter to make a function continuous	67%
Average (36 Quiz Reports)	69%

Quiz B

Question ▼	Answered Correctly
1. Finding limits from a graph	100%
2. Finding limits for a piecewise-defined function	92%
3. Finding a limit by using the limit laws: Problem type 1	96%
4. Limits at infinity and graphs	96%
5. Infinite limits and graphs	96%
Average (24 Quiz Reports)	96%

Practice ALEKS test

View: [Student Scores](#) | [Per Question Results](#) | [Detailed Student Results](#)

[All Q#1](#) [Q#2](#) [Q#3](#) [Q#4](#) [Q#5](#) [Q#6](#) [Q#7](#) [Q#8](#) [Q#9](#) [Q#10](#)

Question ▼	Answered Correctly
1. Using a graphing calculator to find local extrema of a polynomial function	75%
2. Evaluating functions: Absolute value, rational, radical	83%
3. Union and intersection of intervals	75%
4. Finding limits for a piecewise-defined function	92%
5. Finding a limit by using the limit laws: Problem type 2	92%
6. Determining a parameter to make a function continuous	92%
7. Finding limits from a graph	92%
8. Limits at infinity and graphs	92%
9. Infinite limits and graphs	92%
10. Infinite limits and graphs	92%
Average (12 Quiz Reports)	88%

Connect is Next!

Lecture : Epsilon Delta

What is Math?

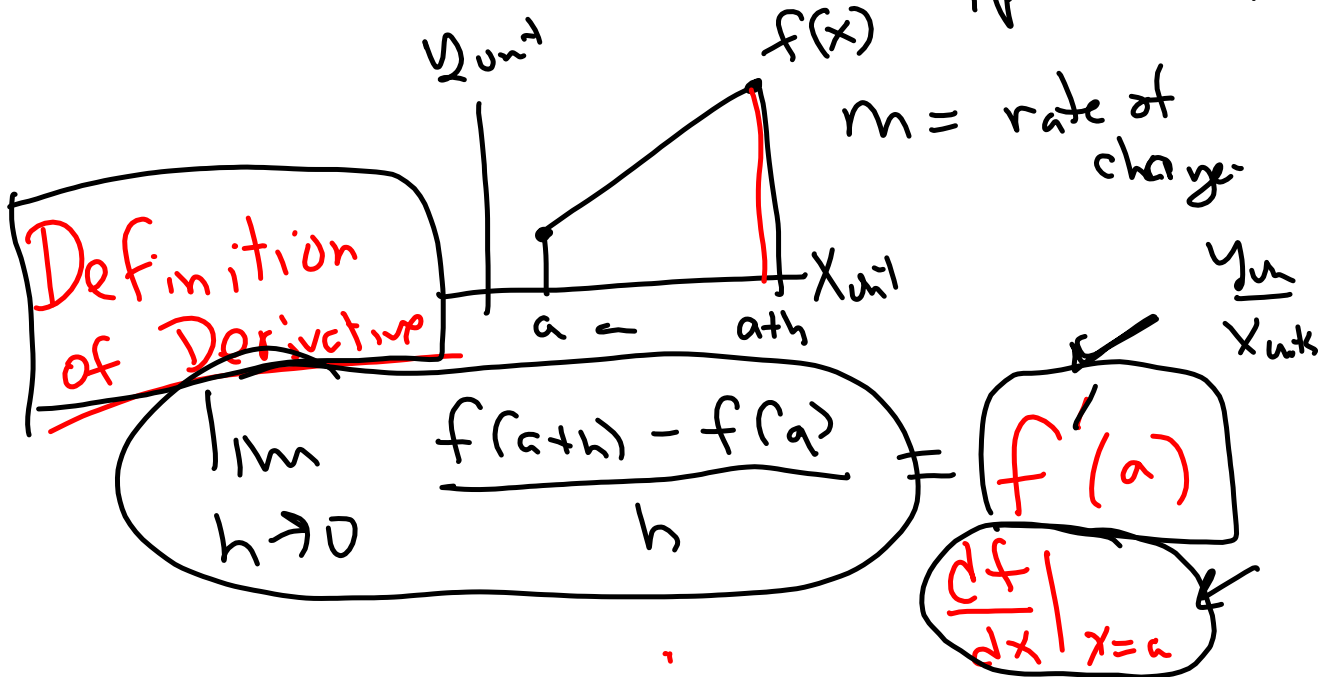
What is Calculus?

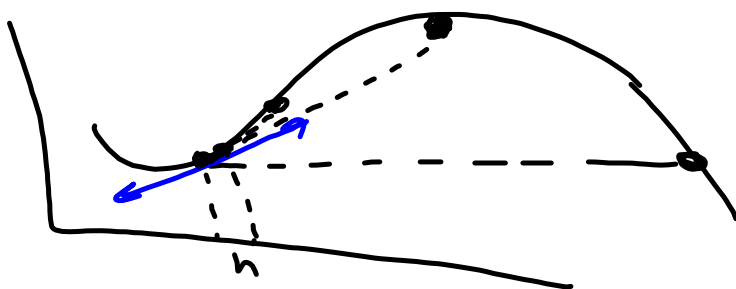
What are the two rates of change?

How do we get from two points to one point?

What is math = Language
 calculus = Study Change

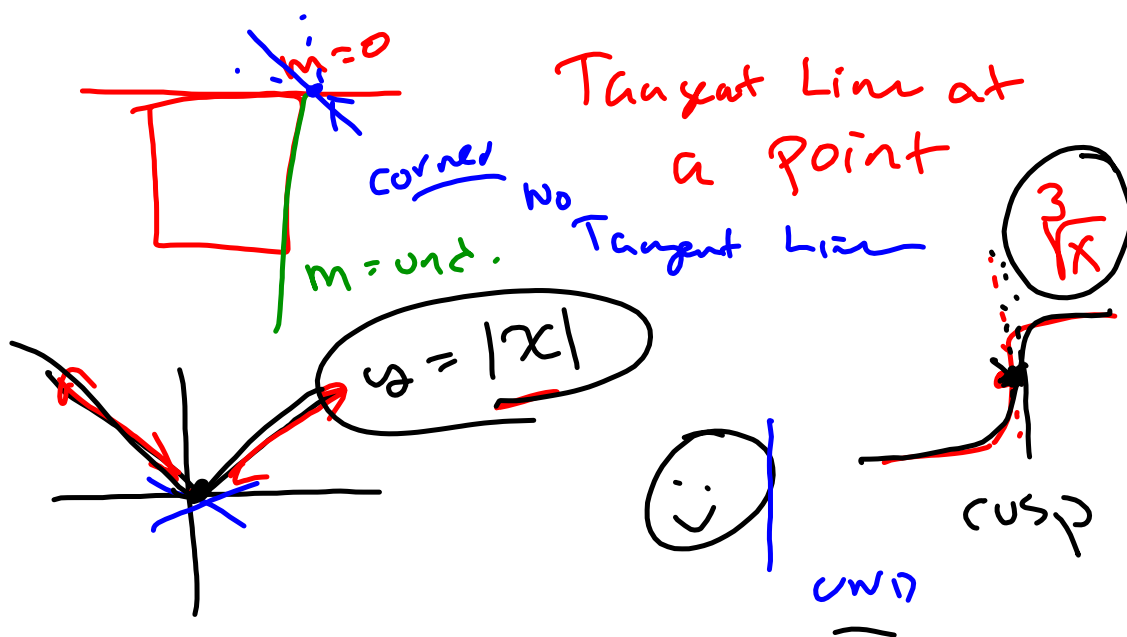
Rates of Change = inten. v. ave.
 1pt 2pts



