Agenda

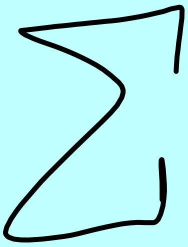
Area with Rectangles

Project Examples

Lecture Fundamental Theorem of Calculus

Project for Fundamental Theorem

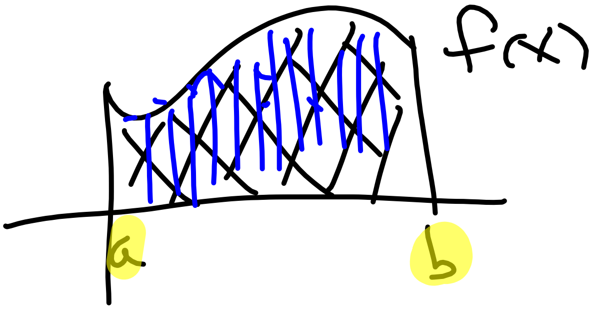
Area with rectangles



Definite Integral

under curve

$$\int_{a}^{b} f(x)\,dx$$



Area $\approx A(n)$

$n = \#$ of rectangles

$$\sum_{i=1}^{n} i = (1+n)\left(\frac{n}{2}\right) = \frac{n^2}{2} + \frac{n}{2}$$

Ex $\underline{1+2+3} = (1+3)\left(\frac{3}{2}\right) = 6$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+3n^2+n}{6}$$

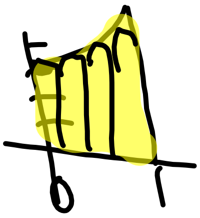
Ex $1^2+2^2+3^2 = \frac{3\cdot 4\cdot 7}{6} = 14$

Check $\sum_{i=1}^{3} i^2 = 1+4+9=14 = \frac{3(4)(7)}{6}$

$$\sum_{i}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{n^2+n}{2}\right)^2 = \underline{n^4+\ldots}$$

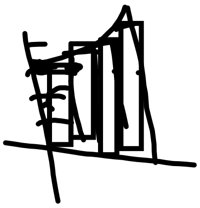
Example

Area under curve -

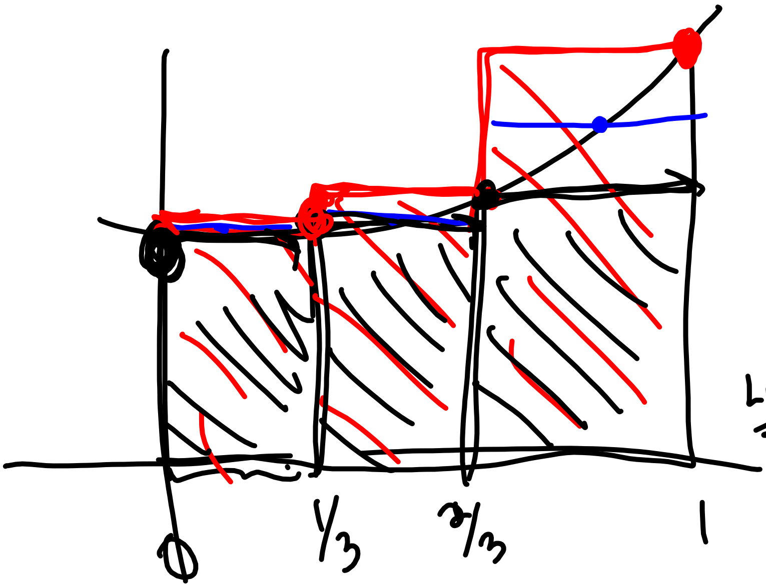


$y = x^2 + 2x + 3$

From 0 to 1



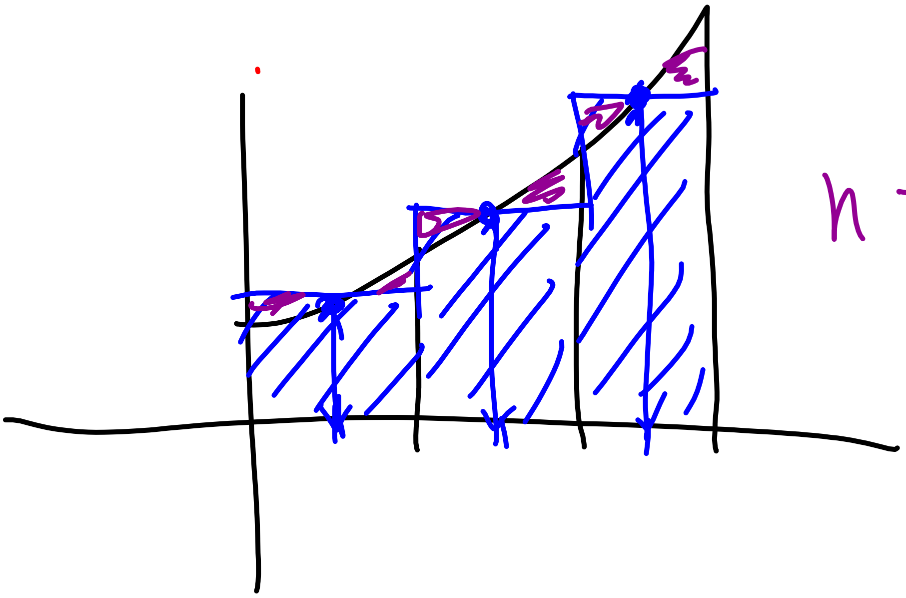
"n" - rectangle to approximate



$n=3$

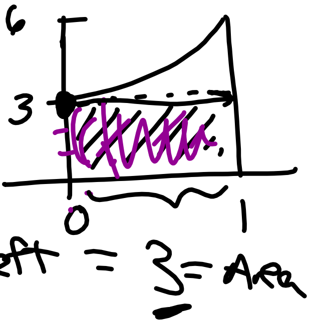
Left $\frac{1}{3}[f(0)+f(\frac{1}{3})+f(\frac{2}{3})]$

Right $\frac{1}{3}[f(\frac{1}{3})+f(\frac{2}{3})+f(1)]$

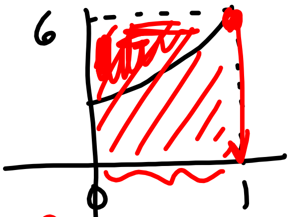


n = 3

$n=1$



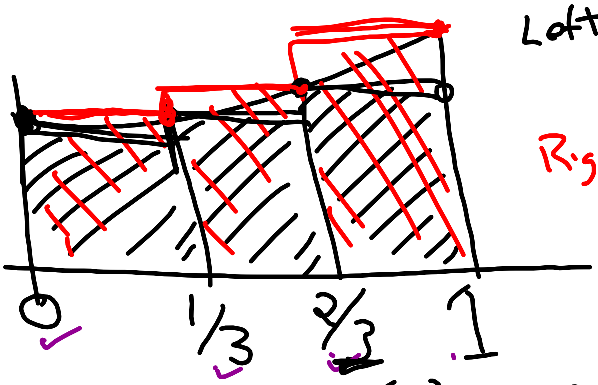
Left = 3 = Area



x^2+2x+3

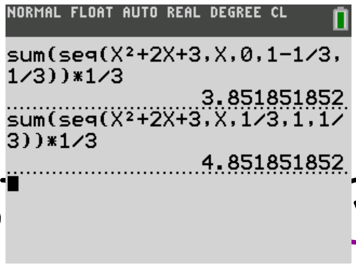
Right = 6 = Area

$n=3$



Left side start: 0
End $1-1/3$

Right side start: $1/3$
End: 1



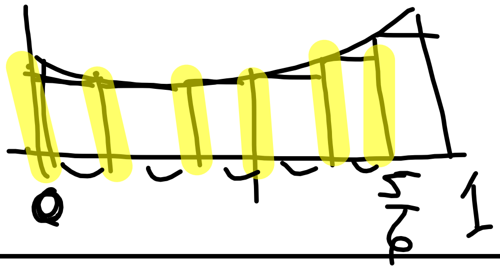
```
NORMAL FLOAT AUTO REAL DEGREE CL
sum(seq(X²+2X+3,X,0,1-1/3,
1/3))*1/3
                3.851851852
sum(seq(X²+2X+3,X,1/3,1,1/
3))*1/3
                4.851851852
```

