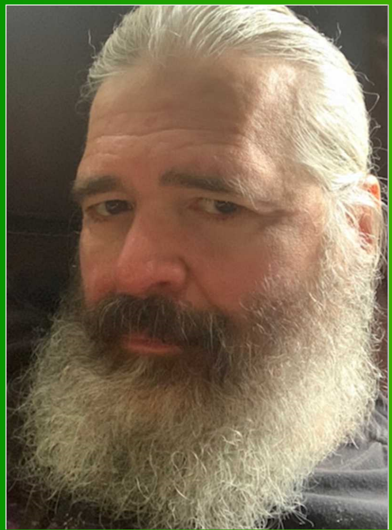
MAT 151 Calculus 1

Prof. Porter



Agenda

Homework #1 Review

Lecture Limits

Tangent Line

Groupwork

151d3

HW 1* ARC and Limits

11 / 11 questions assigned | 10.00 points

Enter student instructions (optional)

add questions | organize assignment | view: list | individually

edit pools » | set all point values... 100

question	question type		points
Sec. Ex. 14 - Limits	Multipart Answer	a	0.90
Sec. Ex. 7 - 1.2.7	Multipart Answer		0.90
Sec. Ex. 8 - 1.2.8	Multipart Answer		0.90
Sec. Ex. 5 - 1.3 Section Exercise 5	Multipart Answer	a	0.90
Sec. Ex. 13 - Limits	Multipart Answer	a	0.90
Sec. Ex. 17 - Limits	Multipart Answer	a	0.90
Sec. Ex. 39 - 1.4 Section Exercise 39	Multipart Answer	a	0.90
Sec. Ex. 11 - 1.5 Section Exercise 11	Multipart Answer	a	0.90
Concept - 1.5.	Multipart Answer		0.90
Example 5a - 2.1 Example 5a	Multipart Answer	a	0.90
Example 6a - 2.1 Example 6a	Multipart Answer	a	1.00

Question 1 (of 10) — Sec. Ex. 14 - Limits

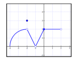
This is an algorithmic question.

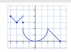
Sec. Ex. 14 - Limits

3 attempts left

Evaluate the indicated limit.

$$\lim_{x \to 256} \frac{x - 256}{\sqrt{x} - 16} = \square$$





Question 4 (of 10) — Sec. Ex. 5 - 1.3 Section Exercise 5

Question 5 (of 10) — Sec. Ex. 13 - Limits

This is an algorithmic question. what's this?

Sec. Ex. 13 - Limits

3 attempts left | Check my work

Use numerical and graphical evidence to conjecture value for the limit.

$$\lim_{x \to 0} \frac{2x^2 + 2x}{\sin(x)}$$

- 0
- 1
- 3
- 2

Question 6 (of 10) — Sec. Ex. 17 - Limits

This is an algorithmic question. what's this? see another version

Sec. Ex. 17 - Limits

3 attempts left | Check my work

Suppose a state's income tax code states that tax liability is 10% on the first \$18000 of taxable earning and 11% on the remainder. Find constants a and b for the tax function

$$T(x) = \begin{cases} a + 0.1x & x \le 18000 \\ b + 0.11(x - 18000) & x > 18000 \end{cases}$$

such that $\lim_{x \to 0^+} T(x) = 0$ and $\lim_{x \to 18000} T(x)$ exists.

$a = \square$, $b = \square$

Question 8 (of 10) — Sec. Ex. 11 - 1.5 Section Exercise 11

This is an algorithmic question. what's this? see another version

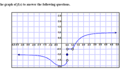
Sec. Ex. 11 - 1.5 Section Exercise 11

3 attempts left | Check my work

Determine the limit (answer as appropriate, with a number, ∞, $-\infty$ or does not exist).

$$\lim_{x \to -\infty} \frac{-8x}{\sqrt{169 + x^2}} = \square$$

Question 9 (of 10) — Concept - 1.5



Question 10 (of 10) — Example 5a - 2.1 Example 5a

This is an algorithmic question. what's this? see another version

Example 5a - 2.1 Example 5a

3 attempts left | Check my work

Suppose that the height of a falling object t seconds after being dropped from a height of given by $f(t) = 279 - 16t^2$ feet. Find the average velocity between times $t = 3$ and $t = 4$.

