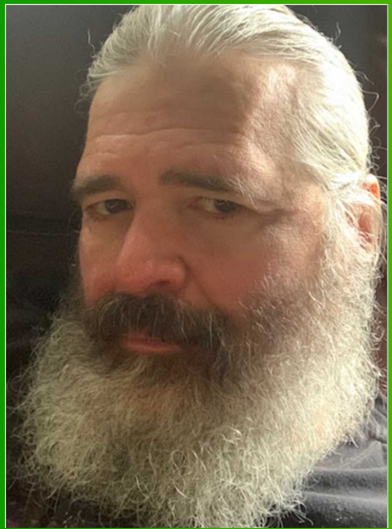
MAT 151 Calculus 1

Prof. Porter



Agenda

Homework Questions

Lecture on Limits

Group Work

151d2

Homework Questions

On odd days I will go over most homework questions, on even days I will just answer specific questions.

Lecture: Limits

What is Math = ?

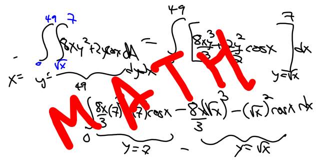
What is Calculus =?

What is the average rate of change?

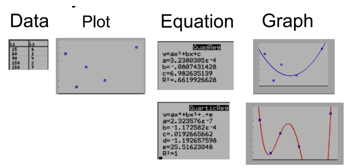
What is the instantaneous rate of change?

What is math?

Language



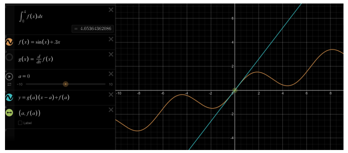
What is Precalculus?



The Study of Functions

What is Calculus?

The Study of Change



What is Average Rate of Change?

How many points are involved?

What is the instantaneous Rate of Change?

How many point(s) are involved/

Limits

a limit is the value that a function f(x) approaches as the input 'x'approaches some value 'a.'

slope of the secant line (m_{sec}) between 'a' and 'a+h' is

$$m_{sec} = \frac{f(a+h)-f(a)}{a+h-a} = \frac{f(a+h)-f(a)}{h}$$

We use limits to find the instantaneous rate of change, you'll want your input 'h' value to go to zero. That means we will be dividing by zero, which won't be a problem here

Limit Notation

a limit is the value that a function f(x) approaches as the input 'x'approaches some value 'a.'

$$\lim_{x \to a} f(x) = L$$

said: "the limit as 'x' goes to 'a' of 'f' of 'x' is 'L' "

NOTE: The function does NOT have to exists at 'a'.

Evaluating a Limit

$$\lim_{x \to a} f(x) = L$$

1. Try just plugging 'a' into f(x), if it works L = f(a)

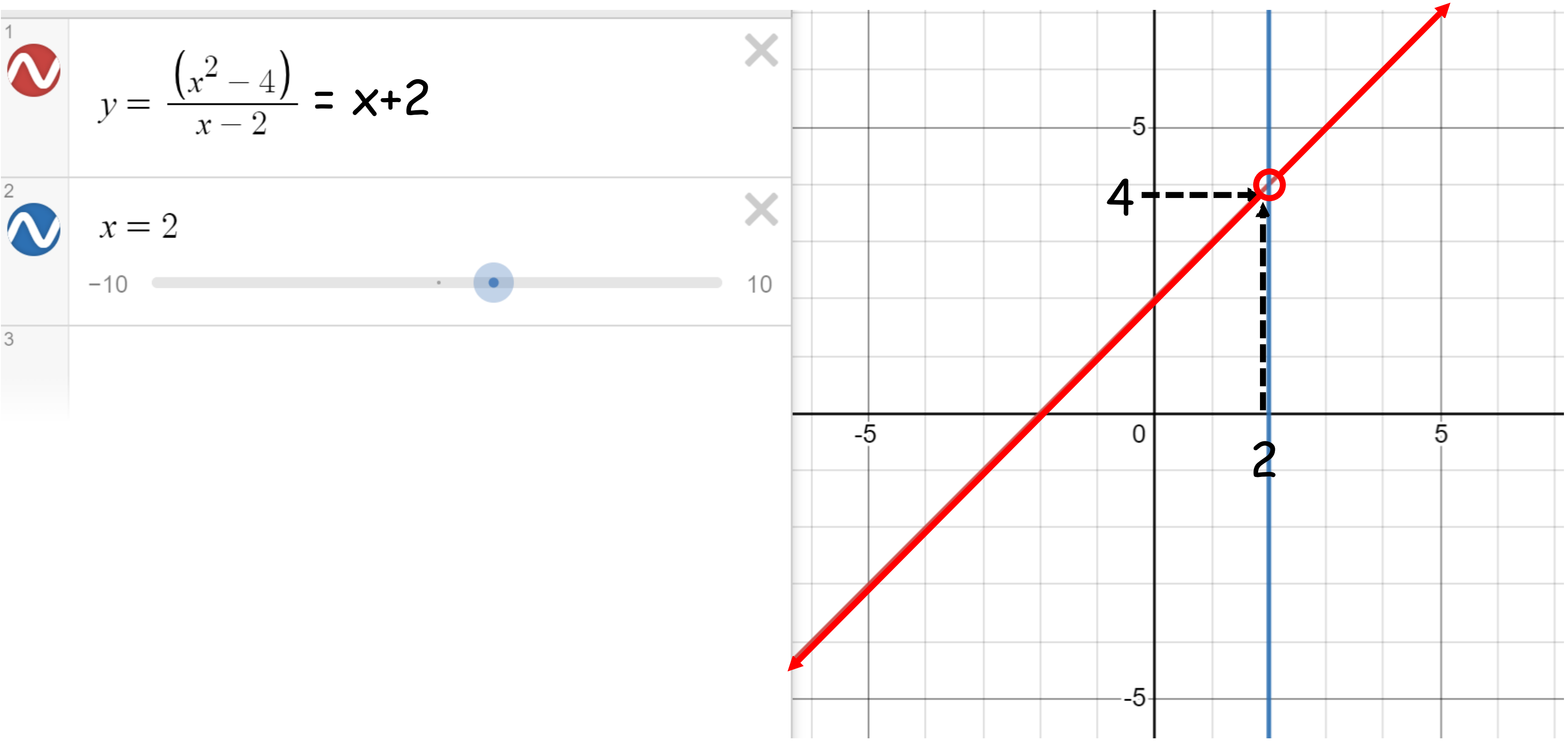
EX: $\lim_{x \to 4} 2x+3 = 2(4)+3 = 11$

2. Try to simplify f(x) by reducing common factors, then plug in 'a'

EX: $\lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{0}{0} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2} = 2+2 = 4$

this function would graph like x+2, but it has a hole the function doesn'e exist at '2'