Left side Sum(Seq($(x^2+2x+3,$ $\frac{1}{n}$

Right side Sum(seq$(x^2+2x+3, x, 1/3, 1, 1/3))*\frac{1}{3}$

$n=6$

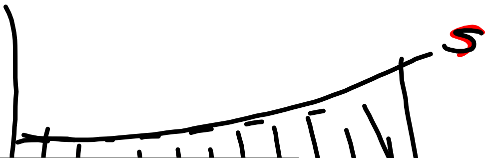
Start: 0 END: $1-1/6$



start: $1/6$ END: 1

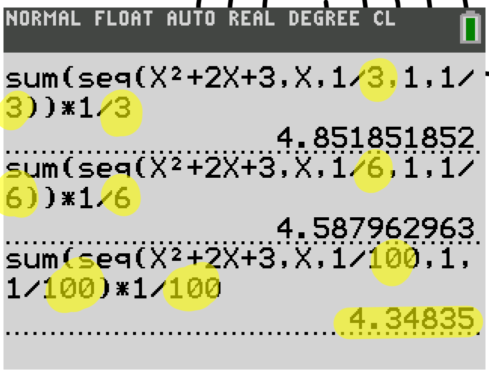
4.58

$n=100$



Start: 0 END: $1-1/100$

start: $1/100$ END: 1



```
NORMAL FLOAT AUTO REAL DEGREE CL
sum(seq(X²+2X+3,X,1/3,1,1/
3))*1/3
                4.851851852
sum(seq(X²+2X+3,X,1/6,1,1/
6))*1/6
                4.587962963
sum(seq(X²+2X+3,X,1/100,1,
1/100))*1/100
                    4.34835
```

$\approx 4\frac{1}{3}$

Left side $\text{sum}(\text{seq}(x^2+2x+3, x, 0, 1-1/n, 1/n))*1/n$

Right side $\text{sum}(\text{seq}(x^2+2x+3, x, 1/n, 1, 1/n)*1/n$

Right Endpoint Area

$$= \text{Sum}(\text{Seq}(x^2+2x+3, x, 1/n, 1, 1/n)) \times \frac{1}{n}$$

$\underbrace{\text{Heights} \quad \text{width}}_{\text{Area}}$

n rectangles

Summation Notation

$$\sum_{i=1}^{n}\left(\overset{x^2}{\downarrow}\left(\frac{i}{n}\right)^2+2\overset{2x}{\downarrow}\left(\frac{i}{n}\right)+\overset{3}{\downarrow}3\right)\cdot\underset{\uparrow\ \text{width}}{\frac{1}{n}}$$

or

$$\left(\sum_{i=1}^{n}\frac{i^2}{n^2}+\frac{2i}{n}+3\right)*\frac{1}{n}$$

or

$$\left(\frac{1}{n^2}\sum_{i=1}^{n}i^2+\frac{2}{n}\sum_{i=1}^{n}i+\sum_{i=1}^{n}3\right)\frac{1}{n}=A(n)$$

$$\sum_{i=1}^{n}3\ \downarrow\ \underbrace{3+3+3+\dots 3}_{3n}$$

$\sum_{i}^{n} i^2$ - or - $\sum_{i}^{n} i$ $\sum_{1}^{n} 3$

$$\left[\frac{1}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right)+\frac{2}{n}\left((1+n)\left(\frac{n}{2}\right)\right)+3n\right]\frac{1}{n}$$

$$A(n)=\left(\frac{\boxed{2n^{3}}+3n^2+n}{\boxed{6n^{3}}}\right)+\left(\frac{\boxed{n^{2}}+n}{\boxed{n^{2}}}\right)+3$$

$$\boxed{\lim_{n\to\infty} A_n}=\frac{1}{3}+1+3=4\tfrac{1}{3}$$

4.33...

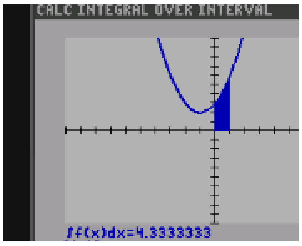
$Y_1 = x^2 + 2x + 3$

Calc 7: $\int f(x)\,dx$

Lower: 0

Upper: 1

$\int f(x)dx = 4.3333....$



CALC INTEGRAL OVER INTERVAL

∫f(x)dx=4.3333333

$$\underline{A}x^2 + \underline{B}x + \underline{C}$$

sum(seq(Ax^2+Bx+C, X, 1/n, 1, (1/n)) 1/n

$$\frac{1}{n}\left(\sum A\left(\frac{i^2}{n}\right) + \frac{B}{2}\left(\frac{i}{n}\right) + C\right)$$

$$A(n) = A\left(\frac{2n^3}{6n^3} + \dots\right) + \frac{B}{2}\left(\frac{n^2}{n^2} + \dots\right) + C$$

$$A(n) \overset{①}{=} \left(\frac{2n^3+\dots}{6n^3}\right) \overset{②}{\frac{+}{2}}\left(\frac{n^2+\dots}{n2}\right) + \textcircled{3}$$

$$\lim_{n\to\infty} A\left(\nearrow^{1/3}\right) + \frac{B}{2}\left(\nearrow^{1}\right) + C$$

$$= A\cdot(1/3) + B(1/2) + C$$

$$\int_0^{\textcircled{1}} x^2 + 2x + 3 \, dx \simeq \frac{Ax^3}{3} + \frac{Bx^2}{2} + C$$

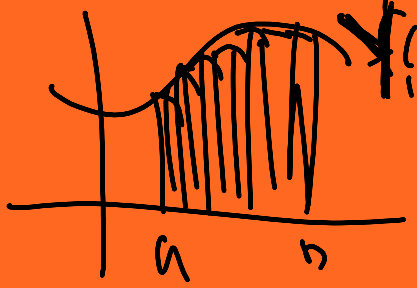
$$= F(1)$$

$$\frac{1}{3} + \frac{2}{2} + 3 \searrow$$

$$\textcircled{1}\frac{x^3}{3} + \textcircled{2}\frac{x^2}{2} + \textcircled{3}x$$

$$\frac{1}{3}A + \frac{1}{2}B + C$$

Project Examples



$f(x) = (a, b, E)$

10 rectangles

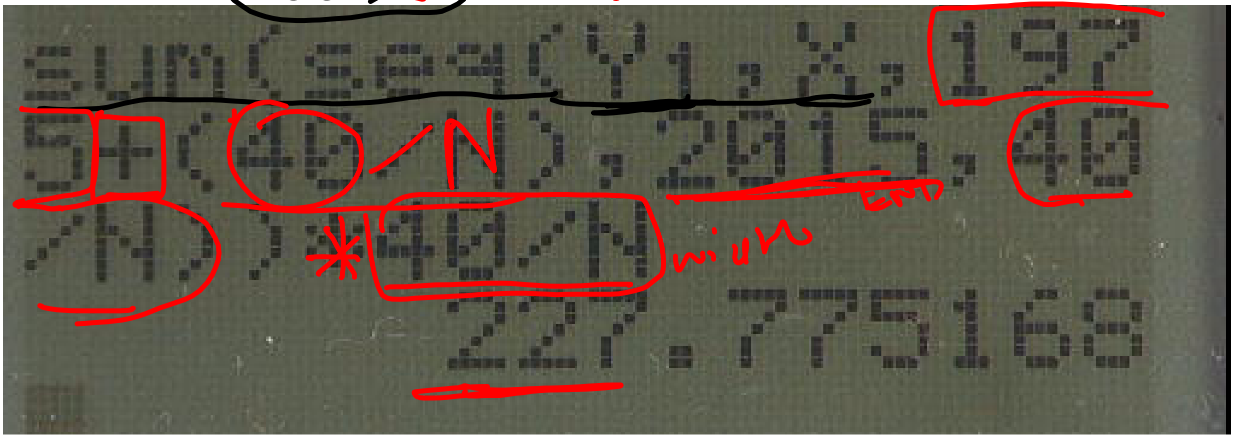
↓

N

≥

100 → N

~~100→N~~ 200→N



```
sum(seq(Y1,X,197
5+(40/N),2015,40
/N))*40/N
          227.775168
```