$\square$ ft/s

Question 11 (of 11) — Example 6a - 2.1 Example 6a

This is an algorithmic question. what's this? see another version

Example 6a - 2.1 Example 6a

3 attempts left | Check my work

If the function $f(t)$ gives the population of a city in millions of people t years after January 1, 1998 interpret the following quantity, assuming that it equals the given number.

$$\frac{f(2) - f(0)}{2} = 0.38$$

- The city's population grew at an average rate of 0.38 million people per year between 1998 and 2000.
- The city's population is 0.38 million people in 2000.
- The city's population grew at an average rate of 0.76 million people per year between 1998 and 2000.
- The city's population is 0.76 million people in 2000.

What is Math?

What is precalculus?

What is Calculus?

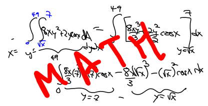
What are the rates of change?

Lecture:

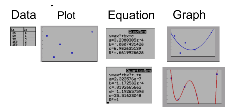
- What is Math? Language
- What is precalculus? Functions
- What is Calculus? Study of Change
- What are the rates of change?

What is math?

Language



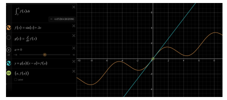
What is Precalculus?



The Study of Functions

What is Calculus?

The Study of Change



What is Average Rate of Change?

How many points are involved?

What is the instantaneous Rate of Change?

How many point(s) are involved/

What mathematics was invented to take two points to one?

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Precise Definition of Limit

ε-δ epsilon-delta definition

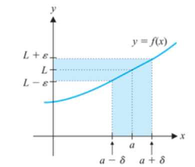
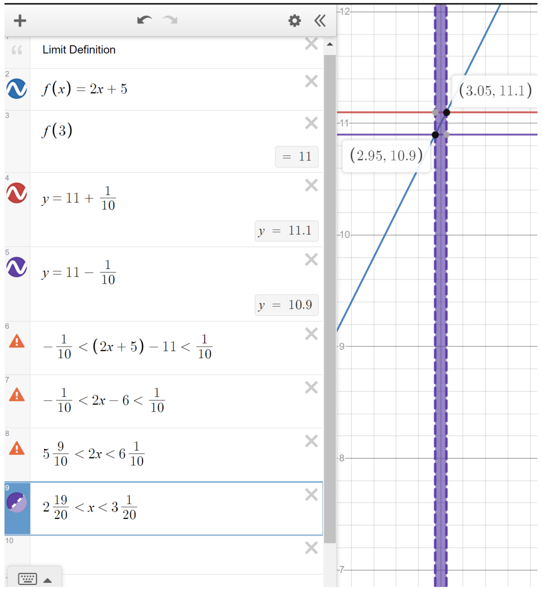


FIGURE 1.47
$a - \delta < x < a + \delta$ guarantees that $L - \varepsilon < f(x) < L + \varepsilon$.

The epsilon-delta definition of limits says that the limit of f(x) at x=c is L if for any ε>0 there's a δ>0 such that if the distance of x from c is less than δ, then the distance of f(x) from L is less than ε. This is a formulation of the intuitive notion that we can get as close as we want to L.

$$\lim_{x \to c} f(x) = L,$$

means that given any $\epsilon > 0$, there exists $\delta > 0$ such that for all $x \neq c$, if $|x - c| < \delta$, then $|f(x) - L| < \epsilon$.


