Agenda

Midterm Review

Project Ave Value

Preview

HonorLock (Google Chrome)

Go to Blackboard.

Lessons: HonorLock

Lessons: midterms

Sec. Ex. 13a - 3.1 Section Exercise 13a

Correct Response:

Give your final answers as reduced improper fractions.

Use Newton's method with the given x_0 to compute x_1 and x_2 by hand.

$x^3 + 3x^2 + 6 = 0,\ x_0 = 1$

$x_1 = -\frac{1}{9}$ and $x_2 = \frac{4349}{459}$

Midterm Review

$X - Y_1 / \text{nderiv}(Y_1)$

$Y_1 = x^3 + 3x^2 + 6 \qquad Y(-1/9)$

$= \frac{-1}{9^3} + 3\left(\frac{1}{9^2}\right) + 6 =$

-1/9³+3/81+6
6.035665295
Ans▶Frac
4400/729

$Y_1' = 3(x)^2 + 6x$

$Y_1'(-1/9) = 3\left(\frac{1}{9^2}\right) + 6\left(-\frac{1}{9}\right)$

3/81-6/9
-.6296296296
Ans▶Frac
-17/27

$X_2 = -\frac{1}{9} - \frac{4400/729}{-17/27}$

$-\frac{1}{9} + \frac{4400}{729} \cdot \frac{27}{17} =$

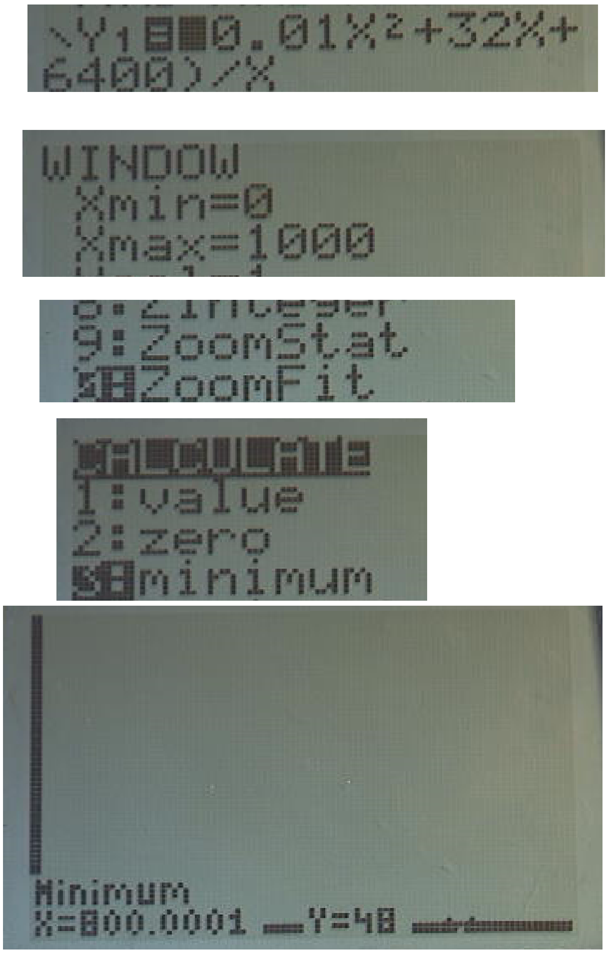
4400*27/729/17
9.586056645
Ans-1/9
9.474945534
Ans▶Frac
4349/459

Suppose that it costs a company $C(x) = 0.01x^2 + 32x + 6400$ dollars to manufacture x units of a product. For this cost function, the average cost function is $\overline{C}(x) = \frac{C(x)}{x}$.

Find the value of x that minimizes the average cost. The cost function can be related to the efficiency of the production process.