end

width

Y_1 = regression between 1975→2015 = 40 years

N = # of rectangles

step $\frac{40}{N}$

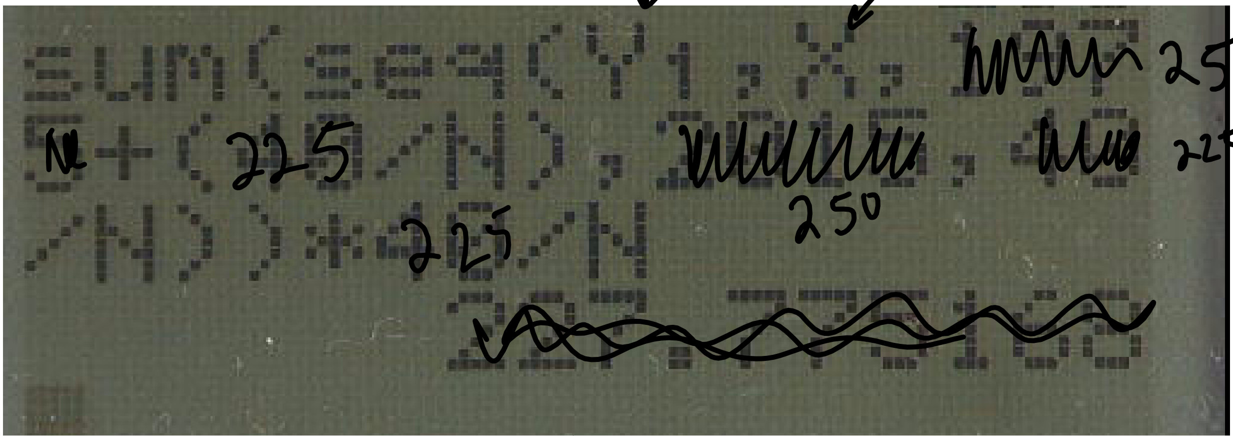
Billions of people * yrs.

$$\boxed{\frac{227}{50_{71}} =}$$

100→N

regression

variable



```
sum(seq(Y1,X,[redacted] 25
N+(225/N),[redacted] 250,[redacted] 225/N
/N))*225/N
[redacted]
```

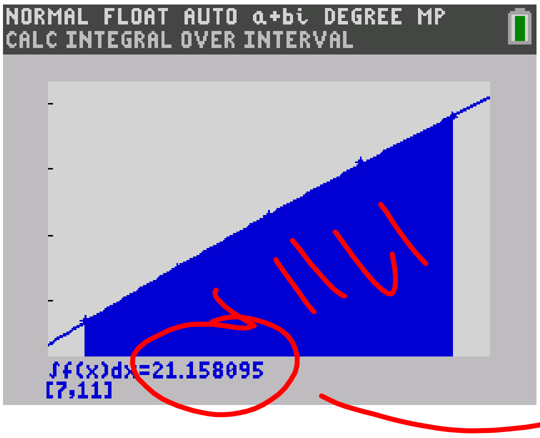
Start: 25 K
END: 250K

$250-25=225$

$25+\frac{225}{N}$

N=100

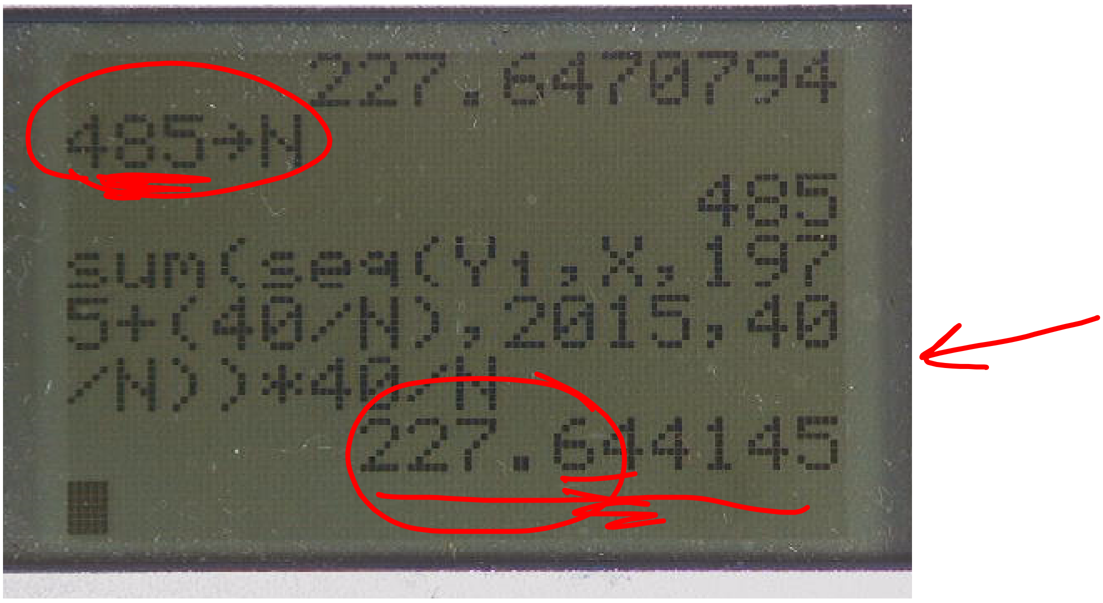
25+2.25
27.25



```
NORMAL FLOAT AUTO a+bi DEGREE MP
CALC INTEGRAL OVER INTERVAL
∫f(x)dx=21.158095
[7,11]
```


1975 + 40/N
1975
2015
40yrs/N

```
∫f(x)dx=227.50476
```



```
      227.6470794
485→N
              485
sum(seq(Y₁,X,197
5+(40/N),2015,40
/N))*40/N
       227.644145
```

a= _1975   b=_2015   y1= regression

$\Delta X = \underset{\text{END}}{b} - \underset{\text{START}}{a}$

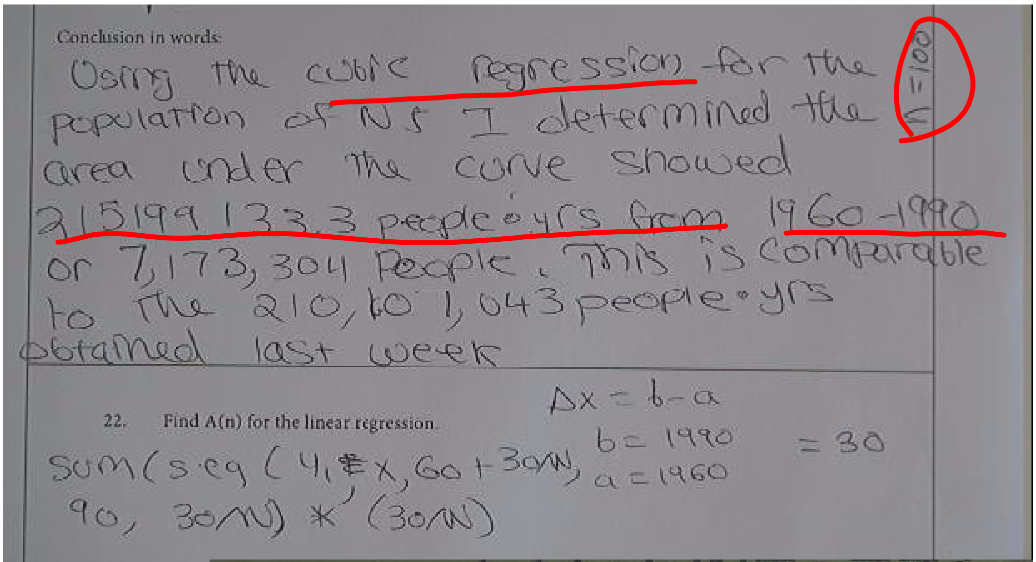
$\text{sum}(\text{seq}(Y_1, X, a+\Delta X/N, b, \Delta X/N)) * \frac{\Delta X}{N}$