Limit Definition
f(x) = 2x + 5
f(3)
= 11
y = 11 + 1/10
y = 11.1
y = 11 − 1/10
y = 10.9
−1/10 < (2x + 5) − 11 < 1/10
−1/10 < 2x − 6 < 1/10
5 9/10 < 2x < 6 1/10
2 19/20 < x < 3 1/20
(3.05, 11.1)
(2.95, 10.9)
12
11
10
9
8
7

Limits at Infinity

For a function *f* defined in some open interval containing *a* (but not necessarily at *a* itself), we say

$$\lim_{x \to a} f(x) = \infty,$$

if given any number *M* > 0, there is another number δ > 0, such that 0 < |*x* − *a*| < δ guarantees that *f*(*x*) > *M*. (See Figure 1.53 for a graphical interpretation of this.)

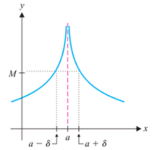


FIGURE 1.53
$\lim_{x \to a} f(x) = \infty$

For a function *f* defined in some open interval containing *a* (but not necessarily at *a* itself), we say

$$\lim_{x \to a} f(x) = -\infty,$$

if given any number *N* < 0, there is another number δ > 0, such that 0 < |*x* − *a*| < δ guarantees that *f*(*x*) < *N*. (See Figure 1.54 for a graphical interpretation of this.)

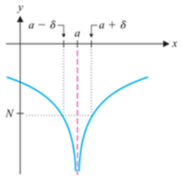


FIGURE 1.54
$\lim_{x \to a} f(x) = -\infty$

For a function *f* defined on an interval (*a*, ∞), for some *a* > 0, we say

$$\lim_{x \to \infty} f(x) = L,$$

if given any ε > 0, there is a number *M* > 0 such that *x* > *M* guarantees that

$$|f(x) - L| < \varepsilon.$$

(See Figure 1.55 for a graphical interpretation of this.)

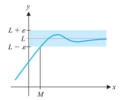


FIGURE 1.55
$\lim_{x \to \infty} f(x) = L$

EX:

Symbolically find a $\delta > 0$ in terms of ε for $\lim_{x \to 3} (1 + x/2) = 2.5$.

a f(x) L

$| f(x) - L | < \varepsilon$

$| 1 + x/2 - 2.5| < \varepsilon$

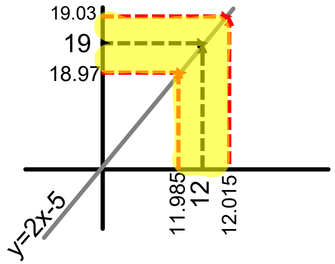
$| x/2 - 1.5| < \varepsilon$

$(1/2)\ | x - 3| < \varepsilon$

$| x - 3| < \boxed{2\varepsilon = \delta}$

EX: Given epsilon = .03 and f(x) = 2x -5 and x approaches 12.

$\lim_{x \to 12} 2x-5 = 19$ find δ.



$| f(x) - L | <$ epsilon

$| 2x - 5 - 19 | < .03$

$| 2x - 24 | < .03$

$2 | x - 12 | < .03$

$|x - 12| < .03/2 = .015 = \delta$

Tangent Line

slope of the tangent line

is the derivative

is the instantaneous rate of change

is the velocity

is dy/dx

can be represented as a value or a function

EX:

Function(x)	Value (3)
$f(x) = 2x^2-5x+7$	f(3)=2*9-5*2+7=15
$m(x)= 4x - 5$	m(3) = 4*3-5 = 7

m(x) is the function that will tell you the slope of the tangent line at a specific point x=a. So m(a) is that value. Sometimes refer to 'a' as a value, not a variable.