$x =$ ______

$$= \frac{0.01x^2 + 32x + 6400}{x}$$



y = .01x+32+6400x^-1

y'=.01-6400x^-2=0

.01=6400x^-2

.01/6400=x^-2

x^2=6400/.01=640000

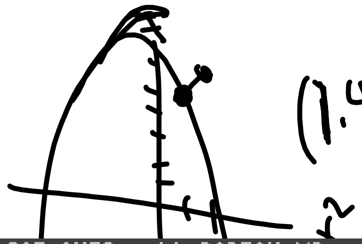
x=800

Min Distance from Point

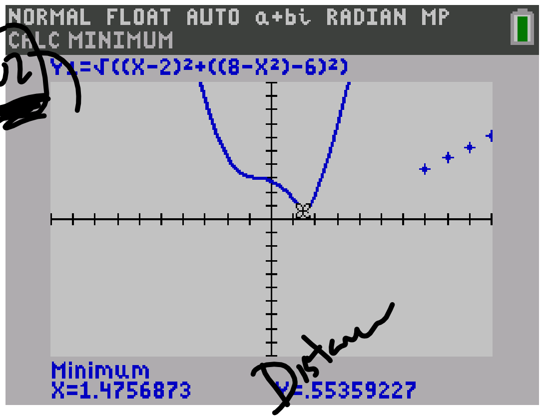
$(2, 6)$ To $y = 8 - x^2$

$d(x, \cancel{y}) = \sqrt{(x-2)^2 + (y-6)^2}$

$d(x) = \sqrt{(x-2)^2 + (8-x^2-6)^2}$



$(1.47\ldots, \boxed{5.82})$



Distance

NORMAL FLOAT AUTO a+bi RADIAN MP

Ans▶Frac

.2923873938

.29238739▶Frac

.29238739

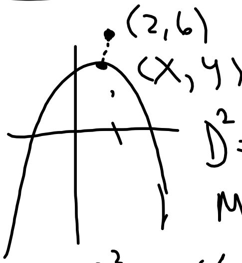
$8-X^2$

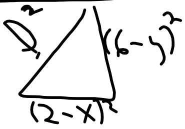
5.822346961

X

1.475687311

Min Distance from Point $(2,6)$ to $y = 8 - x^2$





$$D^2 = (6-y)^2 + (2-x)^2$$

Minimize This Main Idea D^2

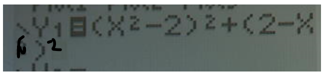
$$D^2_{(x,y)} = (6-y)^2 + (2-x)^2$$

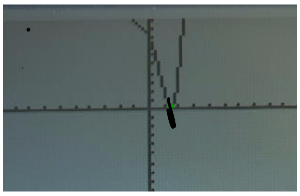
Constraint: $(6-(8-x^2))^2 + (2-x)^2$

$$D^2_{(x)} = (x^2-2)^2 + (2-x)^2$$

$$= x^4 - 4x^2 + 4 + 4 - 4x + x^2$$

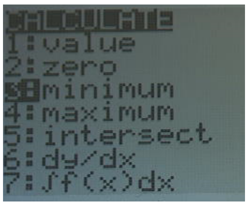
$$D^2 = x^4 - 3x^2 - 4x + 8$$

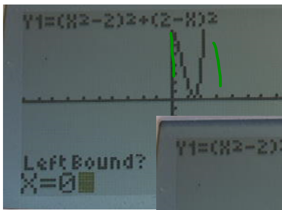


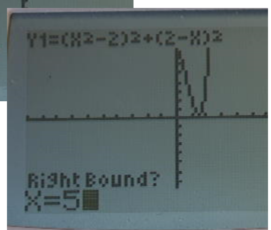


zoom 6:

(zoom 0: if this doesnt work



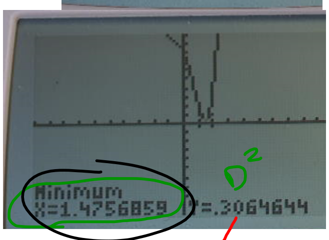




know x = 1.4757

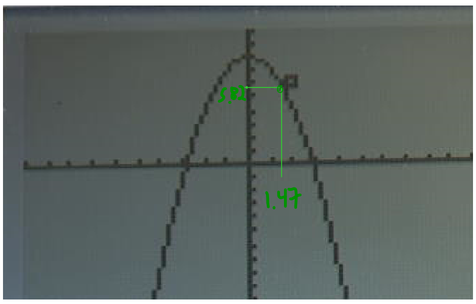
y = 8 - (1.4757)^2

= 5.8223

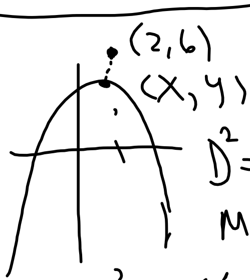


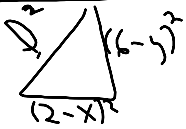
D^2= .3064644

D= .5536



Min Distance from Point $(2,6)$ To $y = 8 - x^2$





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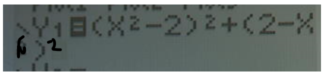

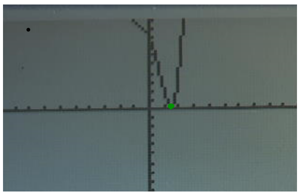

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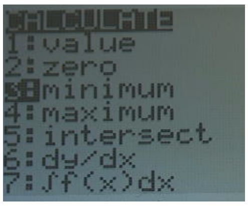

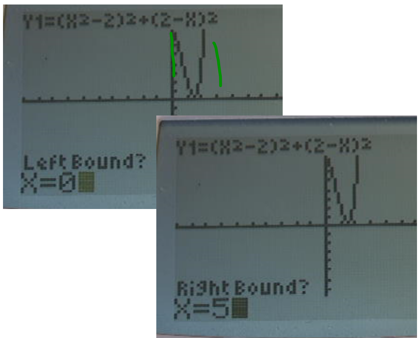

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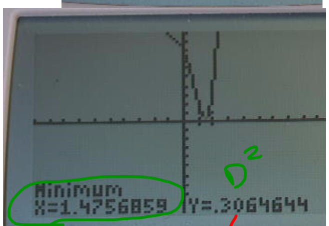

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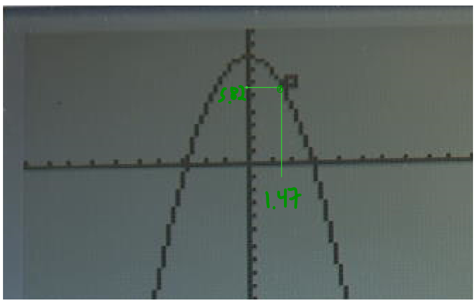


Determine the intervals where the graph of the given function is concave up and concave down.

$f(x) = 16x^{4/3} + 3x^{1/3}$

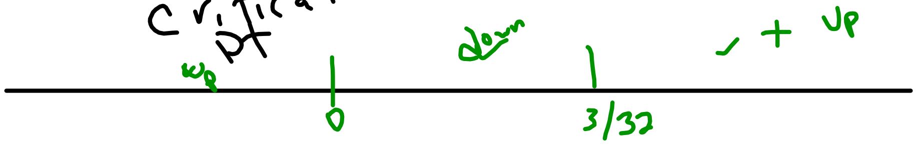
Concave up: $x >$ [3/32] and $x <$ [0], concave down: [0] $< x <$ [3/32]