N=50

sum(seq(y1,x,1975+40/50,2015,40/50)*40/50

N=#

sum(seq(y1,x,1975+40/N,2015,40/N)*40/N



Conclusion in words:

Using the cubic regression for the population of NS I determined the area under the curve showed 215199133.3 people•yrs from 1960-1990 or 7,173,304 People. This is comparable to the 210,601,043 people•yrs obtained last week

n = 100

22. Find A(n) for the linear regression.

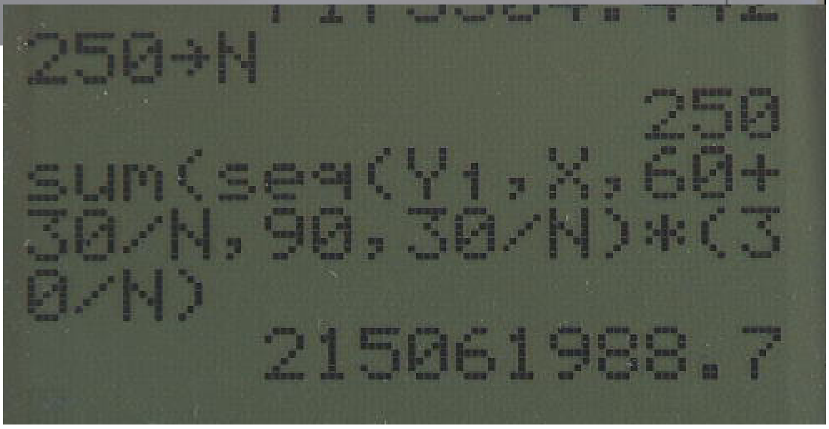
sum(seq(Y1,X,60+30/N, 90, 30/N) * (30/N)

$\Delta x = b - a$

b = 1990

a = 1960

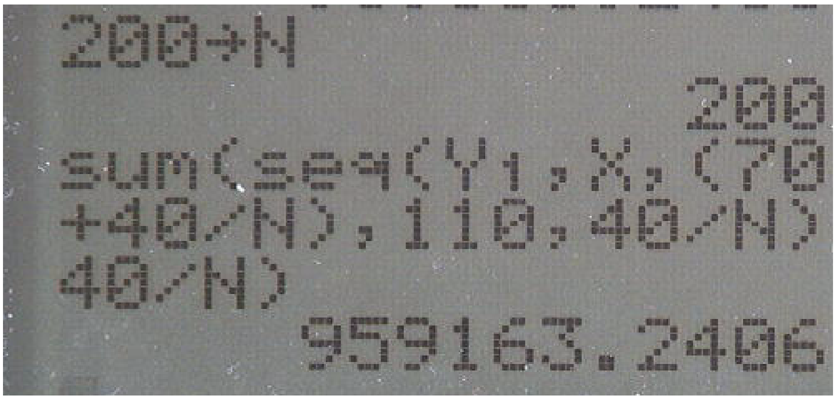
= 30



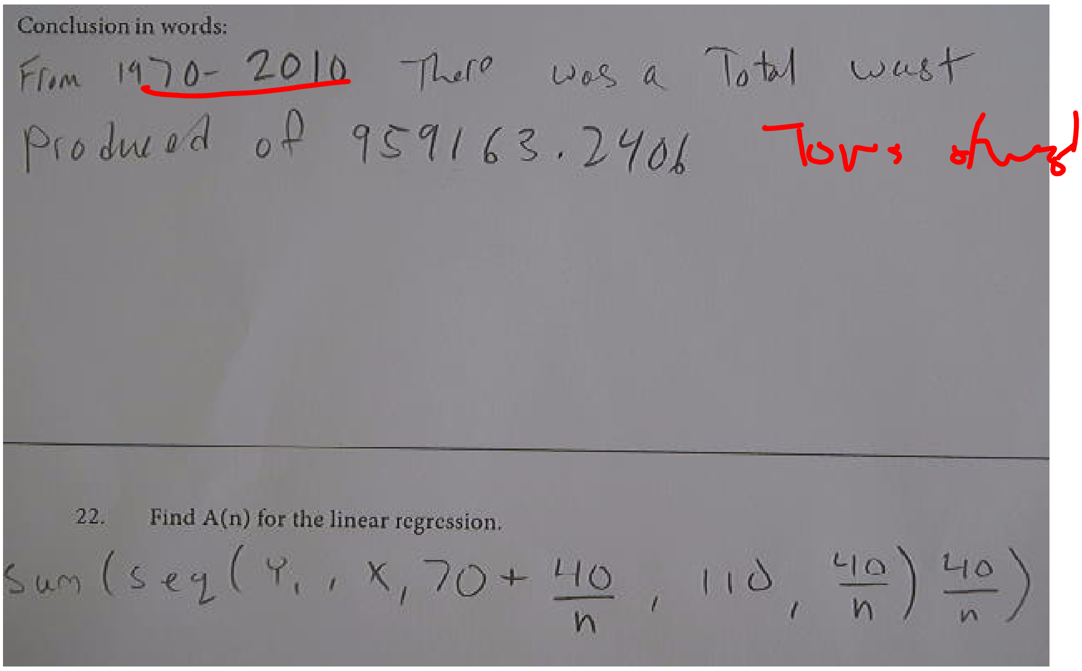
```
250→N
                    250
sum(seq(Y1,X,60+
30/N,90,30/N)*(3
0/N)
          215061988.7
```

Math

$$\sum_{i=1}^{100} Y_1\left(1960+\frac{30}{100}i\right)\frac{30}{100}$$



```
200→N
                 200
sum(seq(Y1,X,(70
+40/N),110,40/N)
40/N)
         959163.2406
```

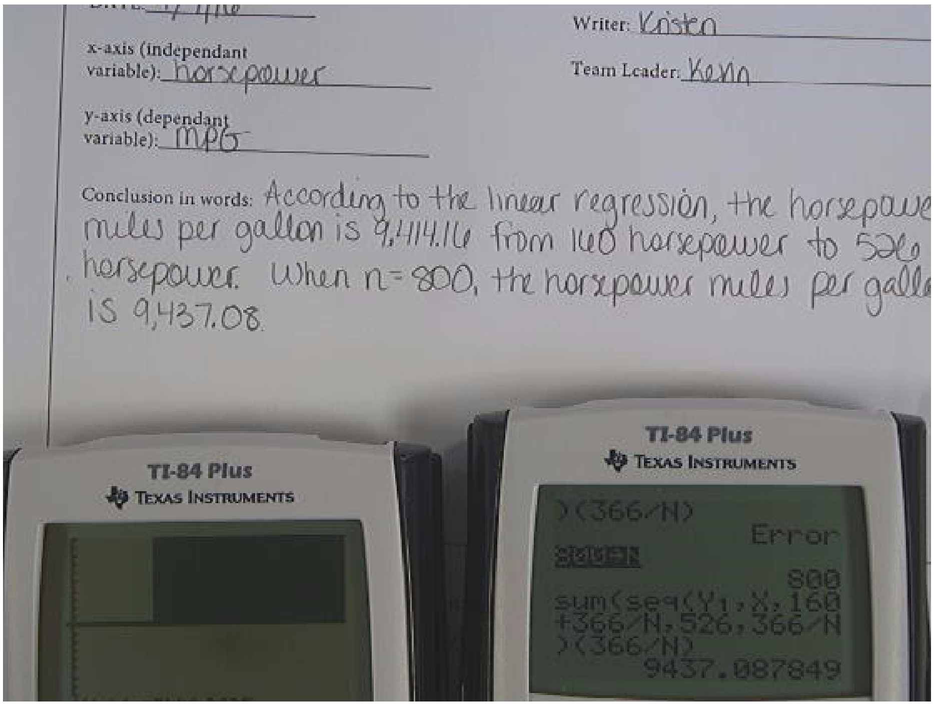


Conclusion in words:

From 1970- 2010 There was a Total wast produced of 959163.2406 Tons of waste * yrs.