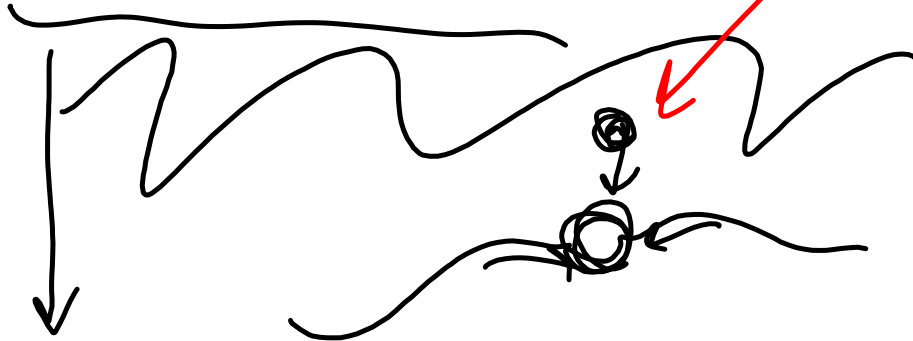


$$m_{\text{tan}} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Differentiability



Continuity



Removable Discontinuity

- ① limit exists
- ② function exists
- ③ limit = function

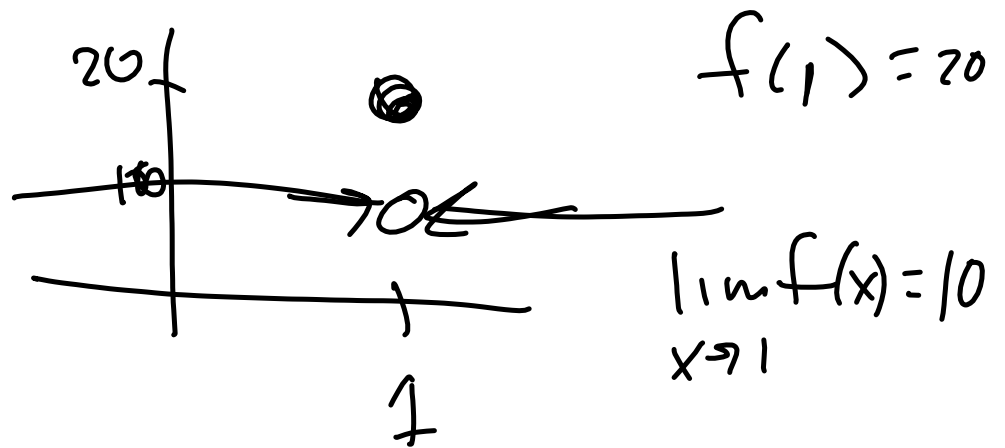
Limit

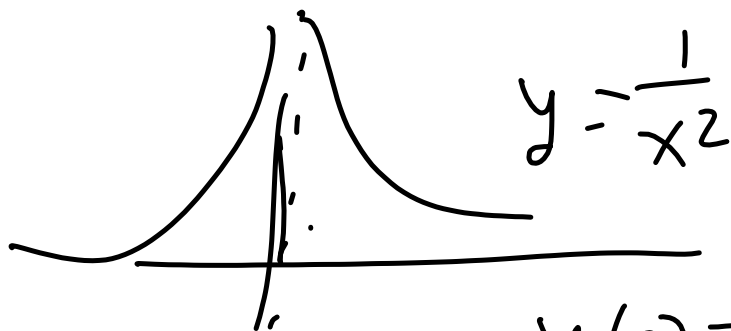
Limit

Allows $\lim_{x \rightarrow 4} \left(\frac{x-4}{x-4} \right) = 1$

\uparrow

Note





$$y = \frac{1}{x^2}$$

$$y(0) = \text{DNE}$$

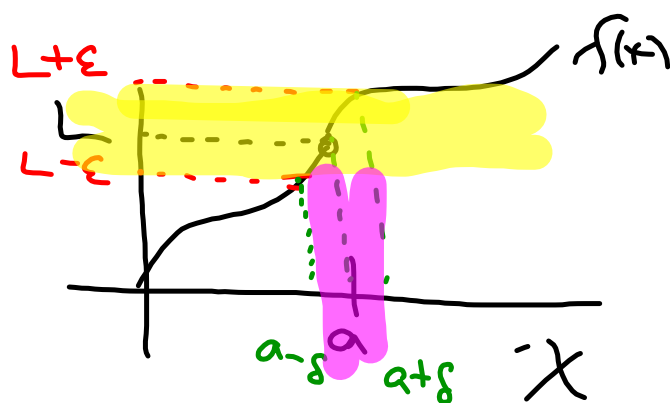
$$\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$$