In this example, we have both the point on the graph (3,15)

and the slope of the tangent line m=7

How to find equation of tangent line using the definition:

$$m(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x+h) = 2(x+h)^2 - 5(x+h) + 7$

$= 2x^2 + 4xh + 2h^2 - 5x - 5h + 7$

$- f(x) = -2x^2 + 5x - 7$

$$\frac{f(x+h) - f(x)}{h} = 4x + 2h - 5$$

$$\lim_{h \to 0} 4x + 2h - 5 = 4x - 5$$

$m(3) = 7$

$f(3) = 15$

Point (3,15) Slope 7

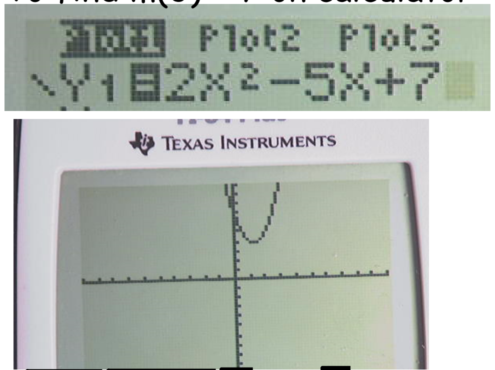
Point Slope Formula of line:

$y - y_0 = m(x - x_0)$

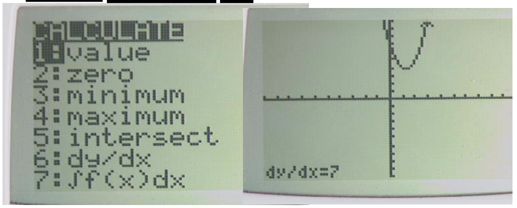
$y - 15 = 7(x - 3)$

*Don't do this unless asked to. There are easier ways!

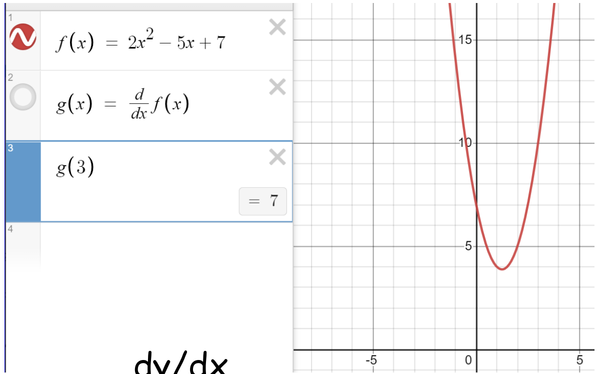
To find m(3) = 7 on calculator:



2nd Trace 6 3



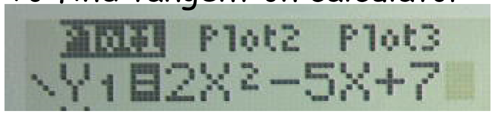
To find m(3) = 7 on DESMOS



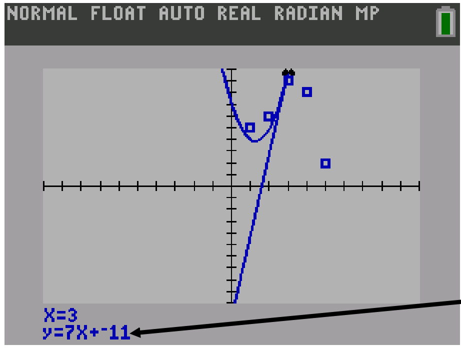
dy/dx

or d/dx are derivatives

To find tangent on calculator:

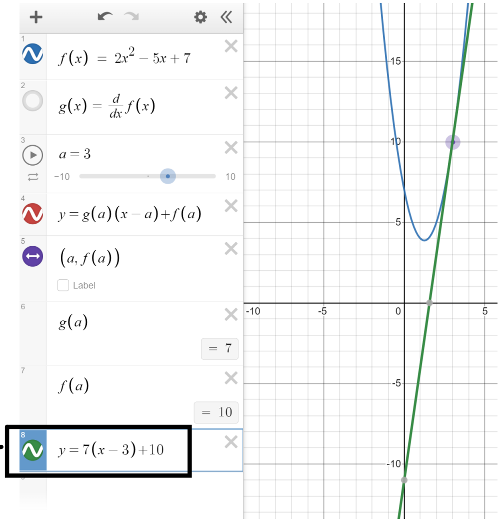


graph Draw 5:Tangent 3



draw is 2nd Prgm

To find m(3) = 7 on DESMOS



EX: Find the derivative for f(x) = 1/ (x-3)