22. Find A(n) for the linear regression.

$\text{Sum}(\text{seq}(Y_1, X, 70+\frac{40}{n}, 110, \frac{40}{n})\frac{40}{n})$



Writer: Kristen

x-axis (independant variable): horsepower

Team Leader: Kevin

y-axis (dependant variable): MPG

Conclusion in words: According to the linear regression, the horsepowe miles per gallon is 9,414.16 from 160 horsepower to 526 horsepower. When n=800, the horsepower miles per galla is 9,437.08.

```
)(366/N)
                Error
800→N
                  800
sum(seq(Y₁,X,160
+366/N,526,366/N
)(366/N)
          9437.087849
```

Fundamental Theorem of Calculus

Antiderivatives

$$\begin{tikzcd} \int f(x)\,dx \arrow[r, bend left] & F(x) \end{tikzcd}$$

indefinite integral

$$F' = f$$

$$\begin{tikzcd}
4 \arrow[r, bend left] & 4n & \int 4\,dn \arrow[l] \\
n \arrow[r, bend left] & \frac{n^2}{2} & \int n\,dn \arrow[l] \\
n^2 \arrow[r, bend left] & \frac{n^3}{3} & \int n^2\,dn \arrow[l]
\end{tikzcd}$$

$$\int_a^b f(x)\,dx$$

definite integral

Area Under Curve

f(x) between a and b

Anti-derivative

$$F(x) = \int f(x)\,dx$$

$$F'(x) = f(x)$$

Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

area = antiderivative

$$\int_a^b f(x)\,dx = F(\underline{b}) - F(a)$$

Area

$$\int f(x)\,dx = \underline{F}(x)$$

Notation $$\boxed{\int_a^b \underline{f}(x) = \underline{F}(b) - F(a)}$$

= F(b) - F(a) written $F(x)\Big|_a^b$ $$F(x)\Big|_{x=a}^{b}$$

$$\text{Ex} \int_3^5 x^2 dx = \frac{x^3}{3}\Bigg|_3^5 = 5^3 - 3^3 = 125-27$$
$$= \boxed{98}$$

Ex $\displaystyle\int_{x=0}^{2} \underbrace{4-x^{2}}_{f(x)}\,dx = \left. \underbrace{4x-\frac{x^{3}}{3}}_{\boxed{F(x)}} \right|_{x=0}^{2} = 5\tfrac{1}{3}$

$$F(x) = \int 4-x^{2}\,dx = \underline{4x-\frac{x^{3}}{3}}$$

$$F(2) = 4(2) - \frac{2^{3}}{3} = 5\tfrac{1}{3}$$

$$F(0) = 4(0) - \frac{0^{3}}{3} = 0$$

Ex

$$\int_0^1 x^2+2x+3 \, dx = \left.\frac{x^3}{3}+x^2+3x\right|_0^1$$

F(1) F(0)

$$=\left(\frac{1}{3}+1+3\right)-\left(\frac{0}{3}+0+0\right)$$

$$=4\tfrac{1}{3} = 4.333.$$

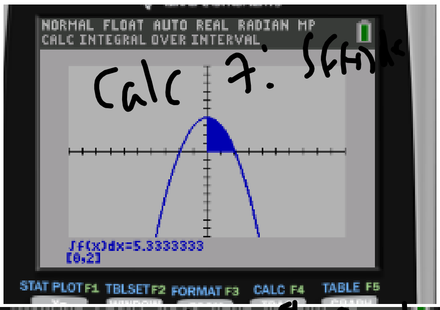
Ex $$\int_0^2 4 - x^2 \, dx = 4x - \frac{x^3}{3}\Big|_0^2$$

$$\left(8 - \frac{8}{3}\right) - \left(0 - 0\right) = 5\tfrac{1}{3}$$

$$F(x) = \int 4 - x^2 \, dx = 4x - \frac{x^3}{3}$$

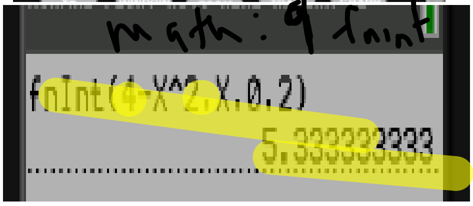
$$F(2) = 4(2) - \frac{2^3}{3} = 5\tfrac{1}{3}$$

$$F(0) = 4(0) - \frac{0^3}{3} = 0$$



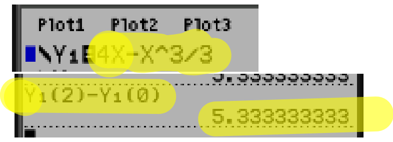
Calc 7: ∫f(x)dx

```
NORMAL FLOAT AUTO REAL RADIAN MP
CALC INTEGRAL OVER INTERVAL
∫f(x)dx=5.3333333
[0,2]
```



math: 9 fnInt

```
fnInt(4-X^2,X,0,2)
                5.333333333
```



```
Plot1  Plot2  Plot3
Y1=4X-X^3/3
Y1(2)-Y1(0)
                5.333333333
```

Written also

$$\int_{x=0}^{2} 4 - x^2 . dx = 4x - \frac{x^3}{3} \Bigg|_{x=0}^{2}$$

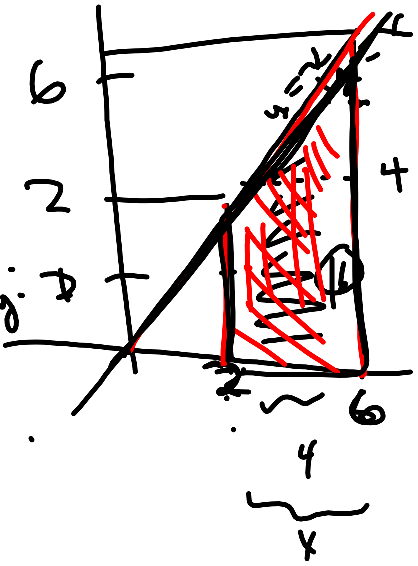
That means

$$\left(4(2) - \frac{2^3}{3}\right) - \left(4(0) - \frac{0^3}{3}\right)$$

ex.

$$\int_{2}^{6} x^{1}\,dx = \boxed{16}$$

Geometry:



$$\Big\downarrow \text{FTC}$$

$$= \frac{x^2}{2}\Bigg|_{2}^{6}$$

$$= \frac{6^2}{2} - \frac{2^2}{2} = 18 - 2 = \boxed{16}$$

$$\boxed{\int_a^b f(x)\,dx} = F(b) - F(a)$$

$$\int_a^b f\underline{(u)\,du}^{\text{Substitution}}$$

FTC Project

your regression: P_4

1:find area from a to b

$$Y_1 = Ax^4 + Bx^3 + Cx^2 + Dx + E$$

zoom 9

ANTI DERIV

calc7 a to b

$$Y_2 = Ax^5/5 + Bx^4/4 + Cx^3/3 + Dx^2/2 + Ex$$

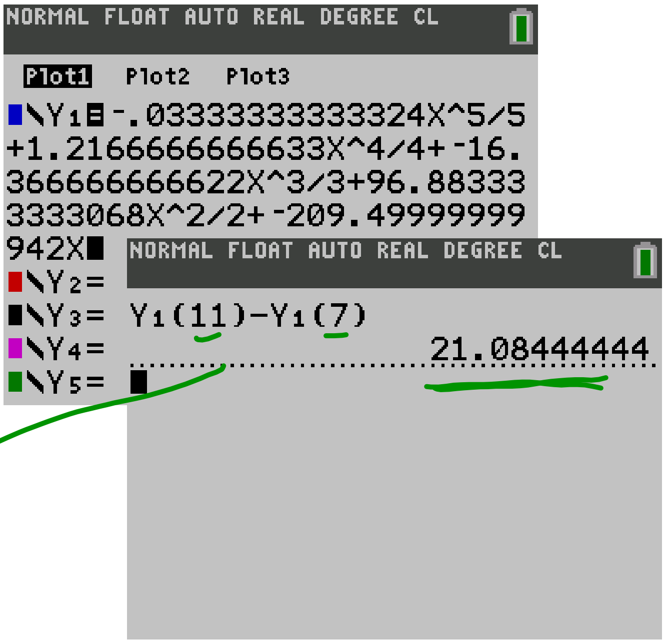
Area between a & b

$$\boxed{Y_2(b) - Y_2(a)} =$$

area under quartic regression between "a" and "b"