ϵ - δ definition of Limit

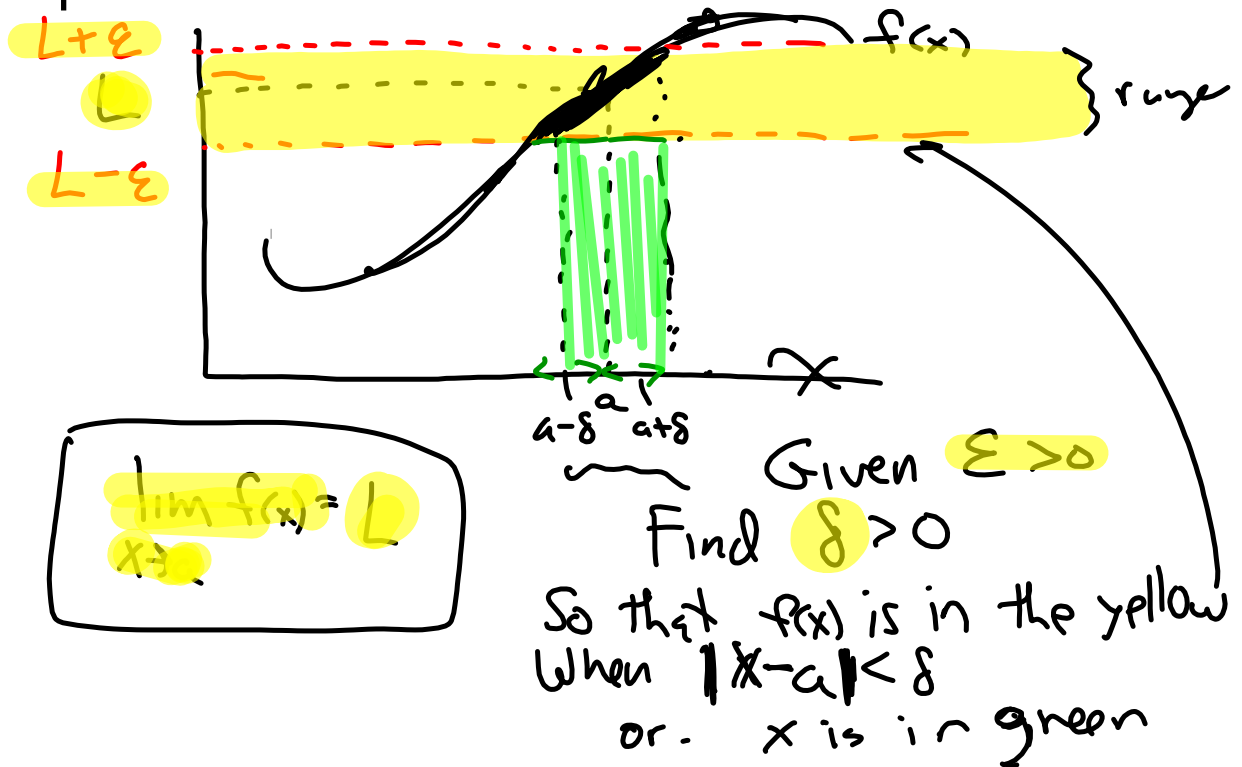
$$\lim_{x \rightarrow a} f(x) = L$$

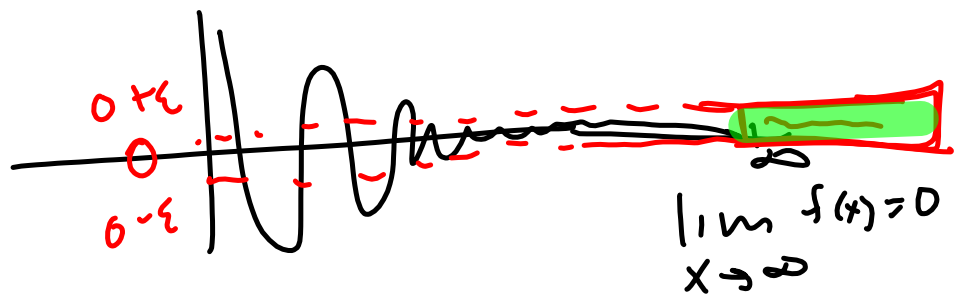
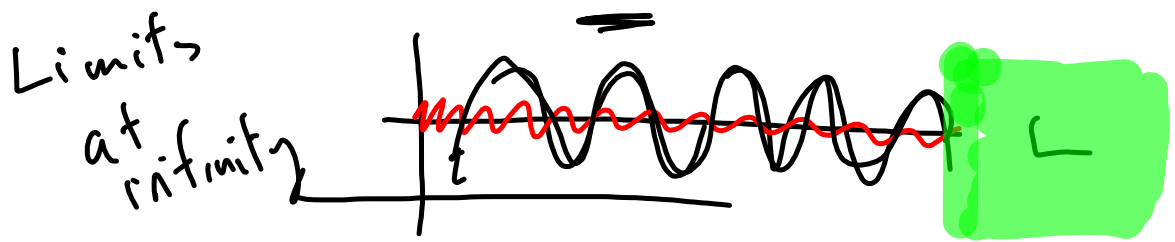
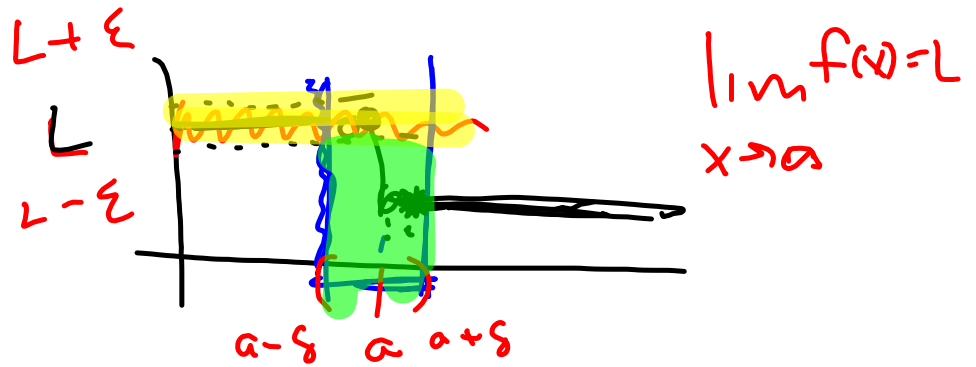
Given any $\epsilon > 0$ (usually small)
You can find a $\delta > 0$ so that

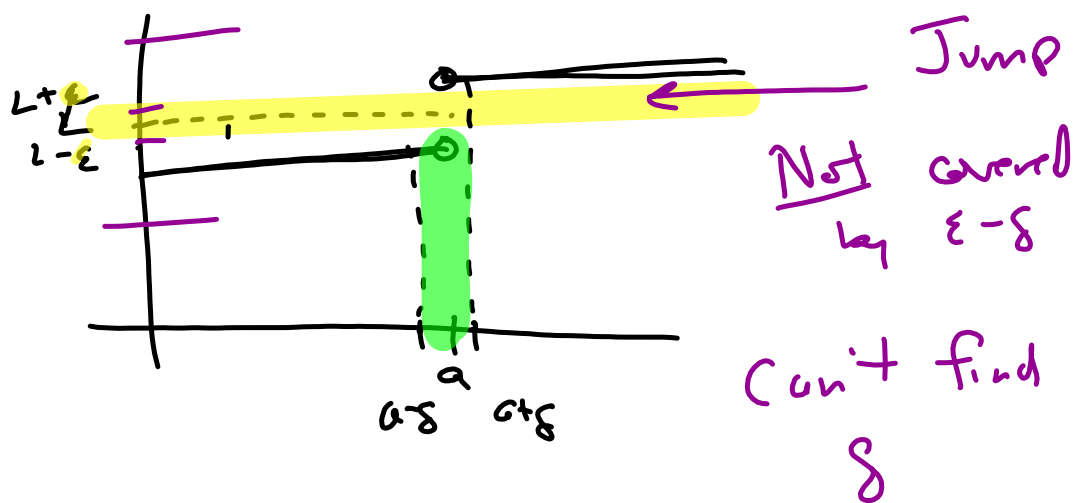
$$|f(x) - L| < \epsilon \text{ whenever } |x - a| < \delta$$