3. Graph the function, evaluate the limit graphically.

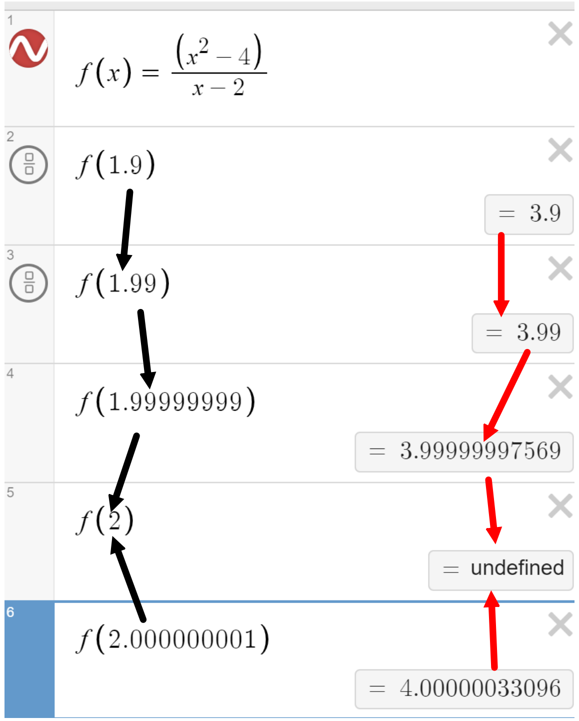


4. Try plugging in input values closer and closer to 'a'

input

approaching

'2'



output

approaching

'4'

Some algebra tricks....

EX: $f(x)=3x^2+7x-8$ find $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

$$f(a+h) = 3(a+h)^2+7(a+h)-8$$

$$=3a^2+6ah+3h^2+7a+7h-8$$

$$f(a) \quad =3a^2 \quad +7a \quad -8$$

$$f(a+h)-f(a) = 6ah + 3h^2 + 7h = h(6a + 3h + 7)$$

so... $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{h(6a+3h+7)}{h} = 6a + 7$

Also... rationalizing denominator. multiply by 1

Evaluate the indicated limit.

$$\lim_{x\to 36} \frac{x-36}{\sqrt{x}-6} = \square$$

$$\lim_{x\to 36} \frac{x-36}{\sqrt{x}-6} * \frac{\sqrt{x}+6}{\sqrt{x}+6} = \lim_{x\to 36} \sqrt{x}+6 = 6+6 = 12$$

Conjugates

OR numerically...

$f(x) = \dfrac{x-36}{\sqrt{x}-6}$

$f(35.9999999)$ $= 12.0000002861$

$f(36)$ $=$ undefined

$f(36.00000001)$ $= 11.9999990071$

One Sided Limits

a one sided limit is the value that a function f(x) approaches as the input 'x' either just increases or just decreases to some value 'a.'

Notation

$$\lim_{x \to a^+} f(x) = L$$

said: "the limit as 'x' goes to 'a' from the right side of 'f' of 'x' is 'L' "
means input is decreasing to 'a'

$$\lim_{x \to a^-} f(x) = L$$

said: "the limit as 'x' goes to 'a' from the left side of 'f' of 'x' is 'L' "
means input is increasing to 'a'

NOTE:limit exists at a point 'a' if limits from both sides at that point are the same. Look at breaks in polynomials

Numerically　　　　Graphically

$\lim_{x \to 0^-} f(x) = -1$

$\lim_{x \to 0^+} f(x) = 1$

The limit itself doesnt exist at '0'

$\lim_{x \to 0} f(x) = \text{DNE}$

Suppose a state's income tax code states that tax liability is 18% on the first $18800 of taxable earning and 14% on the remainder. Find constants a and b for the tax function

EX:

$$T(x) = \begin{cases} a + 0.18x & x \le 18800 \\ b + 0.14(x - 18800) & x > 18800 \end{cases}$$

such that $\lim_{x \to 0^+} T(x) = 0$ and $\lim_{x \to 18800} T(x)$ exists.

$a =$ [ ], $b =$ [ ]

Regular one sided limit

$$\lim_{x \to 0^+} T(x) = \lim_{x \to 0^+} a + .18x = \boxed{a = 0}$$

For limit to exist, both sides must be the same

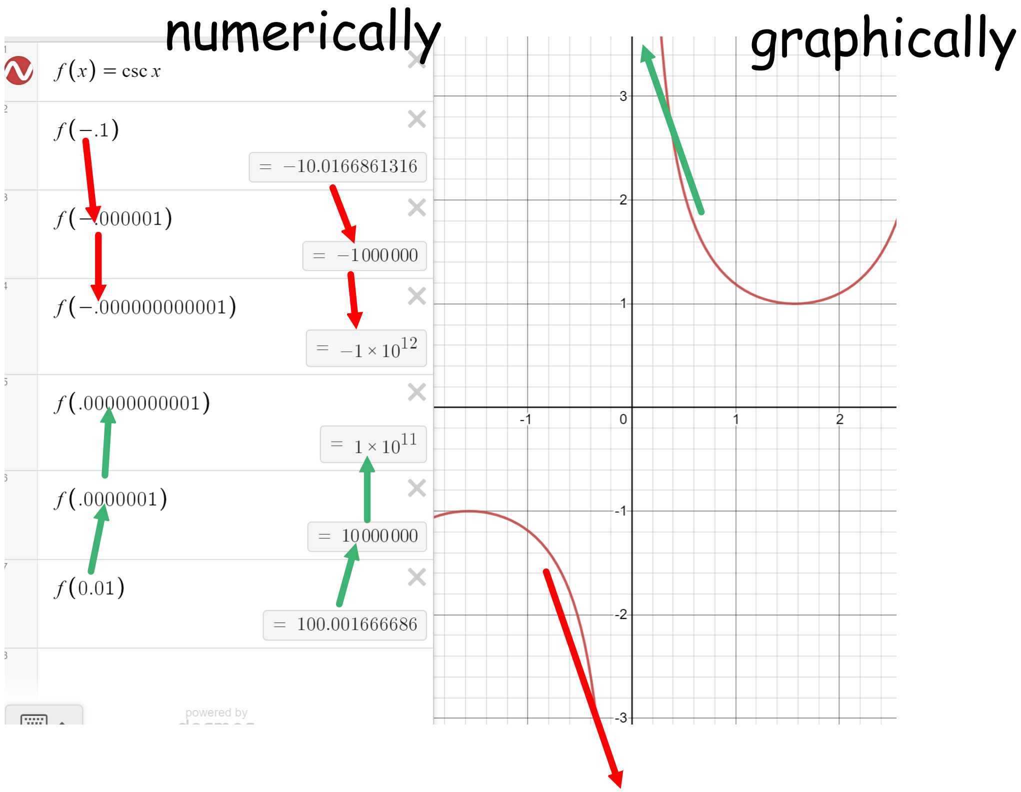
$$\lim_{x \to 18800^-} T(x) = \lim_{x \to 18800^-} a + .18x = .18 * 18800 = 2632$$

$$\lim_{x \to 18800^+} T(x) = \lim_{x \to 18800^+} b + .14(x - 18800) = \boxed{b = 2632}$$