$$y' = \frac{64}{3} x^{1/3} + x^{-2/3}$$

$$y'' = \frac{64}{9} x^{-2/3} - \frac{2}{3} x^{-5/3} = 0$$

$$y'' = \frac{2}{3} x^{-5/3} \left(\frac{32}{3} x^{3/3} - 1 \right) = 0$$

$x = 0$ critical pt $\qquad x = \frac{3}{32}$



$y = x^3$

$y' = 3x^2$

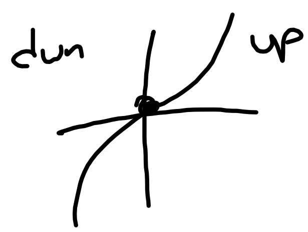
$y'' = 6x = 0$

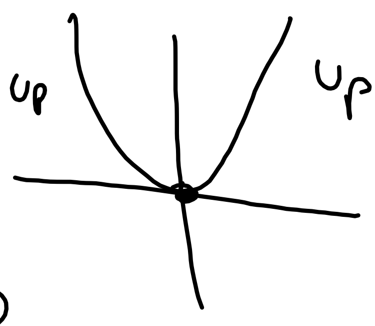
$y = x^4$

$y' = 4x^3$

$y'' = 12x^2 = 0$

Inflection

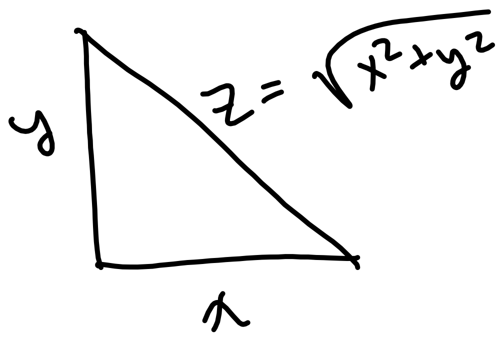




Not Inflection

A car is traveling at 80 mph due south at a point 1/2 mile north of an intersection. A police car is traveling at 70 mph due west at a point 1/4 mile due east of the same intersection. At that instant, the radar in the police car measures the rate at which the distance between the two cars is changing. What does the radar gun register? Round your answer to three decimal places.

Your Answer:



$$x^2+y^2=z^2$$

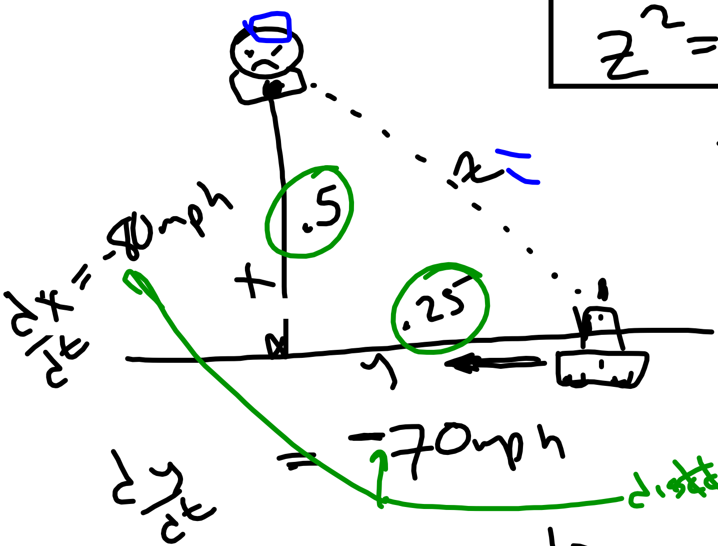
$$2xx'+2yy'=2zz'$$

$$\frac{xx'+yy'}{\sqrt{x^2+y^2}}=z'$$

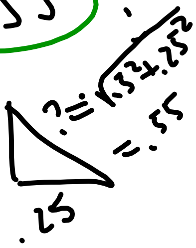
A car is traveling at 80 mph due south at a point 1/2 mile north of an intersection. A police car is traveling at 70 mph due west at a point 1/4 mile due east of the same intersection. At that instant, the radar in the police car measures the rate at which the distance between the two cars is changing. What does the radar gun register? Round your answer to three decimal places.