If $f(x) = \frac{1}{x-3}$

$f(a+h) = \frac{1}{a+h-3}$ and $f(a) = \frac{1}{a-3}$

$f(a+h)-f(a) = \frac{1}{a+h-3} - \frac{1}{a-3} = \frac{a-3-a-h+3}{(a-3)(a+h-3)}$

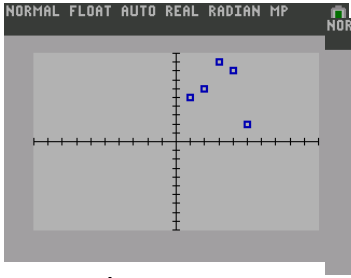
$\lim_{h \to 0} \frac{(a+h)-f(a)}{h} =$

$\lim_{h \to 0} \frac{-h}{h(a-3)(a+h-3)} =$

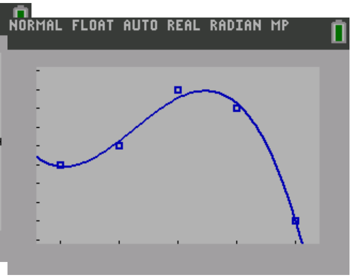
$\lim_{h \to 0} \frac{-1}{(a-3)(a+h-3)} =$

Then dy/dx = $\frac{-1}{(a-3)^2}$

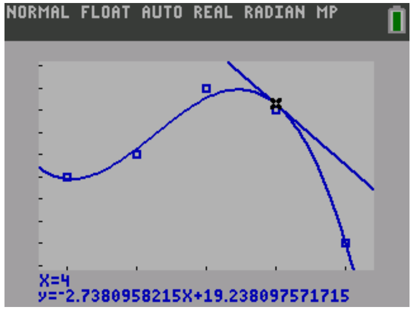
Tangent Line on Data Regression



Data



Regression



Tangent Line

Groupwork : Limits

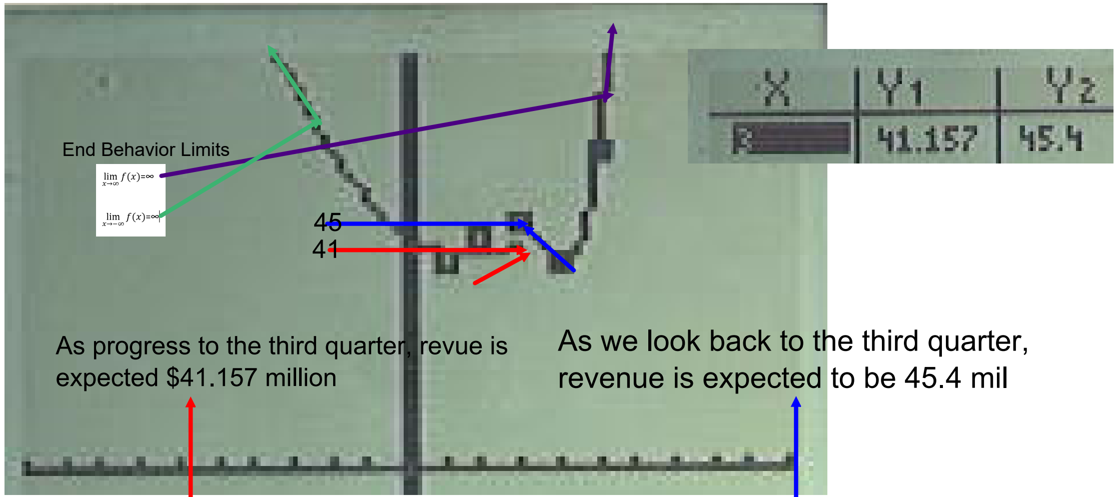
Find four limits at a point of Interests 'a'

$$\lim_{x \to a^-} f(x)$$

$$\lim_{x \to \infty} f(x)$$

$$\lim_{x \to a^+} f(x)$$

$$\lim_{x \to -\infty} f(x)$$



$$\lim_{x\to 3^-} f(x) = \$41\text{mil} \qquad \lim_{x\to 3^+} f(x) = \$45\text{mil}$$