Epsilon Delta Definition







$\epsilon - \delta$
Epsilon delta

$$\lim_{x \rightarrow a} f(x) = L$$

Can Find

$$a \quad f = \frac{\quad}{.05 \cdot 15}$$

So that

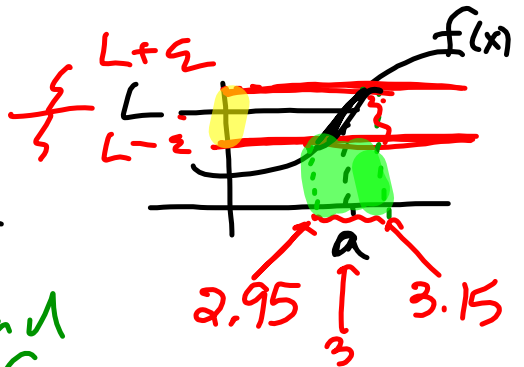
Then

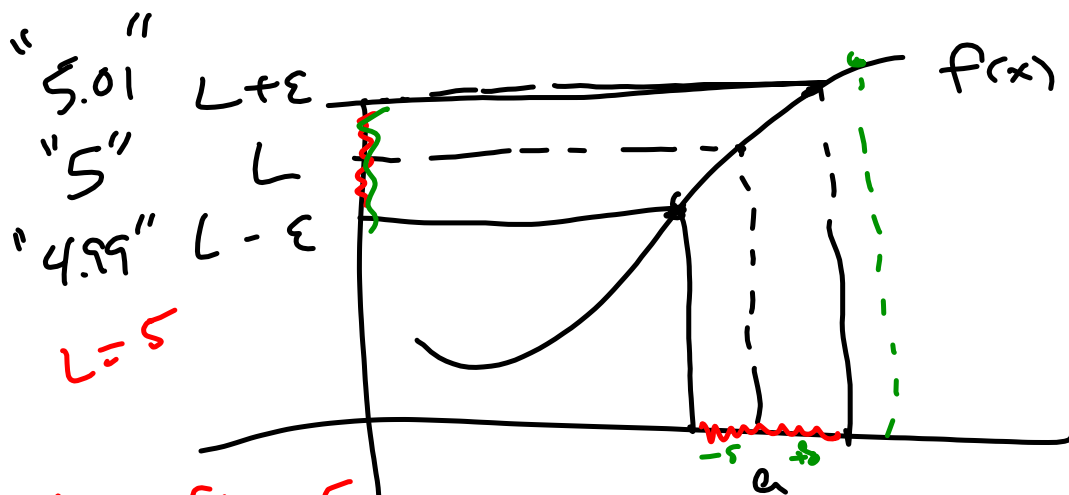
$$|f(x) - L| < \epsilon$$

In
Red
Range.

\Leftarrow distance between $f(x)$ and L value is less than ϵ

Given $\epsilon = .01$





$$\lim_{x \rightarrow 3} f(x) = 5$$

Given $\epsilon = .01$

Find $\delta = .04$

$$2.95 \quad 3 \quad 3.04$$

$$f(2.95) = 4.99$$

$$f(3.04) = 5.01$$

$$f(3 \pm .04) = 4.99 - 5.01$$

$$f(3 \pm .05) = f(3.05) =$$

$$f(3 \pm .03) \Rightarrow 4.97 - 5.01$$

Given $\epsilon = .01$
 $f(x) = 2x + 3$
 Find δ for $a = 4$

$\lim_{x \rightarrow 4} f(x) = 11 \leftarrow L$

$\lim_{x \rightarrow 4} 2x + 3 = 11$

$|f(x) - L| < \epsilon$
 $|2x + 3 - 11| < .01$

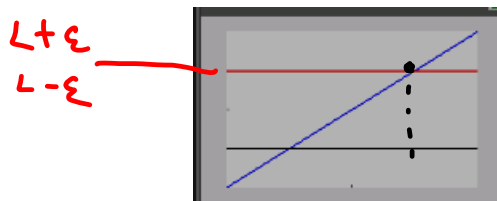
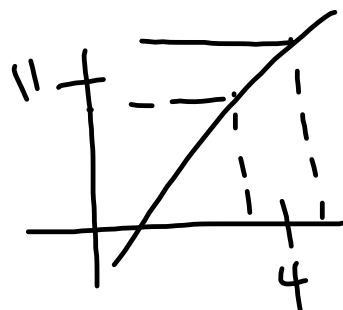
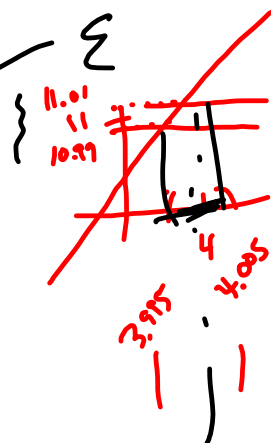
$|2x - 8| < .01$