Your Answer: 102

$$\boxed{z^2 = x^2 + y^2}$$



$$z = \frac{\sqrt{5}}{4} = .559$$



$$\frac{dz}{dt} = ?$$

distance is closing

$$z\frac{dz}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$

$$\frac{(.55)\frac{dz}{dt}}{.55} = \frac{(.5)(-80) + (.25)(-70)}{.55}$$

$$= -102.86 \text{ mph}$$

A company's revenue for selling x (thousand) items is given by $R(x) = \dfrac{168x - x^2}{x^2 + 168}$.

Find the value of x that maximizes the revenue and find the maximum revenue.

$x =$ 12, maximum revenue is $

$$\frac{168\cdot 12 - 12^2}{12^2 + 168} = 6$$

$R' = 0$

$(x^2+168)(168-2x) - (168x - x^2)(2x) = 0$

$-\cancel{2x^3} + 168x^2 - 2(168)x + 168^2 - 2\cdot 168x^2 + \cancel{2x^3} = 0$

$x^2 - 2x + 168 - 2x^2 = 0$

$-x^2 - 2x + 168 = 0$

$x^2 + 2x - 168 = 0$

$(x-12)(x+14) = 0$

$x = 12, -14$

A company's revenue for selling x (thousand) items is given by $R(x) = \dfrac{168x - x^2}{x^2 + 168}$.

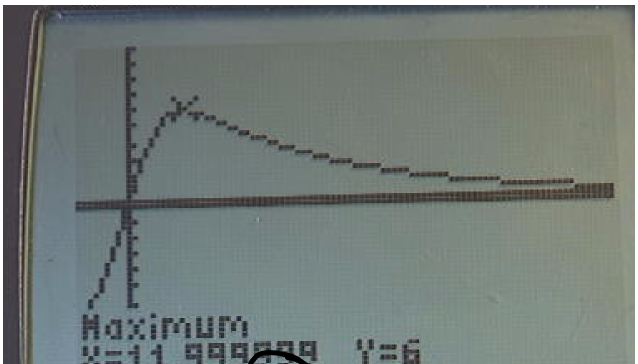
Find the value of x that maximizes the revenue and find the maximum revenue.

x = [ ], maximum revenue is $[ ]

$$R' = (x^2+168)(168-2x) - ((168x-x^2)(2x)) = 0$$

$$\cancel{168x^2} - \cancel{2x^3} + \cancel{168^2} - 336x - \cancel{336x^2} + \cancel{2x^3} = 0$$

$$-168x^2$$



12 6

Find the general antiderivative. Use c as the constant of integration.

$$\int x^{1/4}(x^{5/4}-32)\,dx$$

$$\int x^{6/4} - 32x^{1/4}\,dx$$

$$= \frac{x^{6/4+4/4}}{10/4} - 32\,\frac{x^{1/4+4/4}}{5/4} + C$$

$$\frac{4}{10}x^{10/4} - \frac{4}{5}\cdot 32\,x^{5/4} + C$$

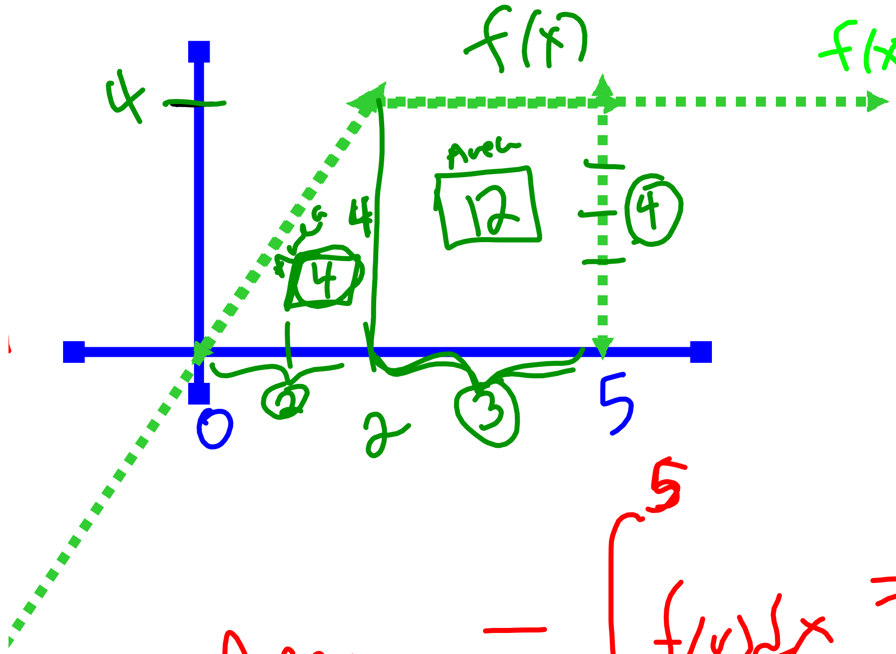
Find the general antiderivative. Use c as the constant of integration.