Infinity as an Output

$$\lim_{x \to 0^+} \csc x = \infty$$

$$\lim_{x \to 0^-} \csc x = -\infty$$

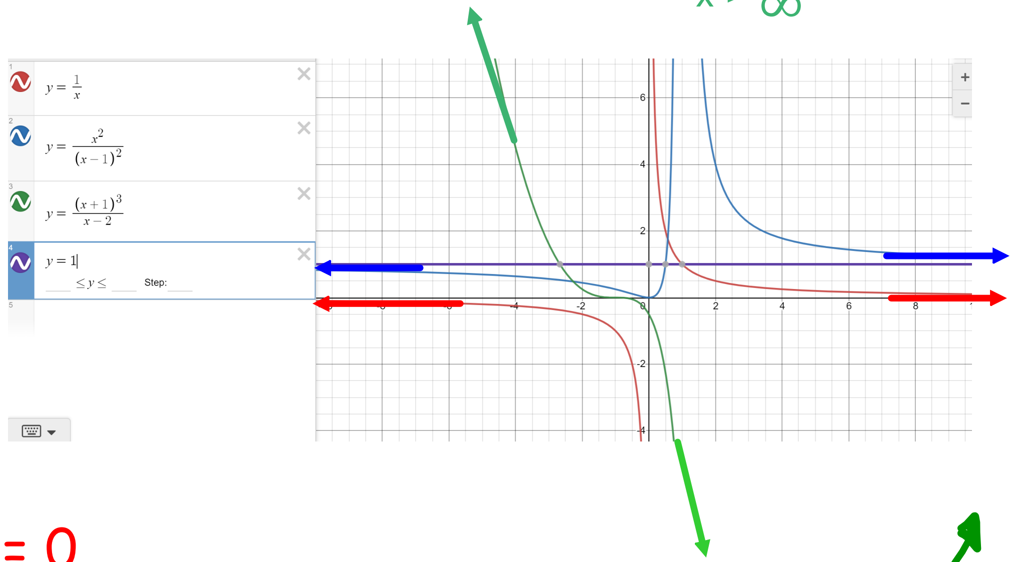


Infinity as a destination

lim f(x) = Left end behaviour

$\lim_{x \to -\infty} f(x)$ = Right end behaviour

EX:



horizontal asymptotes

y=1

y=0

slant asymptote

$y=x^2$ ...

$\lim_{x \to \infty} 1/x = 0$

$\lim_{x \to \infty} x/(x-1) = 1$

$\lim_{x \to -\infty} (x+1)^3/(x-2) = \infty$

$\lim_{x \to \infty} (x+1)^3/(x-2) = \infty$

CONTINUITY

Now that we have limits, we can describe a function as continuous at a point 'a' using the following three conditions:

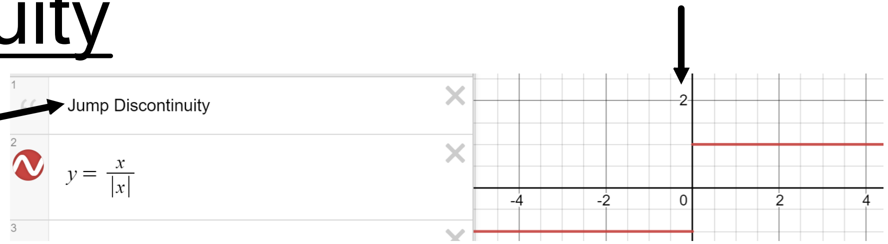
1. LIMIT EXISTS ← means both sides yield same number

2. FUNCTION EXISTS

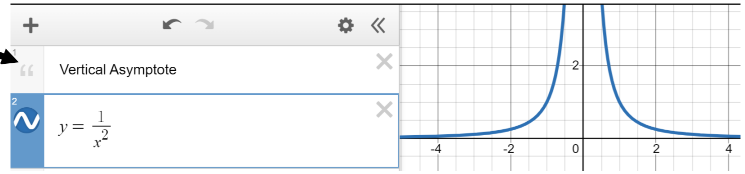
3. LIMIT = FUNCTION

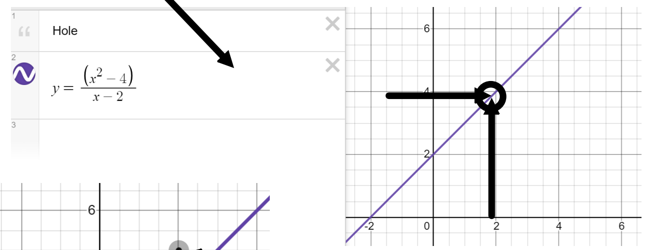
Examples of Discontinuity

1. LIMIT Doesn't EXIST

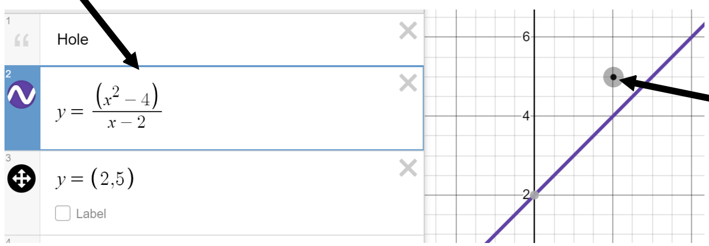


2. FUNCTION Doesn't EXIST





3. LIMIT doesn't equal FUNCTION



called a removable discontinuity if hole can be "plugged"

EX:

Determine values of a and b that make the given function continuous.

$$f(x) = \begin{cases} \dfrac{84\sin(x)}{x} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ b\cos(x) & \text{if } x > 0 \end{cases}$$

$a =$ ☐ and $b =$ ☐

Problem at x=0

<u>Limit has to exist</u>

so $\lim_{x \to 0^-} \dfrac{84\sin(x)}{x} = \lim_{x \to 0^+} b\cos(x)$

so $84 = b \cos 0 = b$

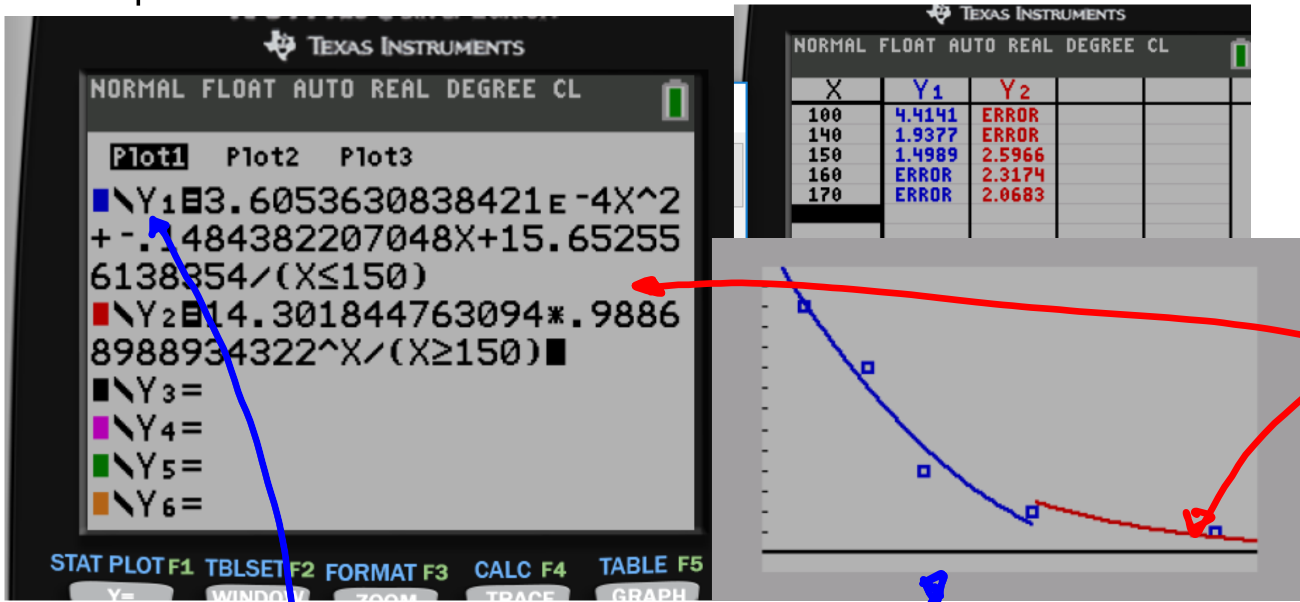
<u>Limit has to equal function</u> f(0)