$2|x - 4| < .01$

$|x - 4| < \frac{.01}{2} = .005 = \delta$

As long as x is between 3.995 and 4.005

Your y -value is between 10.99 and 11.01



Given $\epsilon = .03$

$$f(x) = 2x - 5$$

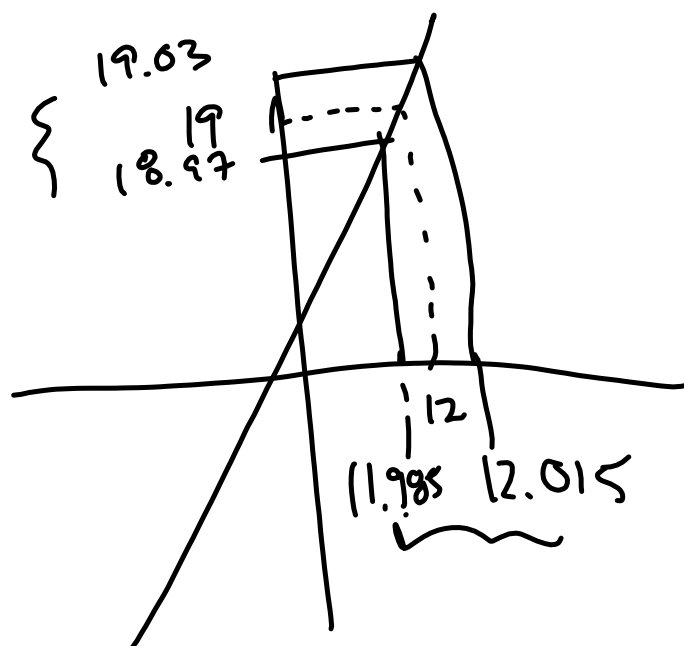
$$\lim_{x \rightarrow 12} 2x - 5 = 19$$

Find δ

$$|f(x) - L| < \epsilon$$
$$|2x - 5 - 19| < .03$$
$$|2x - 24| < .03$$

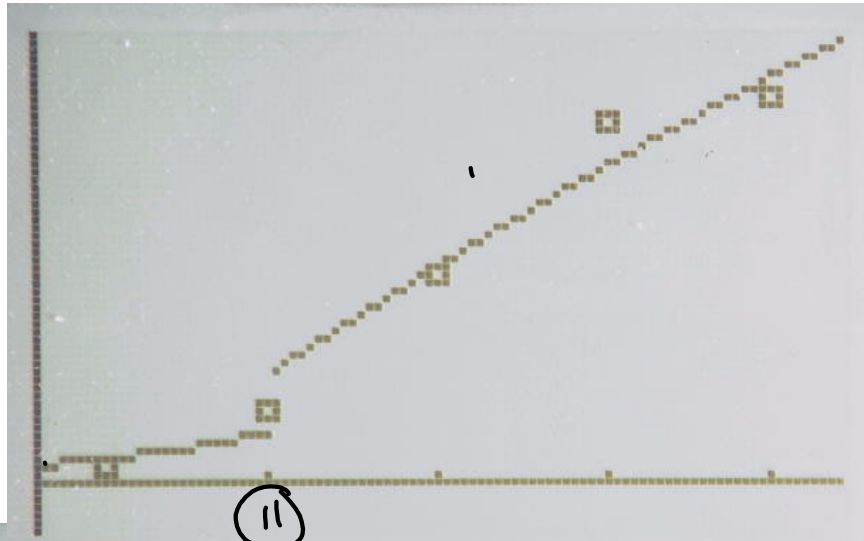
$$2|x - 12| < .03$$

$$\underline{|x - 12|} < .03/2$$



Project

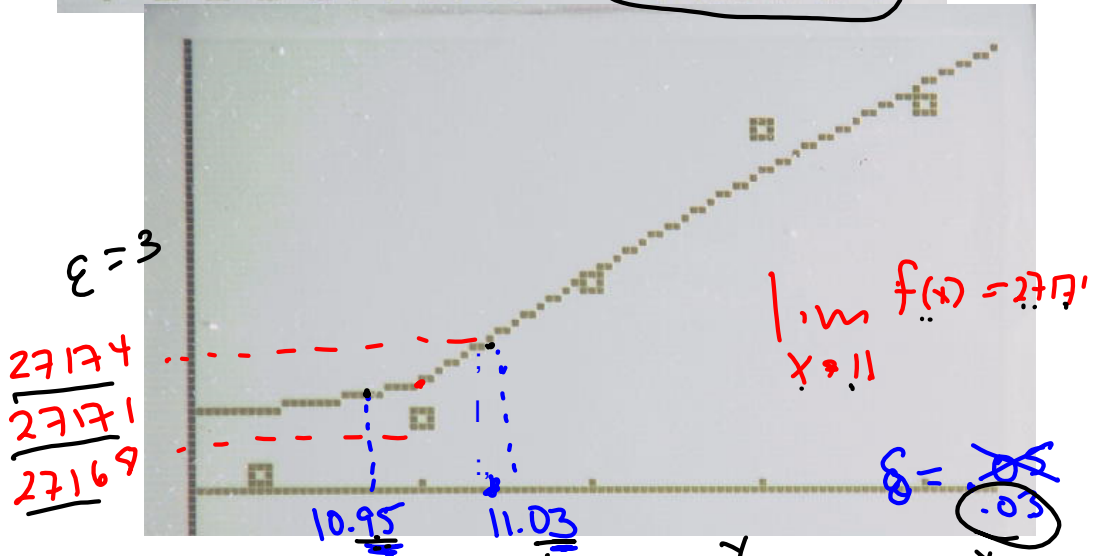
Project epsilon Delta- disc



$$Y_2(11) - Y_1(11) = \underline{14574.46...}$$

```

Plot2 Plot3
Y1 0.96129924517
059*2.3676061244
206^X/(X<11)+145
74.46729
Y2 -717189.1888
8521+310422.4497
75831n(X)/(X>11)+#
  
```



What is $f(11) =$

10	19895	ERROR
11	27171	27171
12	ERROR	54182

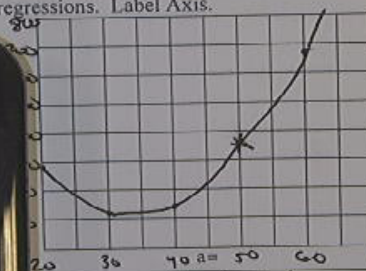
At 50,000 cases of fracking, there are about 367.56 earthquakes. This data is due to global warming and an increase in obesity (which causes more pressure on the earth's surface).

4. For a continuous regression: Given $\epsilon =$ small number Find $\delta > 0$ that satisfies
 Roughly adjust the regressions so the graph is continuous.
 Plot data and graph the regressions. Label Axis.

TI-84 Plus CE

NORMAL FLOAT AUTO REAL DEGREE CL

X	Y1	Y2
50	367.51	367.51



a=split location

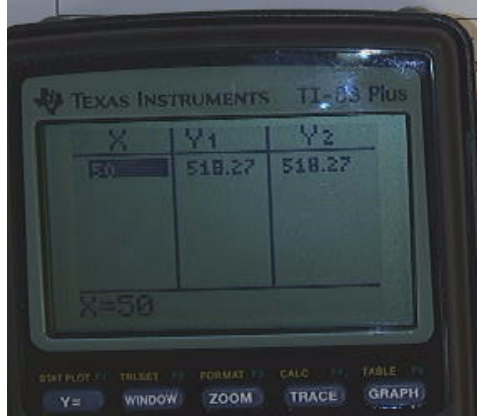
continuity

$$\lim_{x \rightarrow a} f(x) = L$$

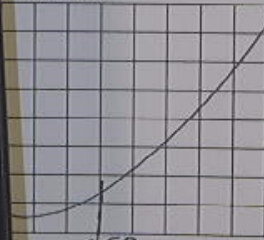
367.51

Conclusion in words:

From our data and quadratic/exponential graphs we predict there will be 518.27 earthquakes caused by 50,000 cases of fracking.



small number Find $\delta > 0$ that satisfies
the graph is continuous.
Label Axis.