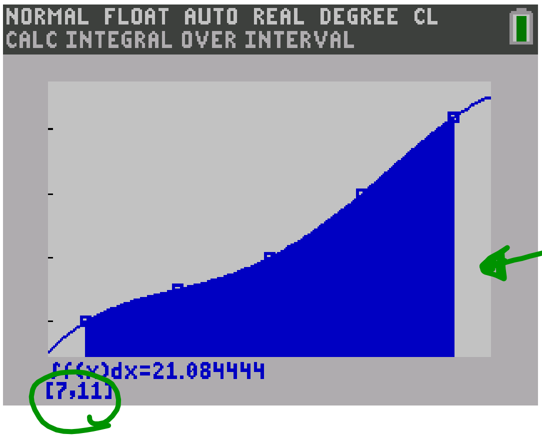
= Area

21.08 . . ~

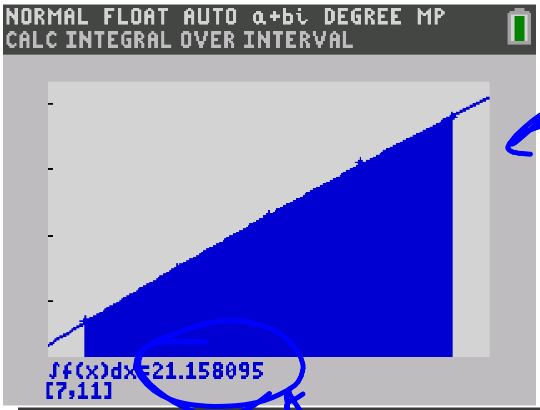


```
NORMAL FLOAT AUTO REAL DEGREE CL
Plot1  Plot2  Plot3
■\Y₁=-.0333333333333324X^5/5
+1.2166666666633X^4/4+-16.
3666666666622X^3/3+96.88333
3333068X^2/2+-209.49999999
942X
■\Y₂=
■\Y₃=
■\Y₄=
■\Y₅=
```

```
NORMAL FLOAT AUTO REAL DEGREE CL
Y₁(11)-Y₁(7)
                    21.08444444
```

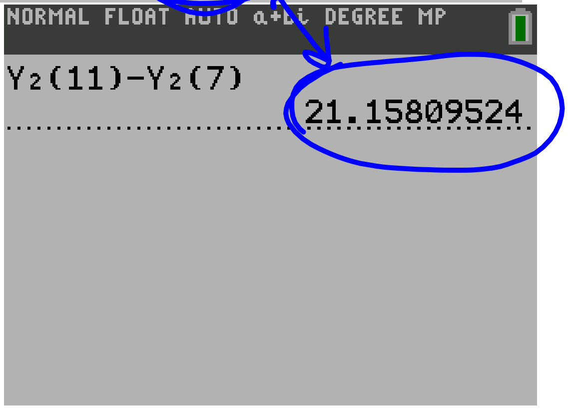


```
NORMAL FLOAT AUTO REAL DEGREE CL
CALC INTEGRAL OVER INTERVAL
∫f(x)dx=21.084444
[7,11]
```



```
NORMAL FLOAT AUTO a+bi DEGREE MP
CALC INTEGRAL OVER INTERVAL
∫f(x)dx=21.158095
[7,11]
```

Calculator

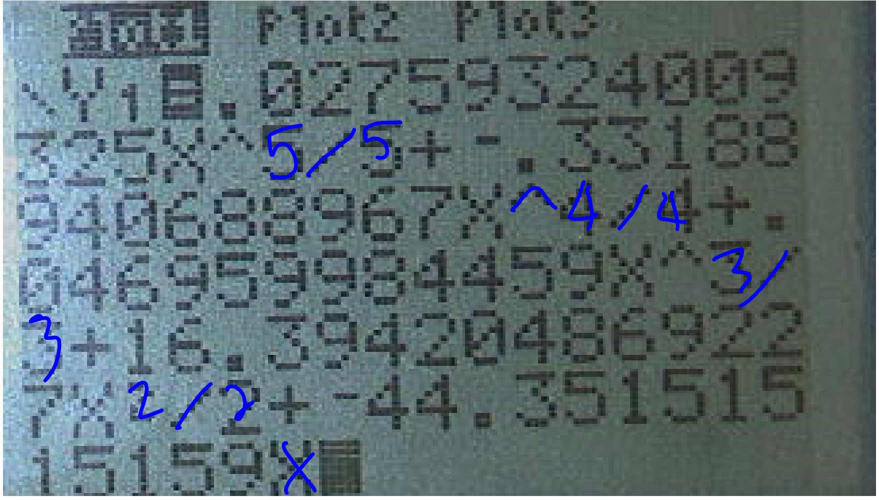


```
NORMAL FLOAT AUTO a+bi DEGREE MP
Y2(11)-Y2(7)
                    21.15809524
```

Verifies F.T.C.

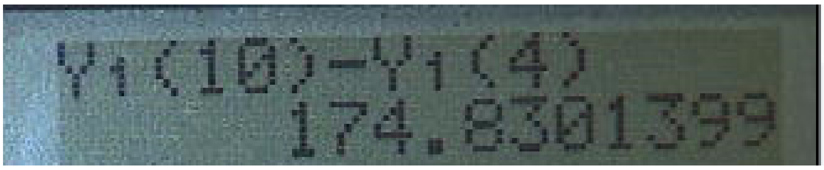
Anti. derivative

edit in calculator



```
Plot1 Plot2 Plot3
\Y1■.02759324009
325X^5/5+-.33188
940688967X^4/4+.
046959984459X^3/
3+16.39420486922
7X^2/2+-44.351515
15159X■
```

exit and evaluate

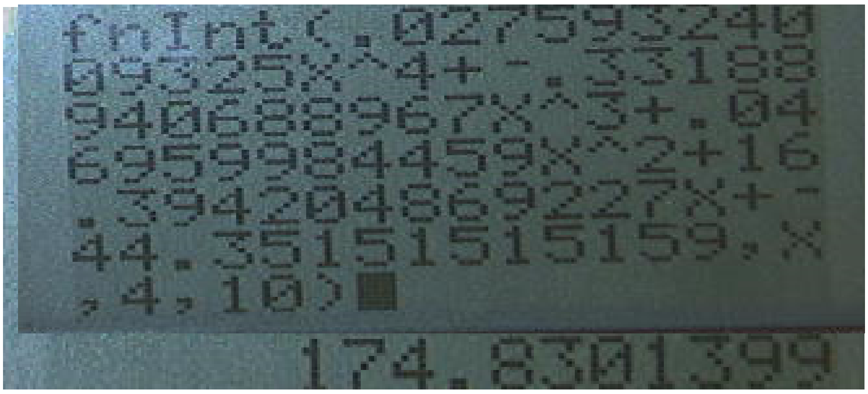


```
Y1(10)-Y1(4)
          174.8301399
```