so $84 = f(0) = a$

Group Work

Split regressions and Limits

Example



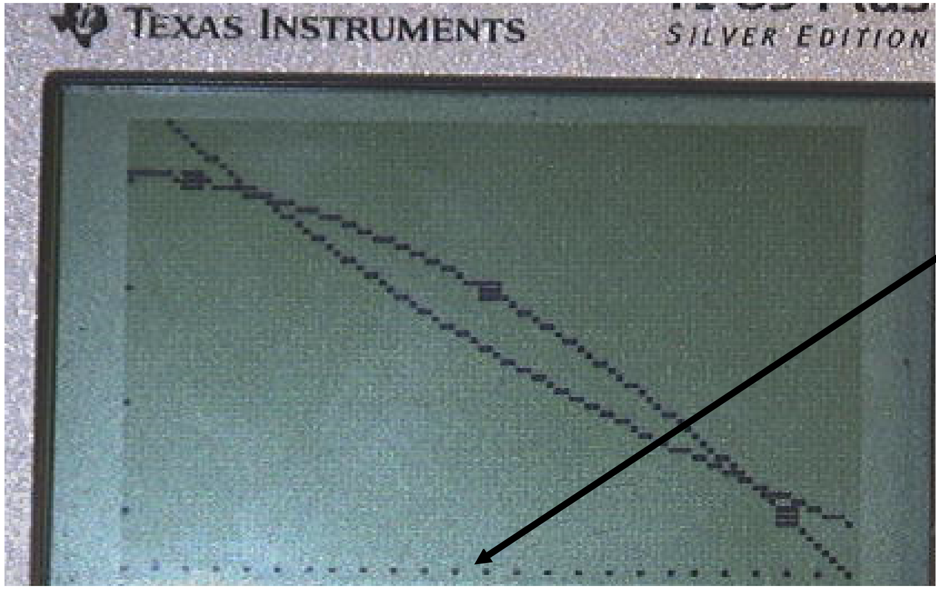
Exponential Regression.

Quadratic Regression

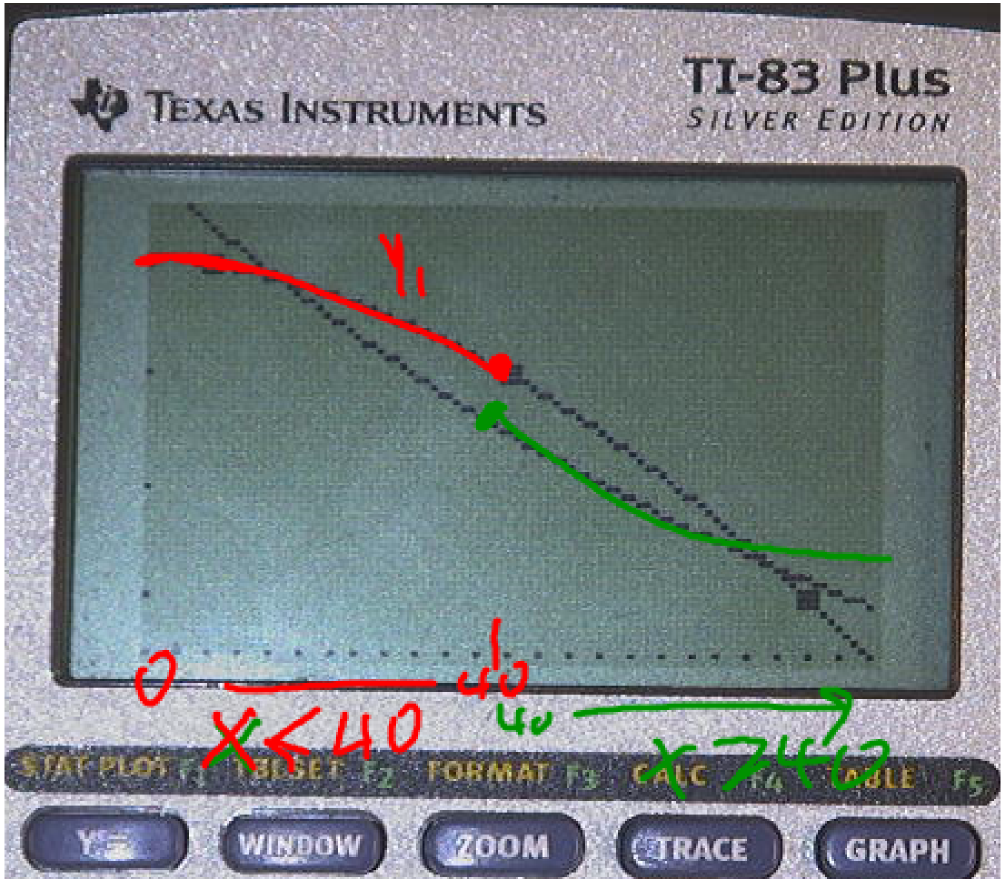
$$\lim_{x\to 150^+} f(x) = 2.5966$$

$$\lim_{x\to 150^-} f(x) = 1.4989$$

Start with two different regressions

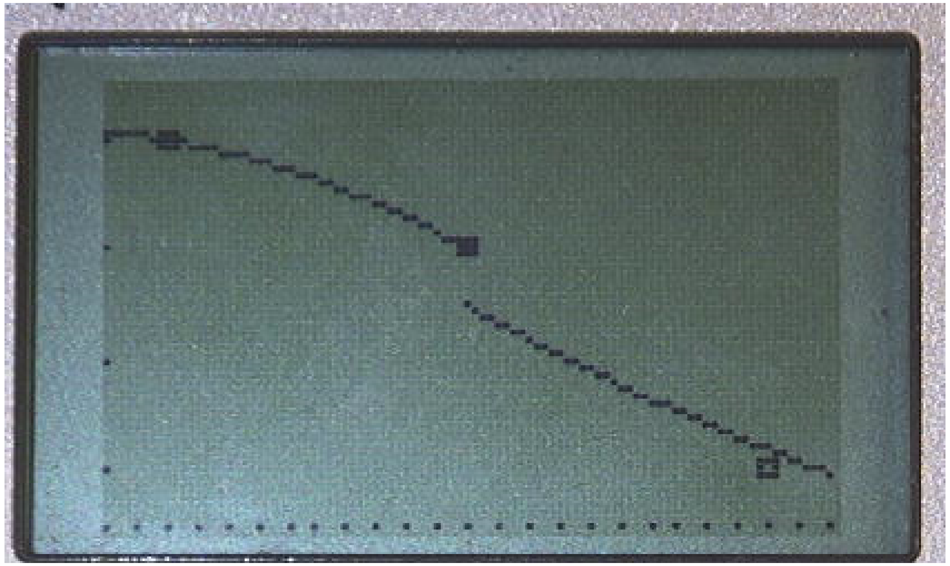


Decide on split (x=40)