Try This yourself

$$\lim_{x \to 75^+} f(x) = \text{368,000 cases}$$

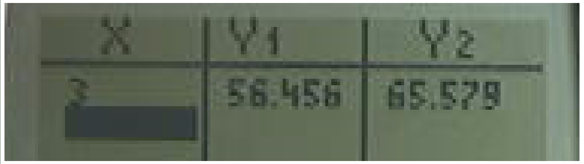
$$\lim_{x \to 75^-} f(x) = \text{434,000cases}$$

As we approach 75 years of age, there will be ≈ 434.38 thousand cases of breast cancer.

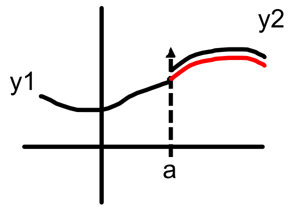
End Behavior Limits

$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x) = \infty$$


X
Y1
Y2
3
56.456
65.578

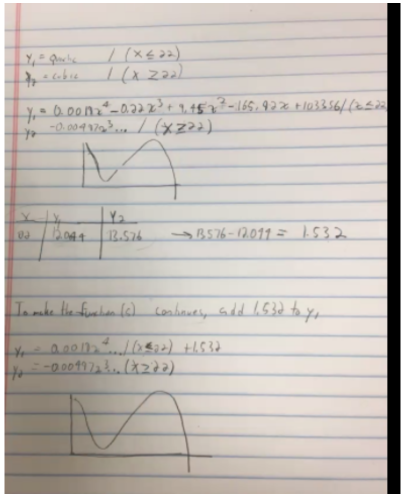
Could make the function continuous (optional)

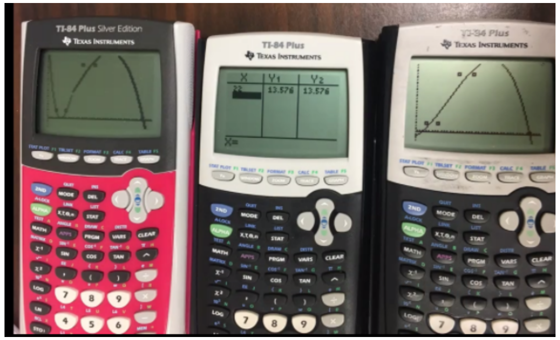


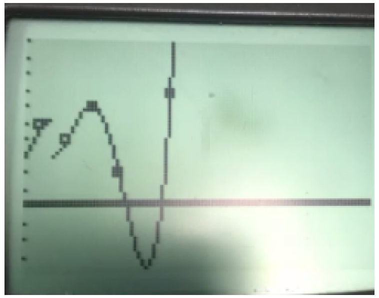
y1=regression 1

y2=regression 2

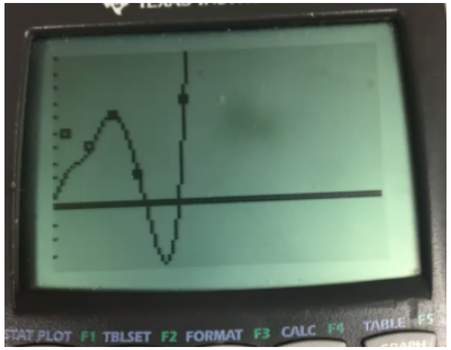
NEW y2=regression 2 - difference y2(a)-y1(a)