$$\int x^{1/4}(x^{5/4} - 32)\,dx$$

$$\int x^{6/4} - 32x^{1/4}\,dx$$

$$\frac{x^{10/4}}{10/4} - \frac{32x^{5/4}}{5/4} + C$$

$$\frac{2}{5}x^{5/2} - \frac{128}{5}x^{5/4} + C$$



$$f(x) = \begin{cases} 2x & x < 2 \\ 4 & x \geq 2 \end{cases}$$

$$\text{Area} = \int_0^5 f(x)\,dx = 12 + 4 = 16$$

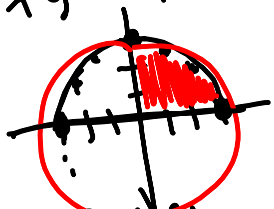
area

Area under

$$\int_0^3 \sqrt{9-x^2}\,dx$$

$y = \sqrt{9-x^2}$

$x^2+y^2=9$



Circle

radius: 3

$$\int_0^3 -\sqrt{9-x^2}\,dx = \frac{1}{4}\,9\pi$$

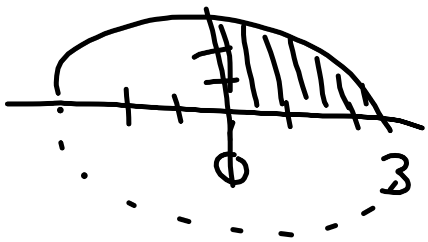
Area a circle

πr^2

9π

$$\int_0^3 \sqrt{9-x^2}\, dx = \frac{9\pi}{4}$$

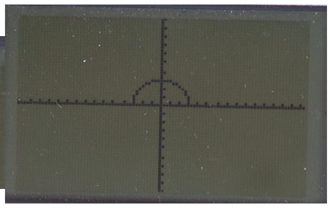
$y = \sqrt{9-x^2}$

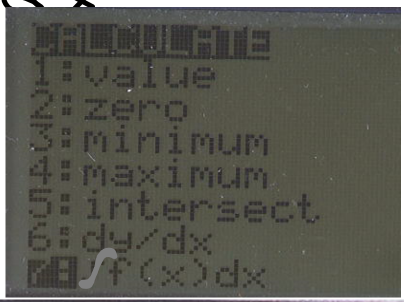


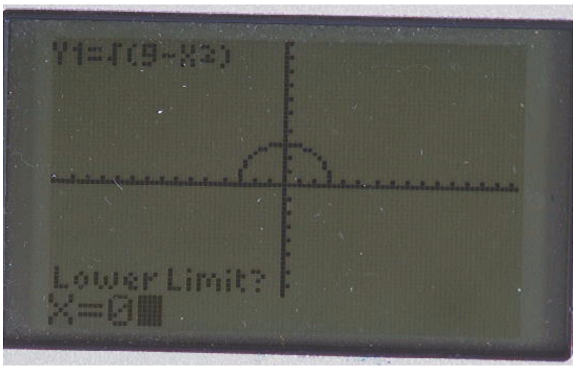
Area circle.
of $r = 3$
$\pi r^2 = 9\pi$