keep y1 for x<40

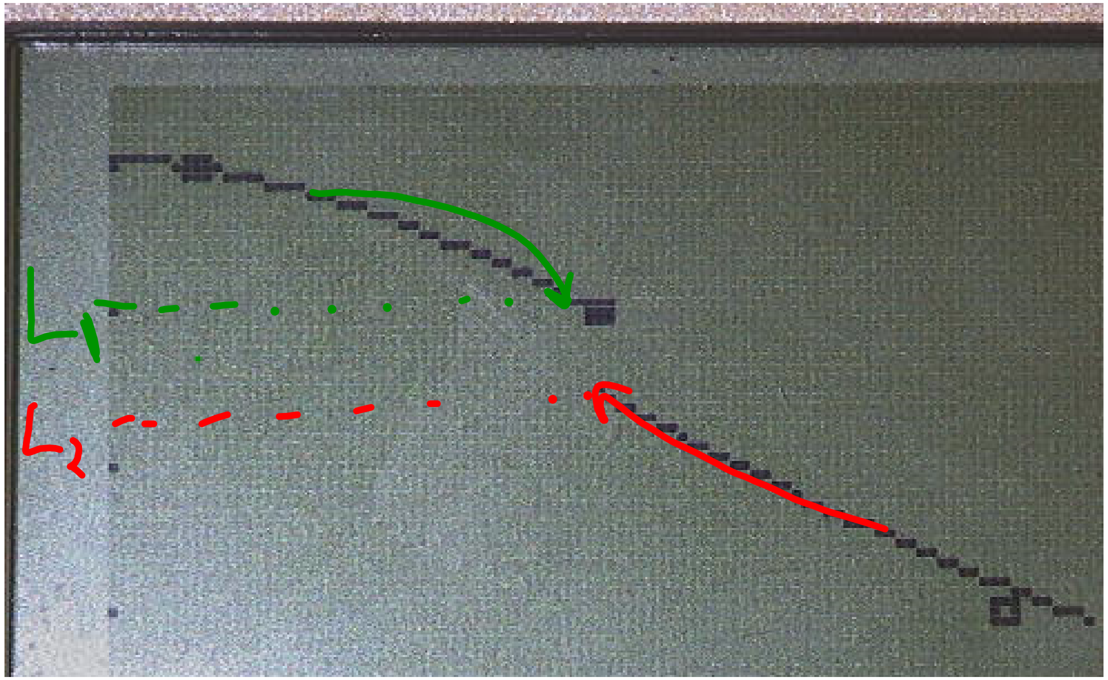
keep y2 for x>40



$Y_1 = reg1 / (x < 40)$

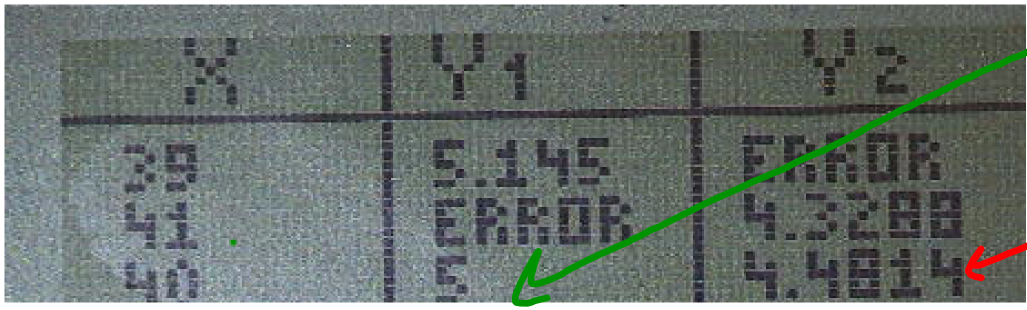
$Y_2 = reg2 / (x > 40)$

2nd math



$$\lim_{x \to 40^-} f(x) = L_1$$

$$\lim_{x \to 40^+} f(x) = L_2$$



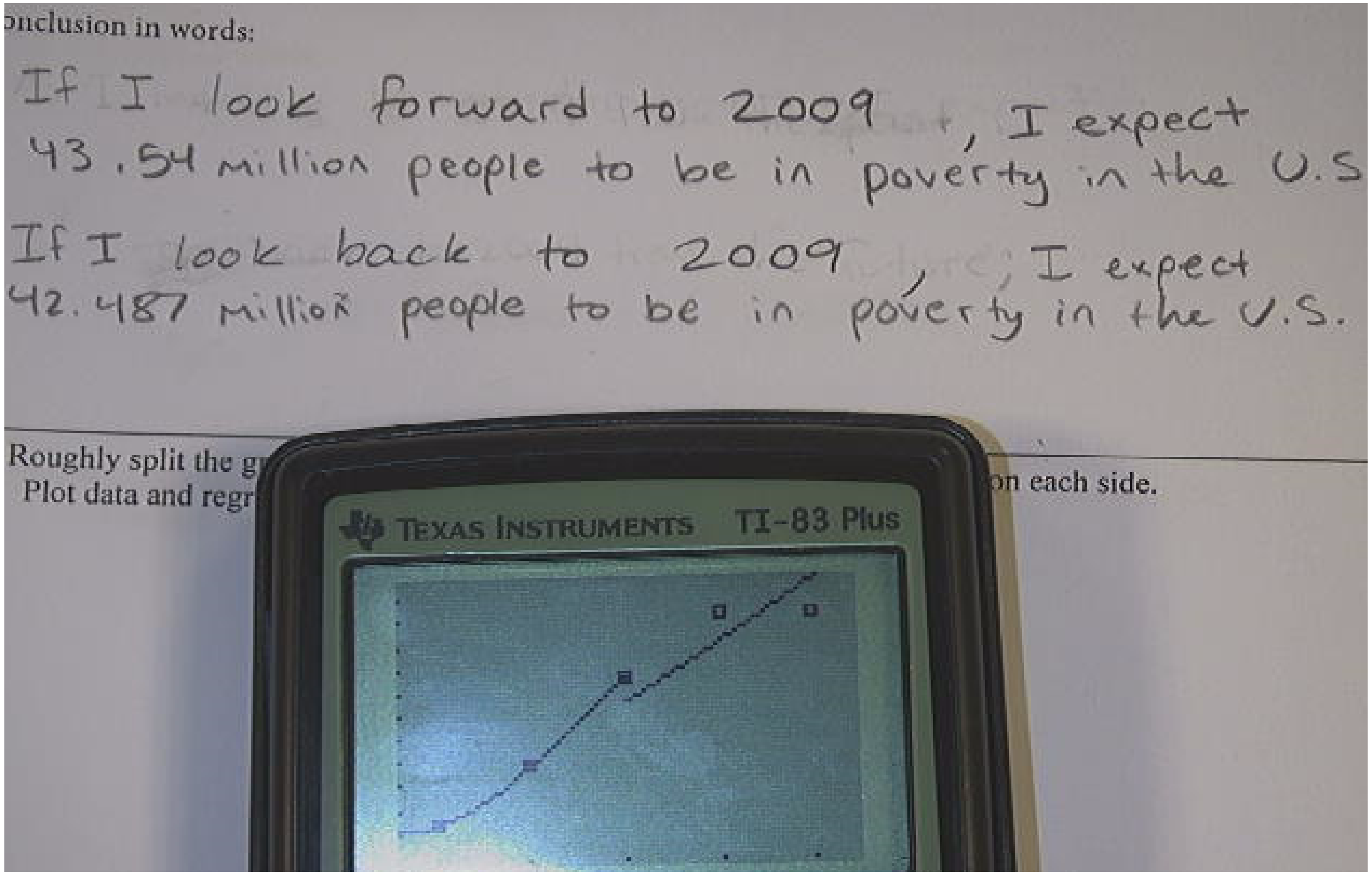
X	Y1	Y2
39	5.145	ERROR
41	ERROR	4.3288
40	5	4.4814