Y1 = quartic / (x ≤ 22)
Y2 = cubic / (x ≥ 22)
Y1 = 0.0012x^4 – 0.22x^3 + 9.45x^2 – 165.92x + 1033.56 / (x ≤ 22
Y2 -0.00497x^3... / (x ≥ 22)
X Y1 Y2
22 12.044 13.576
→ 13.576 – 12.044 = 1.532
To make the function (c) continues, add 1.532 to Y1
Y1 = 0.0012x^4... / (x ≤ 22) +1.532
Y2 = -0.00497x^3... (x ≥ 22)

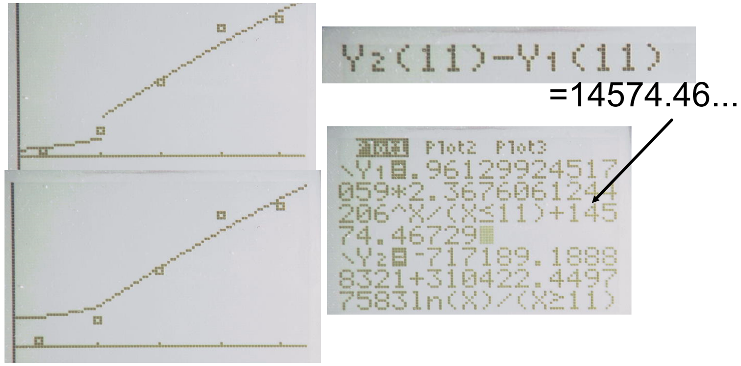

TI-84 Plus Silver Edition
TEXAS INSTRUMENTS
TI-84 Plus
TEXAS INSTRUMENTS
X Y1 Y2
13.576 13.576
TI-84 Plus
TEXAS INSTRUMENTS



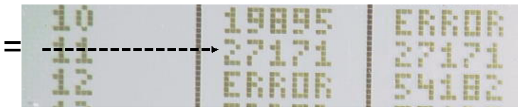
According to the split regression as we approach 250,000 in property costs, the number of available homes is 5.3402

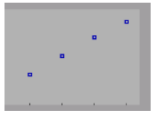


According to the continuous regression, as we approach 250,000 in property costs, the number of available homes is 5.3402

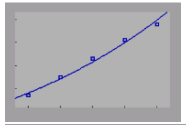

Y2(11)-Y1(11)
=14574.46...
Plot1 Plot2 Plot3
\Y1=.96129924517
059*2.3676061244
206^X/(X≤11)+145
74.46729
\Y2=-717189.1888
8321+310422.4497
75831n(X)/(X≥11)

What is f(11) =


10 19895 ERROR
11 27171 27171
12 ERROR 54182

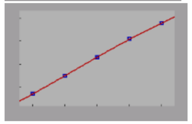


ExpReg
y=a*b^x
a=1.312342647
b=1.164322178
r²=.9881140124
r=.9940392409



X	Y1
5	2.8081
10	6.0087
11.5	7.549
11.7	7.7822
12	8.1456

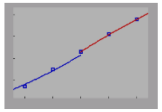
Logistic
y=c/(1+ae^(-bx))
a=17.26455086
b=.3417641063
c=9.53927036



X	Y2
5	2.3118
10	6.0911
11.5	7.124
11.7	7.2452
12	7.419

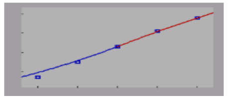
decade v world population

Split Regressions



X	Y1	Y2
0	1.3123	ERROR
5	2.8081	ERROR
9	5.1607	5.3092
10	ERROR	6.0911
11.7	ERROR	7.2452
12	ERROR	7.419

Limits of Continuous Functions



X	Y1	Y2
0	1.4608	ERROR
5	2.9566	ERROR
9	5.3092	5.3092
10	ERROR	6.0911
11.7	ERROR	7.2452
12	ERROR	7.419

According to the continuous regression above, as we approach 1990, we expect world populatioin to be 5.3 billion

Reminders....

STOP

1. Go to Blackboard
2. Post your Limit Groupwork on Discussion Forum
3. Set up Connect by opening an assignment

You <u>must</u> complete a Connect assignment in first two weeks or you will be dropped from class