a=split location

$$\lim_{x \rightarrow a} f(x) = L$$

518.27

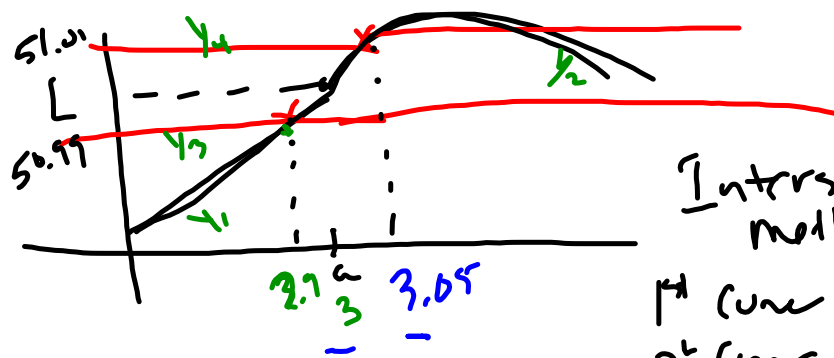
$$Y_1 = \text{res } 1 / (x \leq a)$$

$$Y_2 = \text{res } 2 / (x \geq a)$$

$$Y_3 = 51 - .01$$

$$Y_4 = 51 + .01$$

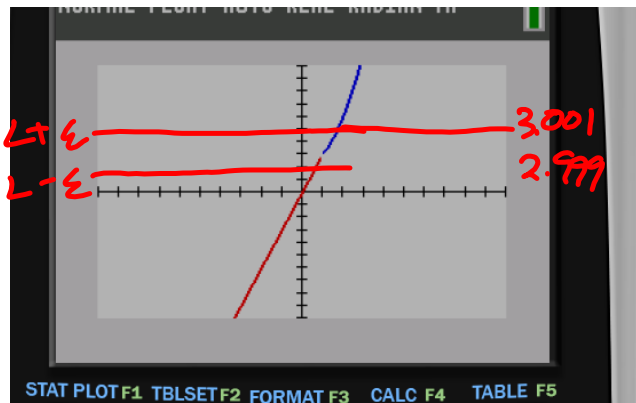
Graph



Intersection method

1 st curve	Y_1	Y_2
2 nd curve	Y_3	Y_4
Gauss	2.9	3.1

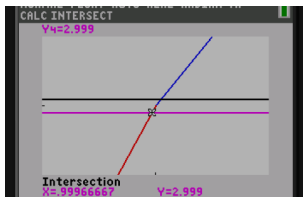
$$\lim_{x \rightarrow 1} S(x) = 3$$



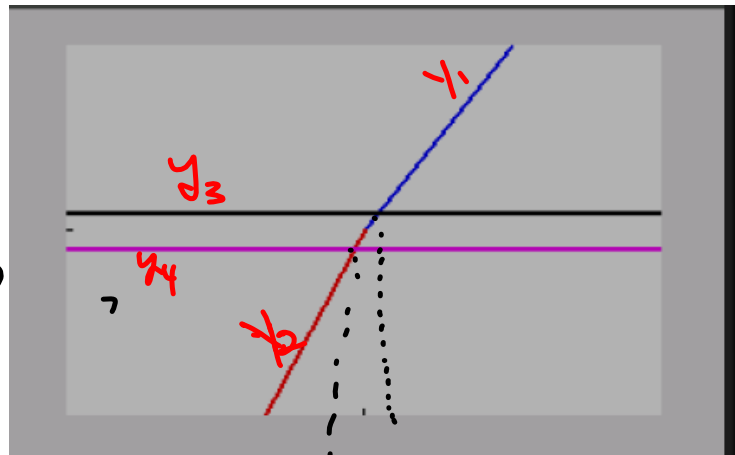
Given $\epsilon = .001$
Find δ so that

Whenever $|S(x) - 3| < .001$
 $|x - 1| < \delta$

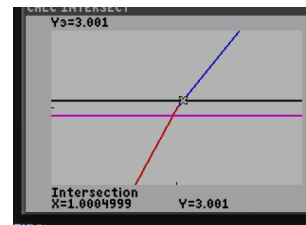
$S(x)$

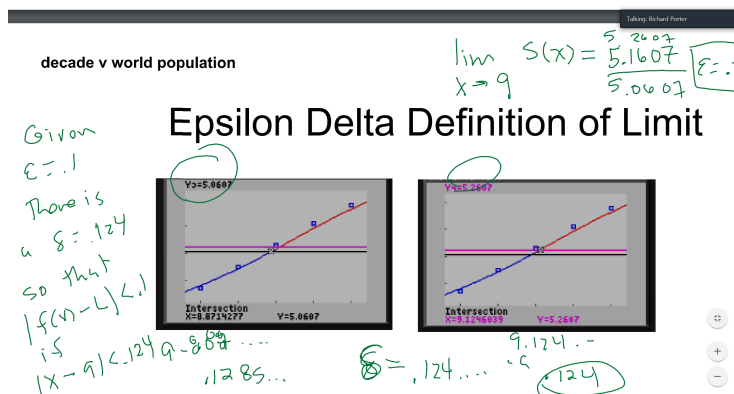


If $\epsilon = .001$
 $\delta = .0003$
 $\delta = .002$



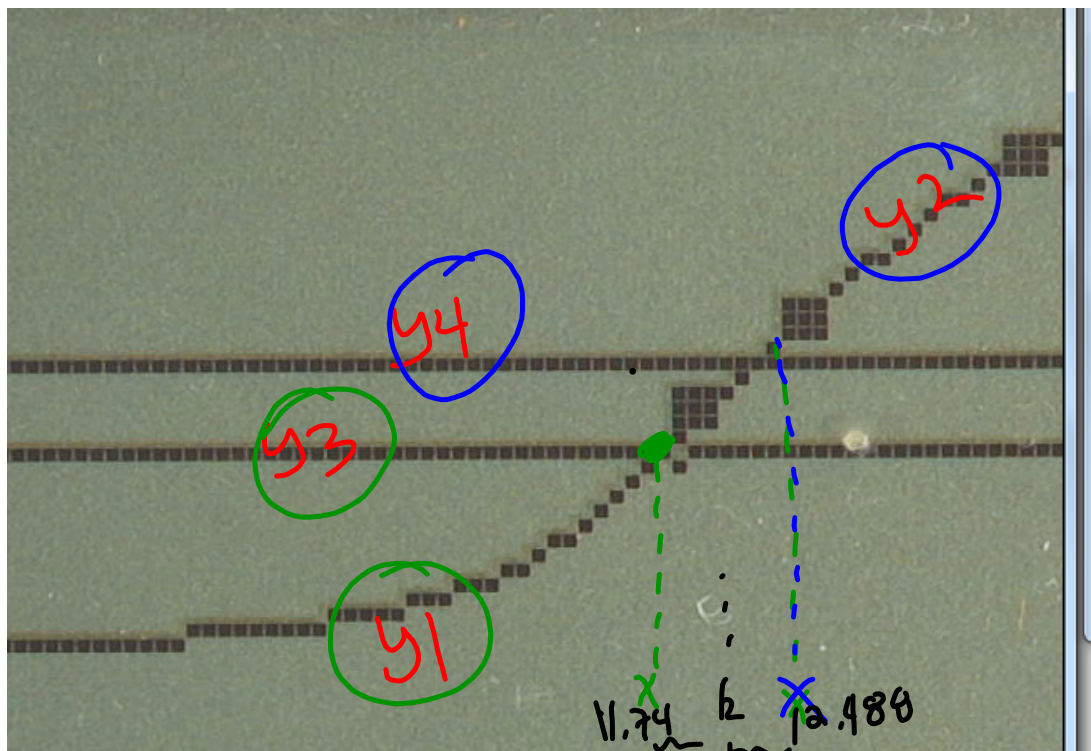
$1.0004999 - 1 = .0004999$
 $1 - .99966667 = .00033333$
 $.99966667$