left regression split at a Y1=vars 5: > > 1: RegEq /(x≤a) right regression Y2=vars 5: > > 1: RegEq /(x≥a)	Left Regression used:	QUAD
	Right Regression used:	EXP
	Location of split (a)	40
Y1(a) Y2(a)	$\lim_{x\to a^-} r(x)$	5
	$\lim_{x\to a^+} r(x)$	4.48

X	Y
Price	Sales

As I raise the price to $40 I expect to sell 5 calcs.

As I lower the price to $40 I expect to sell 4.48 calcs

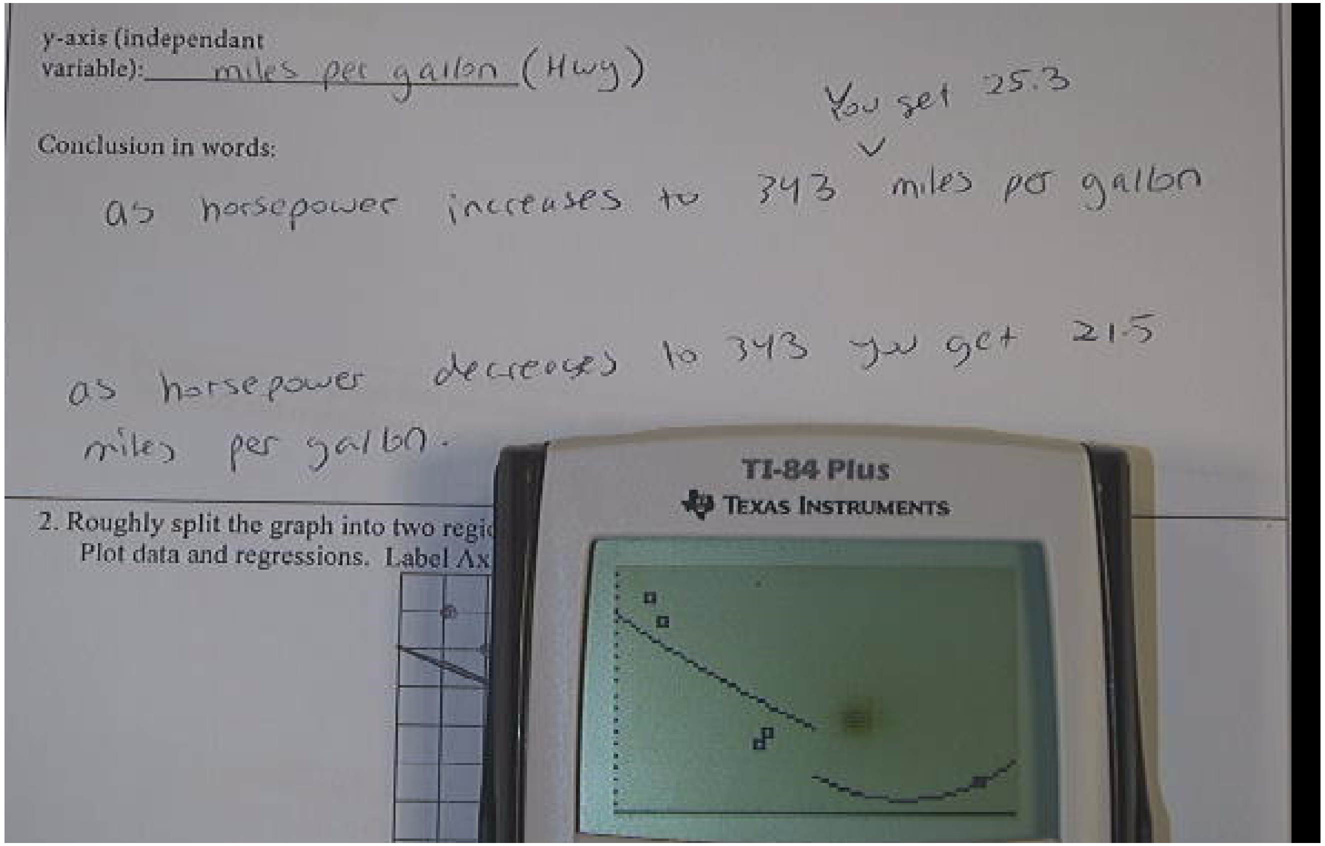


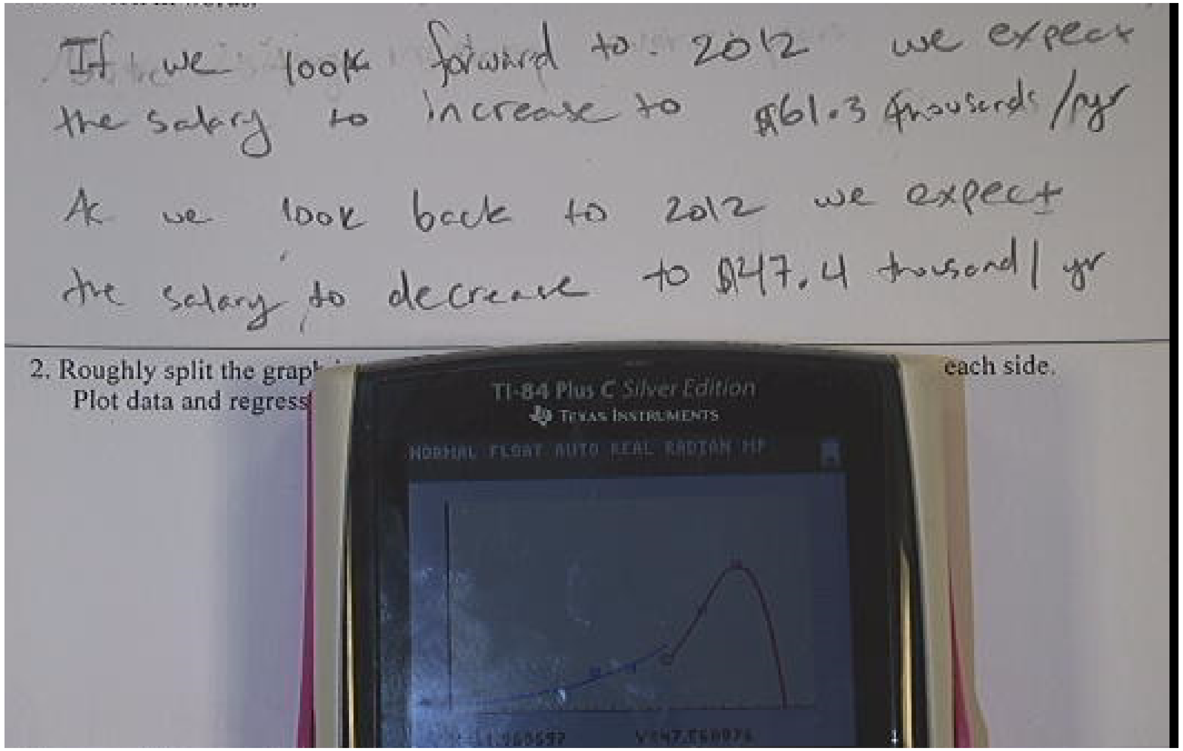
onclusion in words:

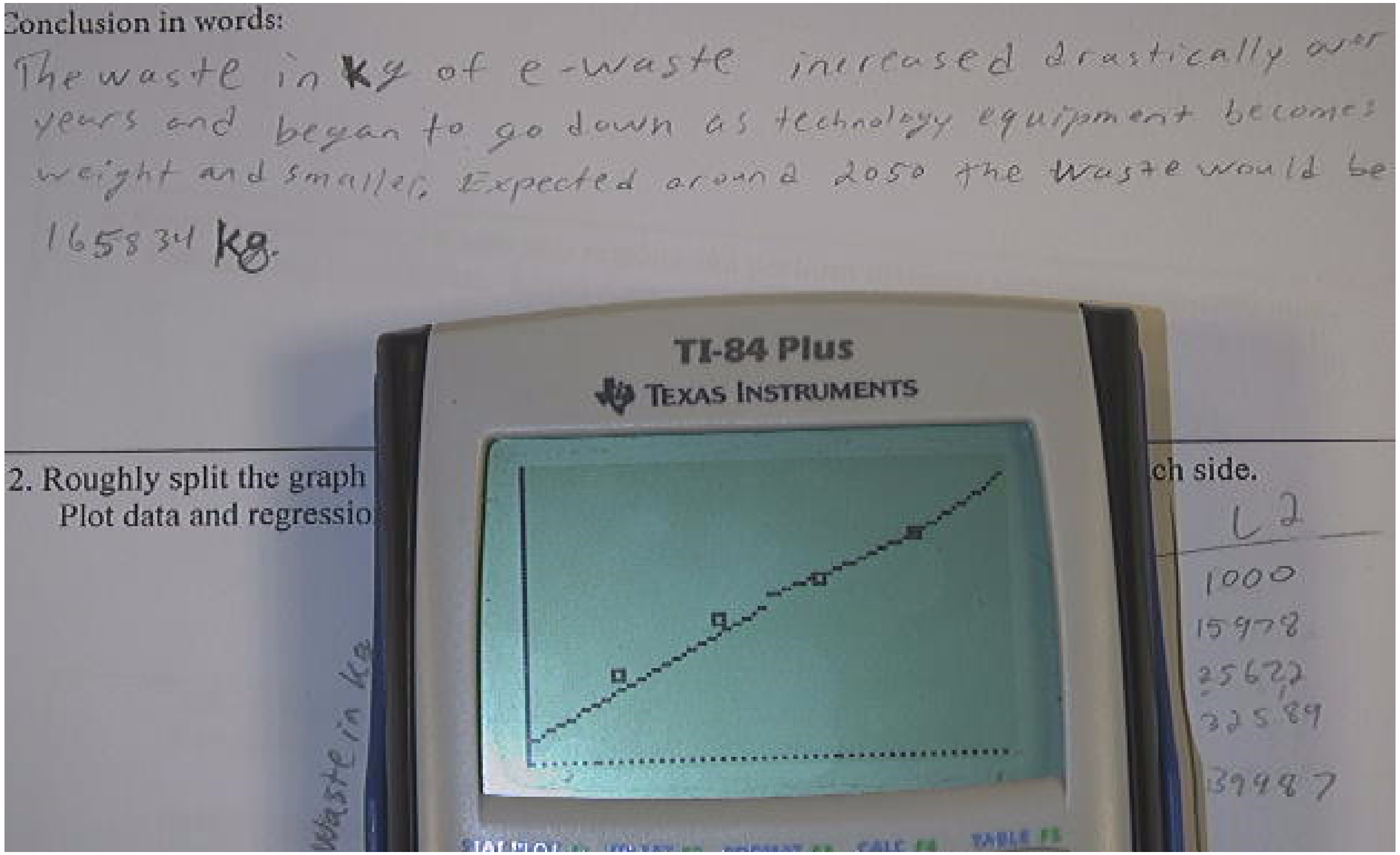
If I look forward to 2009, I expect 43.54 million people to be in poverty in the U.S

If I look back to 2009, I expect 42.487 million people to be in poverty in the U.S.

Roughly split the g... on each side.
Plot data and regr...


y-axis (independant variable): miles per gallon (Hwy)
Conclusion in words:
You get 25.3
as horsepower increases to 343 miles per gallon
as horsepower decreases to 343 you get 21.5 miles per gallon.
2. Roughly split the graph into two regi
Plot data and regressions. Label Ax
TI-84 Plus
Texas Instruments