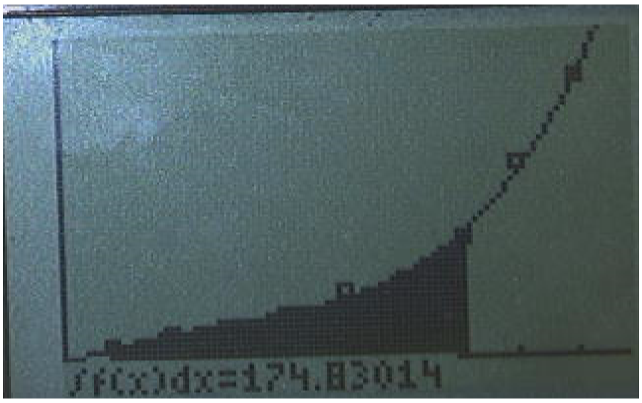
area under quartic regression between 4 and 10 - you use your own points

another method Math:7 fnint

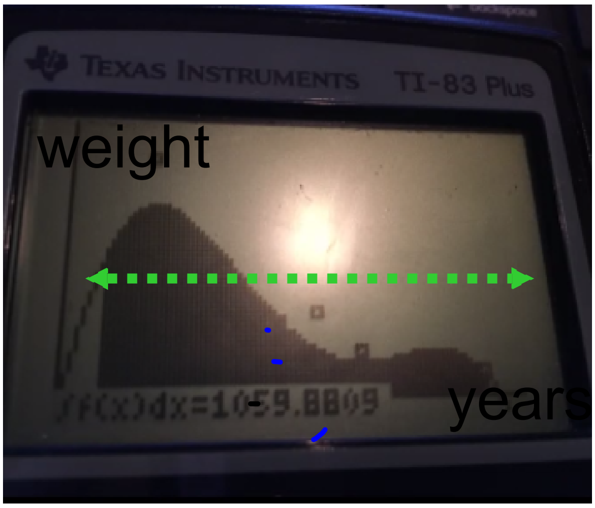
vars 5 >>1

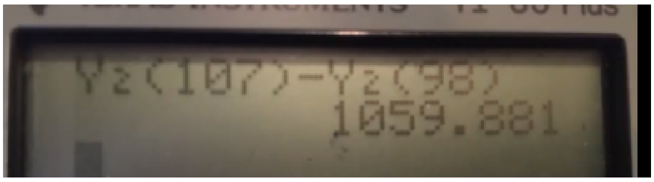


```
fnInt(.0275932400
09325X^4+-.33188
940688967X^3+.04
6959984459X^2+16
.394204869227X+-
44.35151515159,X
,4,10)■
          174.8301399
```



∫f(x)dx=174.83014





```
Y₂(107)-Y₂(98)
                1059.881
```

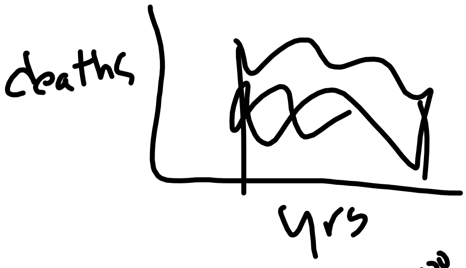
1059 year*lbs

but if b-a=9

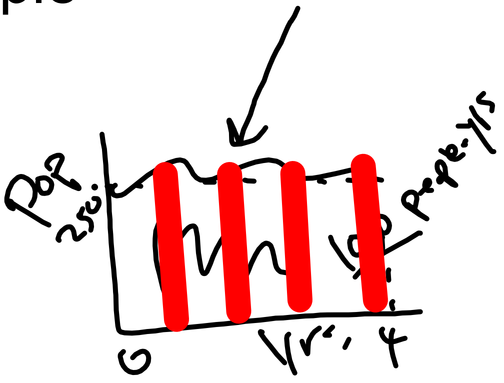
1059/9=117.67 lbs

Average value of weight between the years 1998 and 2007, according to the quartic regression, is 118 lbs

Deaths versus living people



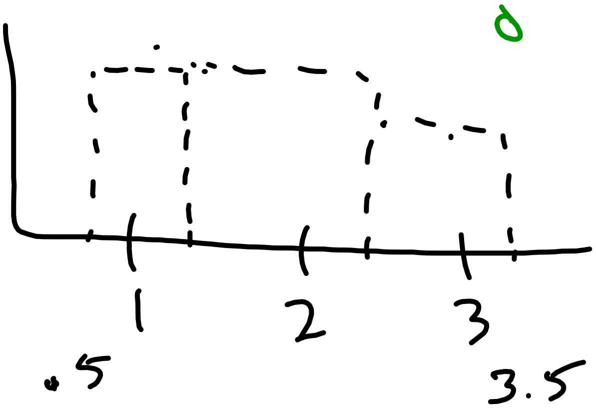
1000 death·yrs. = 1000 deaths



$$\text{Ave Value} = \frac{1000}{4} = 250$$

Years?

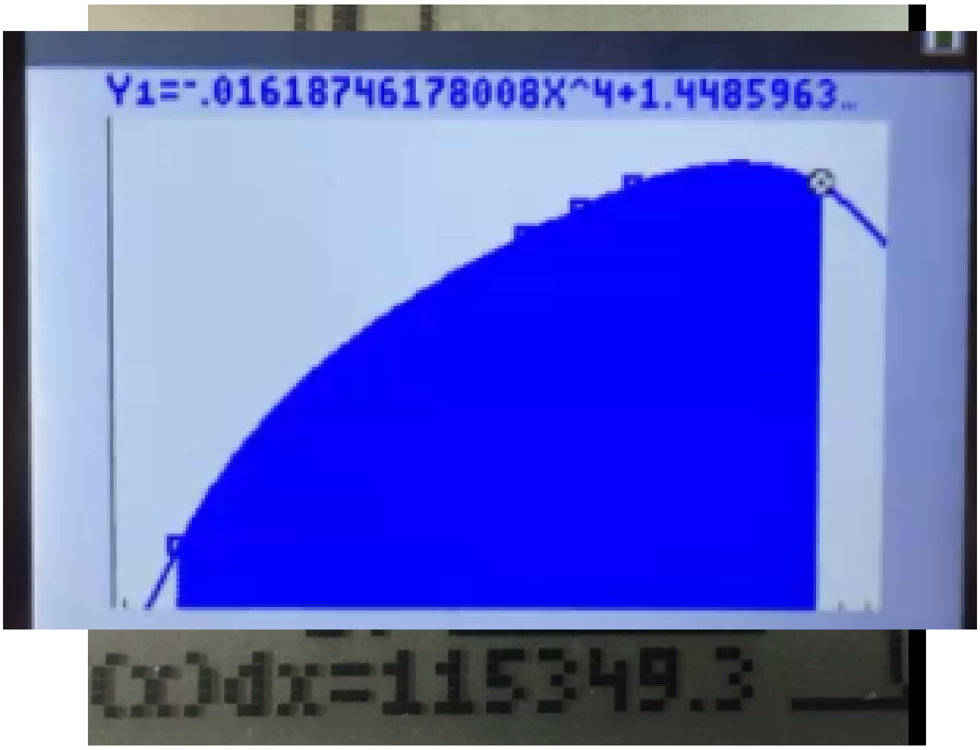
$\int_{0}^{3}$ $\int_{.5}^{3.5}$



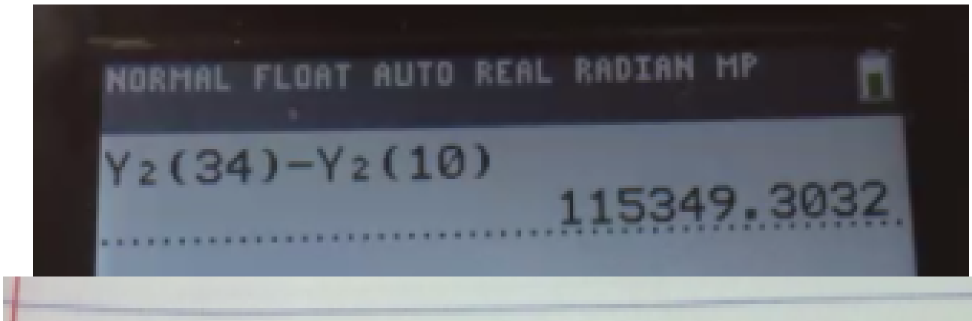
0 - 3 1 - 4

.5 - 3.5

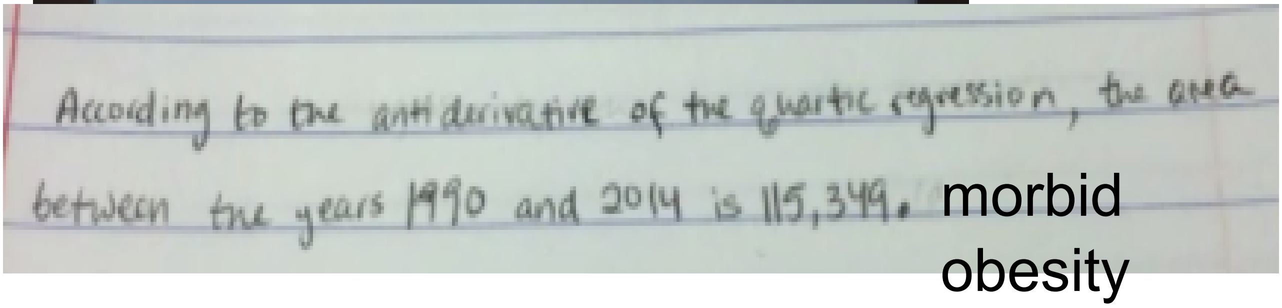
Project Examples



```
Y1=⁻.01618746178008X^4+1.4485963…
(x)dx=115349.3
```