As the decades approach 90's

The world population quickly
 approaches 5.16 Billion people

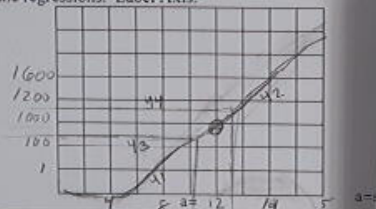


$$(11.74, 12.488)$$

$$12 - .26 \quad 12 \quad 12 + .488$$

As long as x is within .26
of 12,
 $M_n f(x)$ is within ϵ of
 L

4. For a continuous regression: Given ϵ - small number Find $\delta > 0$ that satisfies
 Roughly adjust the regressions so the graph is continuous.
 Plot data and graph the regressions. Label Axis.



regression 1 $f(x \leq a)$

2nd regression $f(x > a)$ + adjust for continuity

$L - \epsilon$ (.01) $984.01 = 100$

$L + \epsilon$ (.01) $984.01 = 100$

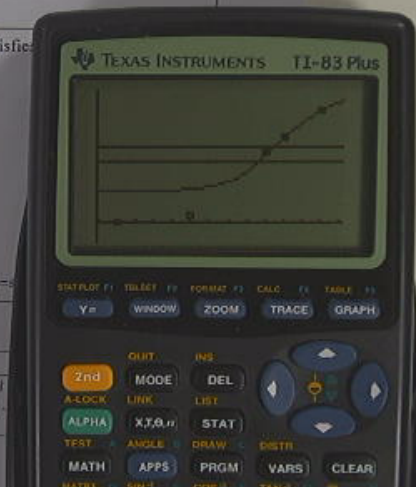
5: intersect $y1$ and $y3$ (or 4) = $x1$ 10.06

5: intersect $y2$ and $y4$ (or 3) = $x2$ 12.16

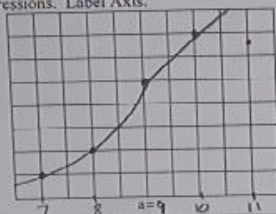
maximum $(|a - x1|, |a - x2|)$

minimum

$\lim_{x \rightarrow a} f(x) = L$	984.01
Given $\epsilon =$	100
Find $\delta =$	$12.16 - 12 = .16$ $12 - 10.06 = 1.94$

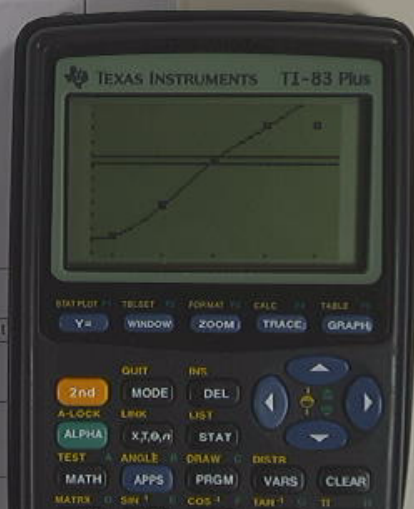


continuous regression: Given ϵ = small number Find $\delta > 0$ that satisfies
 ly adjust the regressions so the graph is continuous.
 ata and graph the regressions. Label Axis.

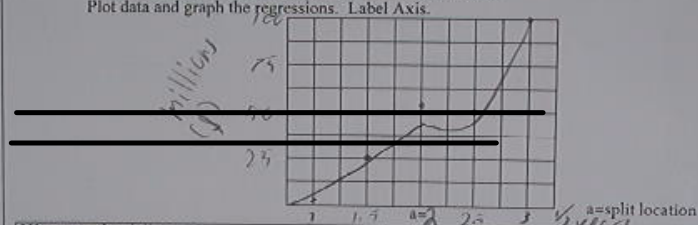


$L(3.54101)$
 $a=split\ location$

$1/(x < a)$	$\lim_{x \rightarrow a} f(x) = L$	43.54
ssion $f(x > a) + \text{adjust for continuity}$		
1) 43.51		
2) 43.57		
ct $y1$ and $y3(\text{or } 4) = x1$		8.917
ct $y2$ and $y4(\text{or } 3) = x2$		9.121
$\{[a-x1], [a-x2]\}$		
	Given $\epsilon =$.03 (0.01 as default)
	Find $\delta =$	0.083, 0.121



4. For a continuous regression: Given $\epsilon =$ small number Find $\delta > 0$ that satisfies
 Roughly adjust the regressions so the graph is continuous.
 Plot data and graph the regressions. Label Axis.



Y1=regression 1 $(x <= a)$

Y2=2nd regression $(x > a)$ + adjust for continuity

Y3 = $L + \epsilon$ (.01) 47.2117

$\lim_{x \rightarrow a} f(x) = L$

Given $\epsilon =$

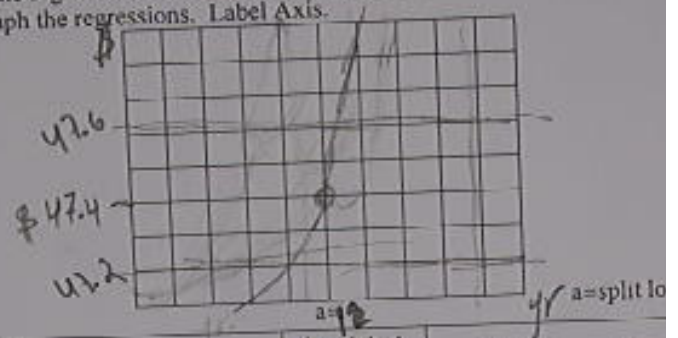
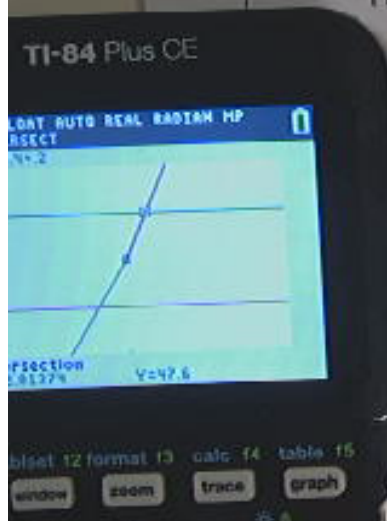
and $\delta =$

(.01 as default)