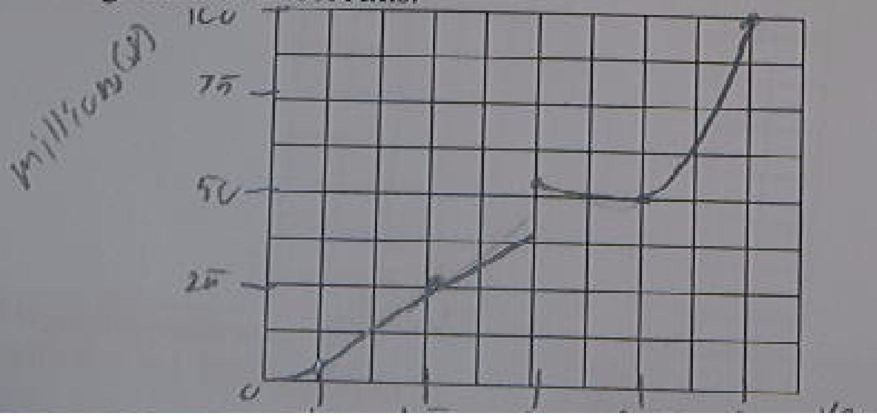

If we look forward to 2012 we expect
the salary to increase to $61.3 thousands/yr
As we look back to 2012 we expect
the salary to decrease to $47.4 thousand/yr
2. Roughly split the grap
each side.
Plot data and regress
TI-84 Plus C Silver Edition
Texas Instruments
NORMAL FLOAT AUTO REAL RADIAN MP


Conclusion in words:
The waste in kg of e-waste increased drastically over years and began to go down as technology equipment becomes weight and smaller, Expected around 2050 the waste would be 165834 kg.
TI-84 Plus
TEXAS INSTRUMENTS
2. Roughly split the graph
Plot data and regressio
ch side.
L2
1000
15978
25622
32589
39987

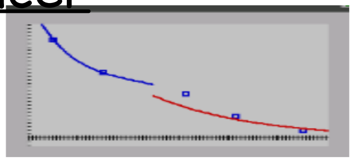
on in words: As we go back in time, after 2 years of revenue we would think that the reven would be 842,714 millions

As we look back at the revenue, we would see the revenue from weed continue to incr at a rapid rate.

ly split the graph into two regions and perform different regressions on each side.
data and regressions. Label Axis.



Producer



X	Y1	Y2
30	22.672	ERROR
45	16.744	ERROR
55	14.711	11.724
65	ERROR	8.4355
75	ERROR	6.0696

Speaker

According to the split regression given, As we increase the price to \$55, we epect sales to be 14.7 Billion. Lowering the price predicts 11.7 Billion units sold.

Writer

$$\lim_{x \to \$55^-} r(x) = 14.7 \text{ billion units sold}$$

$$\lim_{x \to \$55^+} r(x) = 11.7 \text{ billion units sold}$$

DESMOS

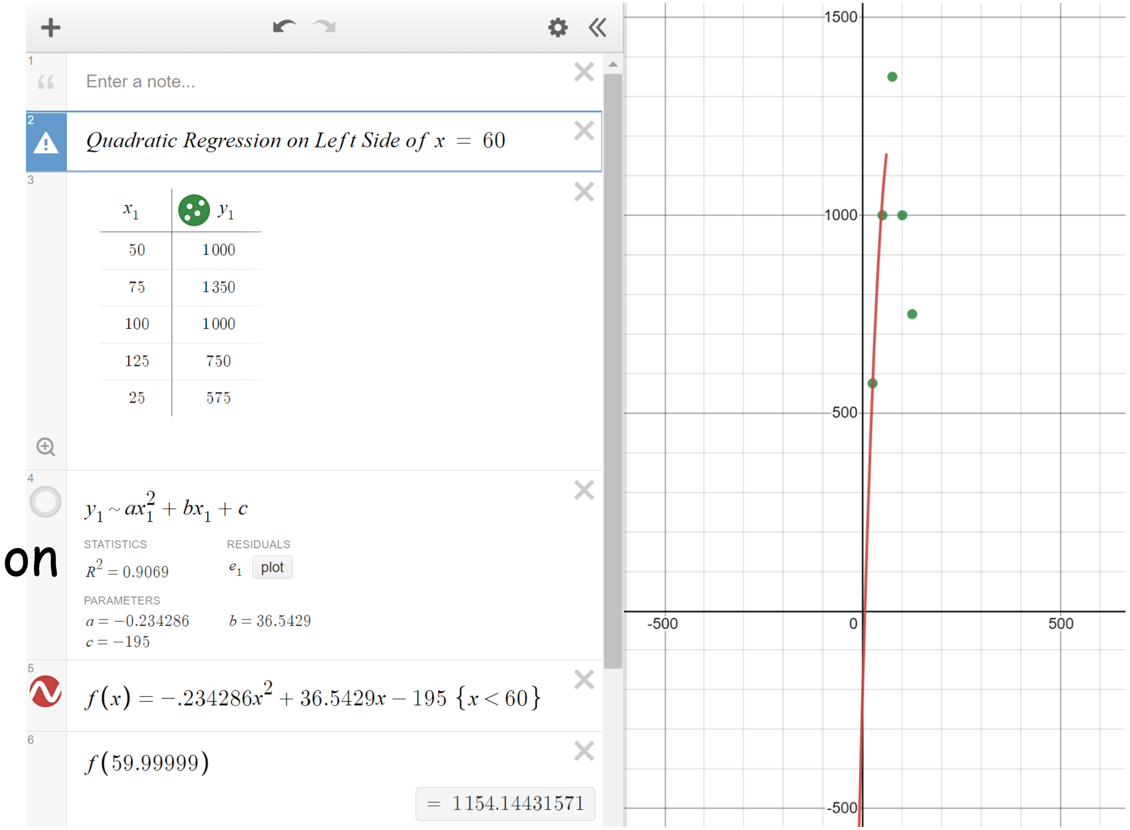
- Easy to reproduce regressions
- Much easier to find information- keyboard commands

Data

Quad Regression

Function

Evaluate



Linear Regression on Right Side of x = 60

Linear Regression

$y_1 \sim bx_1 + c$

STATISTICS

$r^2 = 0.03556$

$r = 0.1886$

RESIDUALS

e_2 plot

PARAMETERS

$b = 1.4$ $c = 830$

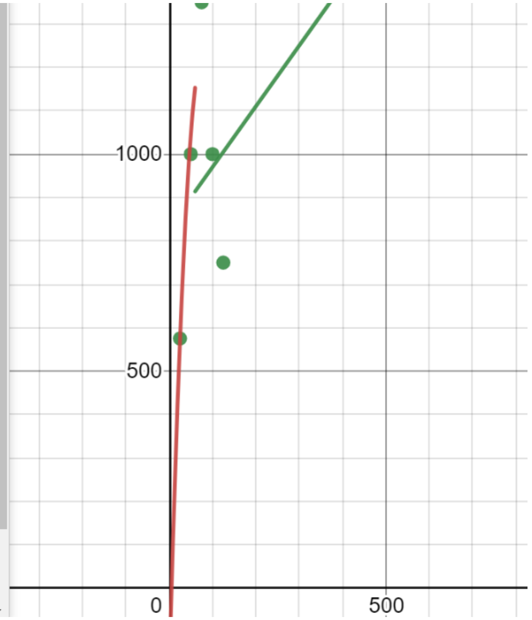
Function

$g(x) = 1.4x + 830\{x > 60\}$

Evaluate

$g(60.0001)$

$= 914.00014$



Reminders....

1. Go to Blackboard

2. Post your Selfie on Discussion Forum

3. Set up Connect by opening an assignment

You must complete a Connect assignment in first two weeks or you will be dropped from class