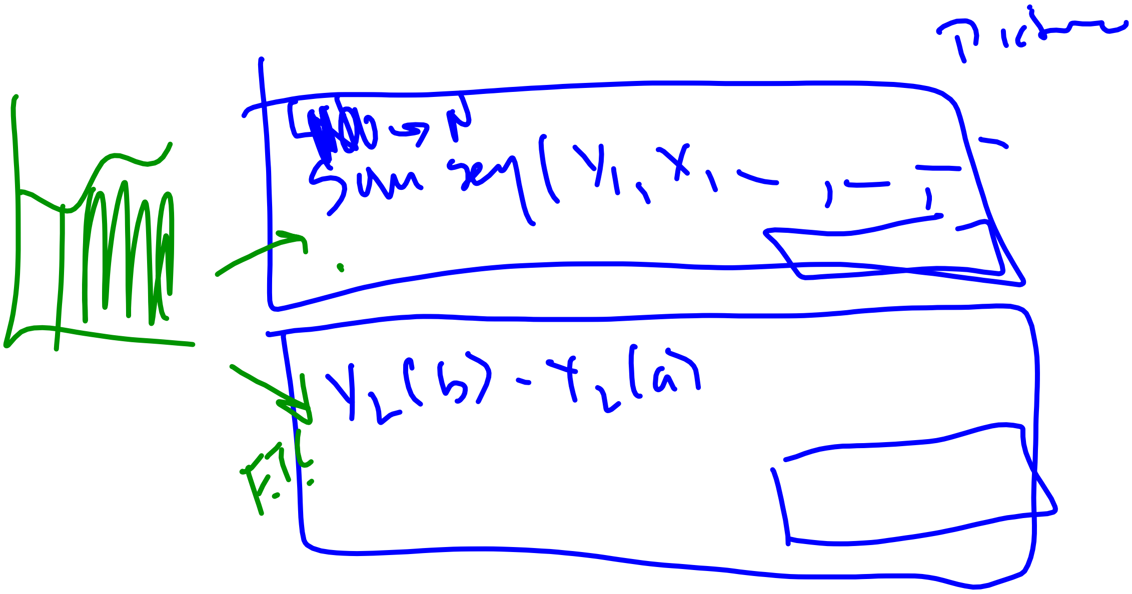
```
NORMAL FLOAT AUTO REAL RADIAN MP
Y2(34)-Y2(10)
                    115349.3032
```



According to the antiderivative of the quartic regression, the area between the years 1990 and 2014 is 115,349.

morbid obesity years

115349/24=4806 cases of MO


Picture
→ N
sum seq(Y1, X1, _ , _ , _)
Y2(b) - Y2(a)
Fill

$$\sum_{i=1}^{400} Y_1(a+\Delta i) \quad \longleftarrow \text{Math}$$

$$\int_a^b Y_1(t)\,dt = Y_2(b) - Y_2(a) \quad \swarrow \text{F.T.C.}$$

Preview of Rest of class

Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$

<u>UNDO IT</u> with substitution.

$$\int f'(g(x)) \cdot \underline{g'(x)\,dx} = \int f'(u)\,du$$

Inside Function $u = g(x)$

$u' = g(x)dx$

$F(u) + c$

$F(g(x)) + c$

Ex

$$\int \underbrace{\sin(\boxed{x^2})}\cdot \underbrace{2x\,dx}$$

$$\|$$

$u = x^2$

$du = 2x\,dx$

replace "u"

$$\boxed{\int \sin(u)\,du}$$

$$= {}^{-}\cos(u) + C$$

$$\boxed{= {}^{-}\cos(x^2) + c}$$

$$\downarrow$$

$\checkmark \quad + \; +\sin(x^2)\cdot 2x$

Ex

$$\int e^{x^2}\cdot 2x\,dx = \int e^u\,du$$

$u = x^2$

$du = \boxed{2x\,dx}$

$$= e^u + c$$

□ $$= \boxed{e^{x^2} + c}$$

$$\int e^{\sinh x}\cosh x\,dx$$

↓

$u = \sinh x$

$du = \cosh x\,dx$

$$\int e^u\,du$$

$$= e^u + c$$

$$\boxed{= e^{\sinh x} + c}$$

Ex

$$\int \frac{\sin x}{\cos x}\,dx = \int \frac{1}{u}(-du)$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$$\downarrow$$

$$-\int \frac{1}{u}\,du$$

$$-\ln|u| + C$$

$$\boxed{-\ln|\cos x| + C}$$

$$\boxed{\int \frac{\sin x}{\cos x}\,dx} = \boxed{\int \tan x\,dx} = \ln|\sec x| + C$$

$$\int \frac{\sin x}{\cos x}\,dx = \int -\frac{du}{u}$$

$$= -\int \frac{1}{u}\,du$$

$$= -\ln|u| + C$$

$$= \boxed{-\ln|\cos x| + C}$$

$$= \ln|\sec x| + c$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$\rightarrow \int \tan x\,dx$

$$\int \tan x\,dx = \ln|\sec x| + c$$

Ex

$$\int \frac{\sin x}{\cos x} dx = \int \frac{-du}{u}$$

$u = \cos x$

$du = -\sin x\, dx$

$-du = \sin x dx$

$$= -\int \frac{1}{u} du$$

$$= -\ln|u| + c$$

$$= -\ln|\cos x| + c$$

What's
the difference
between the
following problems... .

$$\boxed{\int x \cos x^2 \, dx} = \int \cos u \, \frac{du}{2}$$

$$= \frac{1}{2} \int \cos u \, du$$

$$= \frac{1}{2} \sin u + c$$

$$= \boxed{\frac{1}{2} \sin(x^2) + c}$$

$u = x^2$

$du = 2x \, dx$

$\frac{du}{2} = x \, dx$

$$\boxed{\int_{x=0}^{4} x\cos(x^2)\,dx} = \int_{u=0}^{u(4)=16} \cos(u)\,\frac{du}{2}$$

$$u(x) = x^2$$

$$du = 2x\,dx$$

$$\frac{du}{2} = \frac{2x\,dx}{2}$$

$$\frac{du}{2} = x\,dx$$

$$= \frac{1}{2}\int_{0}^{16} \cos(u)\,du$$

$$= \frac{1}{2}\sin(u)\Big|_{0}^{16}$$

$$= \frac{1}{2}\sin(16) - \frac{1}{2}\sin(0)$$

$$= \frac{1}{2}\sin(16)$$

later...

$$u(4) \rightarrow 16$$

$$\int_{x=0}^{4} x\cos(x^2)\,dx = \int_{u=0}^{16} \cos u \,\frac{du}{2}$$

$$u(0) \rightarrow 0$$

$$u = x^2$$

$$\frac{du}{2} = \frac{2\boxed{x\,dx}}{2}$$

$$= +\left.\frac{\sin u}{2}\right|_{0}^{16}$$

$$= +\frac{\sin 16}{2} + -\frac{\cancel{\sin(0)}}{2}$$

$$\int_{-2}^{2} x\sin(x^2)\,dx = \int_{4}^{4} \sin(u)$$

$u = x^2$

$$= 0$$