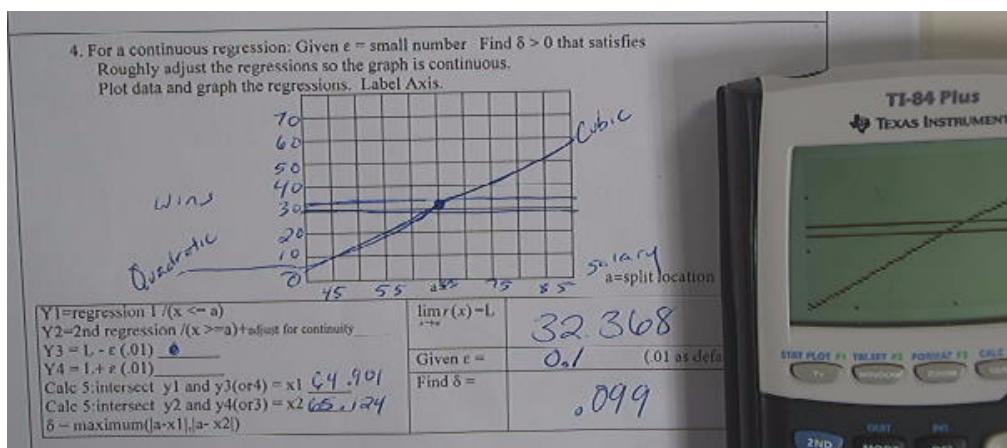
$|2 - 1.989| = .011$



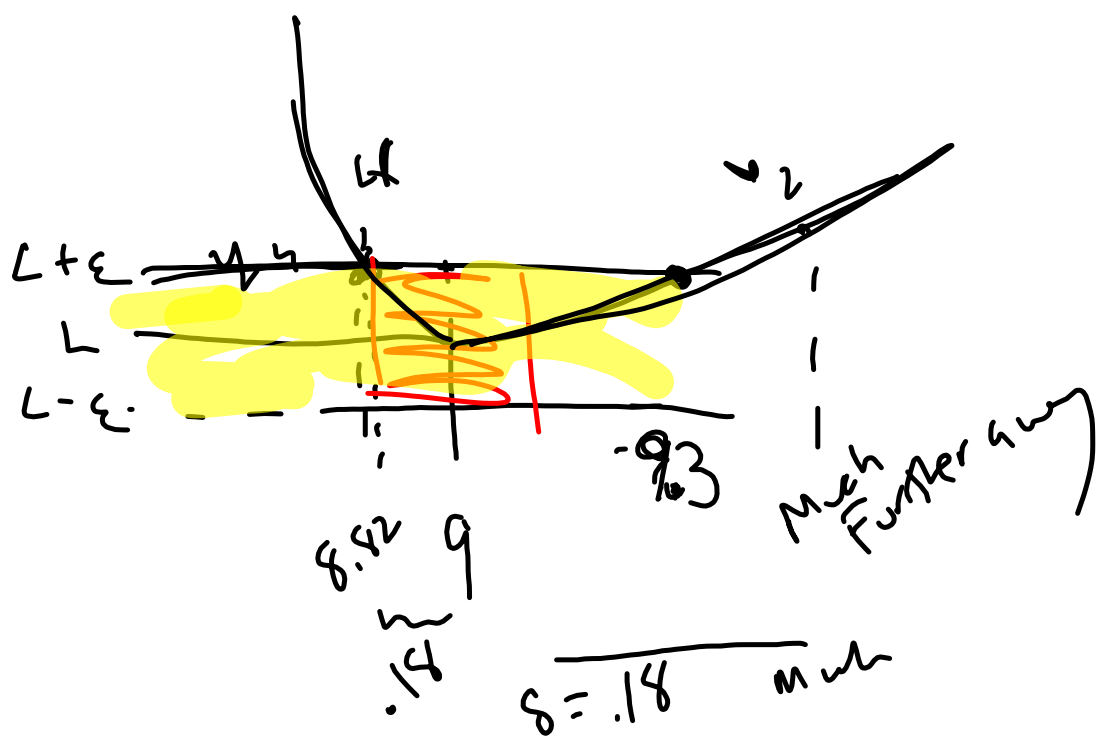
4. For a continuous regression: Given $\epsilon =$ small number Find $\delta > 0$ that satisfies
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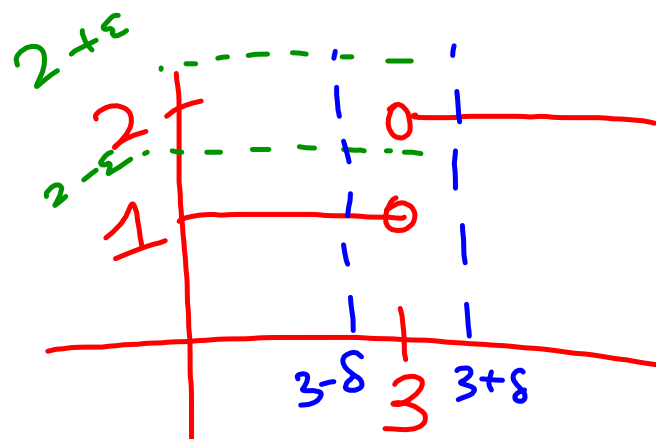


on $1/(x \leq a)$	$\lim_{x \rightarrow 0} r(x) = L$	47.4, $\delta =$
ression $/(x \rightarrow a) + \text{adjust for continuity}$		
01) <u>47.4</u>	Given $\epsilon =$	0.03 (0.1)
01) <u>47.4</u>	Find $\delta =$	0.03
ect $y1$ and $y3$ (or 4) = $x1 \frac{11.98}{12-0.13}$		
ect $y2$ and $y4$ (or 3) = $x2 \frac{12-0.13}{12-0.13}$		
$n(a-x1 , a-x2)$		

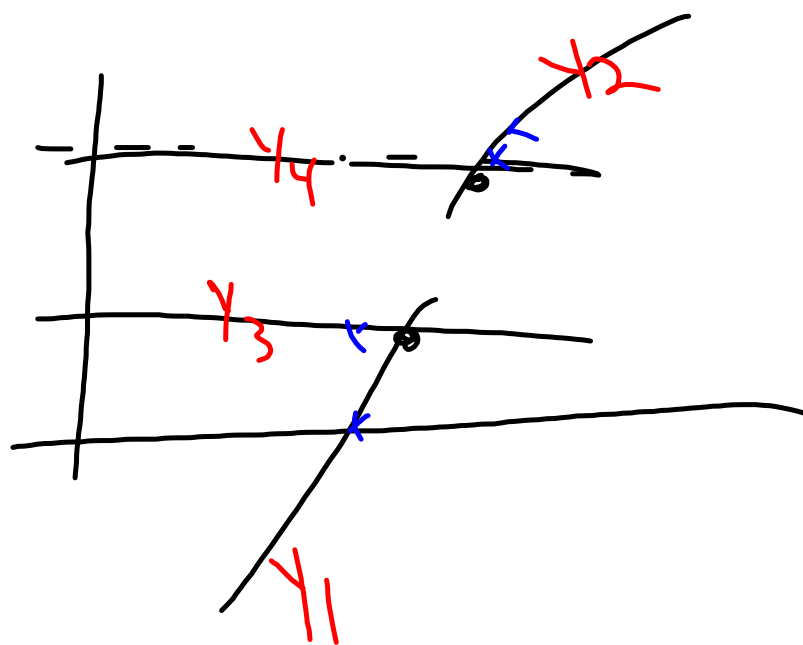


"as long as the salary is within .099 of 65K, the number of wins is within .1 of 32.368"





$$\lim_{x \rightarrow 3} f(x) = 2$$



Project Presentations

Week 1: Average Rate of Change