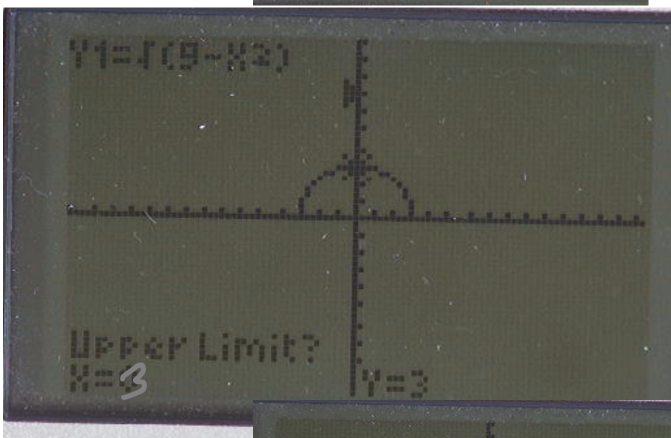
$Y_1 = \sqrt{(9 - x^2)}$

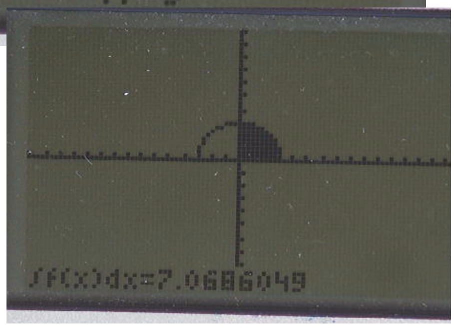
Calc 7: $\int f(x)dx$











Evaluate $\int 4x^3\, dx$. $= 4 \underbrace{\int x^3\, dx}$

$= \cancel{4}\dfrac{x^4}{\cancel{4}} + C$

$= x^4 + C$

Check. $\frac{d}{dx} x^4 = 4x^3$ ✓

$$\frac{d}{dx}\left(\frac{x^n}{n}\right) = \frac{n x^{n-1}}{n}$$

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$$

$$\frac{x^{n+1}}{n+1} + c = \int x^n \, dx$$

Power Rule

$$\boxed{\int x^n \, dx = \frac{x^{n+1}}{n+1} + C}$$

"Product" Rule.

$$\boxed{\int c\, f(x)\, dx = c \int f(x)\, dx}$$

Sum Rule

$$\boxed{\int f(x) + g(x)\, dx = \int f(x)\, dx + \int g(x)\, dx}$$

$$F(x) + G$$

$$\int e^x \, dx = e^x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1}(x) + c$$

Special Rule

$$\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + c$$

Evaluate $\int (5\cos x + 9x^9)\,dx$. Use c as the constant.

$$\int 5\cos x\,dx + \int 9x^9\,dx$$

$$5\int \cos x\,dx + 9\int x^9\,dx$$

$$5\sin x + \frac{9x^{10}}{10} + C$$

Evaluate the given integral.

$$\int e^{14x}\,dx = \frac{e^{14x}}{14} + C$$

$$\frac{d}{dx}\boxed{\frac{e^{14x}}{14}} = \frac{\boxed{e^{14x}} \cdot \cancel{14}}{\cancel{14}}$$

$$\int \cos 5x \, dx = \frac{\sin(5x)}{5} + C$$

$$\frac{d}{dx} \frac{\sin(5x)}{5}$$

$$\frac{\cos(5x)}{\cancel{5}} \cdot \cancel{5}$$

Find the general antiderivative.

$$\int \frac{4x}{x^2+1} = 2\int \frac{2x}{x^2+1}\,dx$$

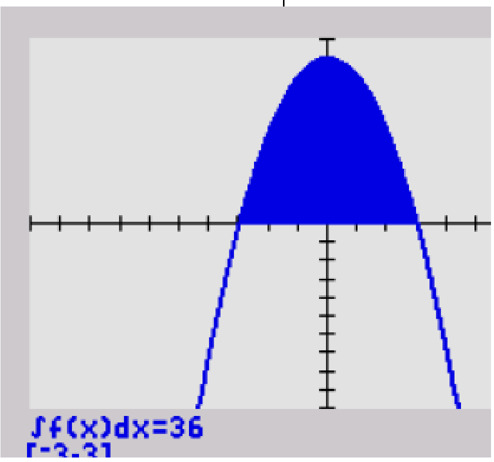
$$= 2\boxed{\int \frac{f'}{f}\,dx \quad \text{special}}$$

$$= 2\ln|x^2+1| + C$$

f
f'

Write the given (total) area as an integral or sum of integrals.

The area above the x−axis and below $y = 9 - x^2$

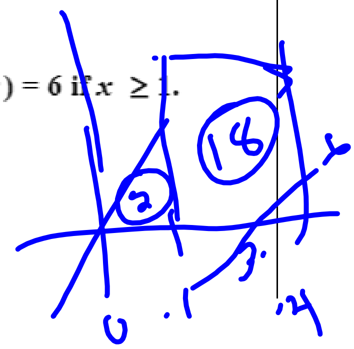


- A. $\int_0^6 -2x \; dx$
- B. $\int_0^6 (9 - x^2) \; dx$
- C. $\int_{-3}^3 (9 - x^2) \; dx$
- D. $\int_{-3}^3 -2x \; dx$

Write your answer in decimal form.

Compute the area of $\int_0^4 f(x)\ dx$ **for** $f(x) = 4x$ **if** $x < 1$**, and** $f(x) = 6$ **if** $x \geq 1$**.**

Your Answer: 20



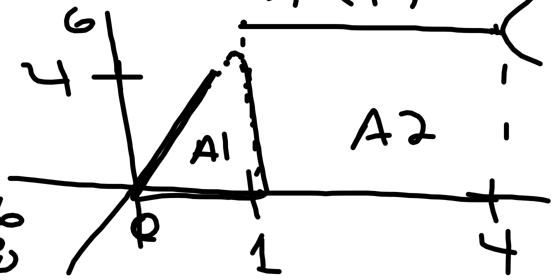
$\frac{1}{2} \cdot 4$

$$f(x) = \begin{cases} 4x & x < 1 \\ 6 & x \geq 1 \end{cases}$$

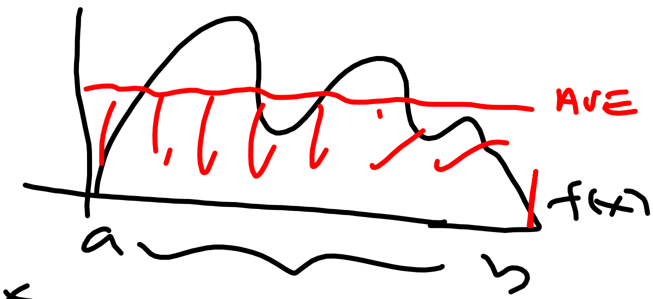
A1= 2

A2=3.6

18



Average Value

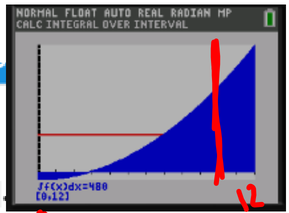


$$\text{Height} = \frac{\text{Area}}{\text{length}} \quad \frac{\int_a^b f(x)\,dx}{b-a} = \underline{\text{ave value}} \quad \text{Height}$$

Round your answer to the nearest whole number.

Compute the average value of the function $f(x) = x^2 - 8$ on the interval [0,12].

Your Answer:



0 12

Use calculator

$Y_1 = x^2 - 8$

Calc 7: $\int f(x)dx$

Lower: 0

Upper: 12

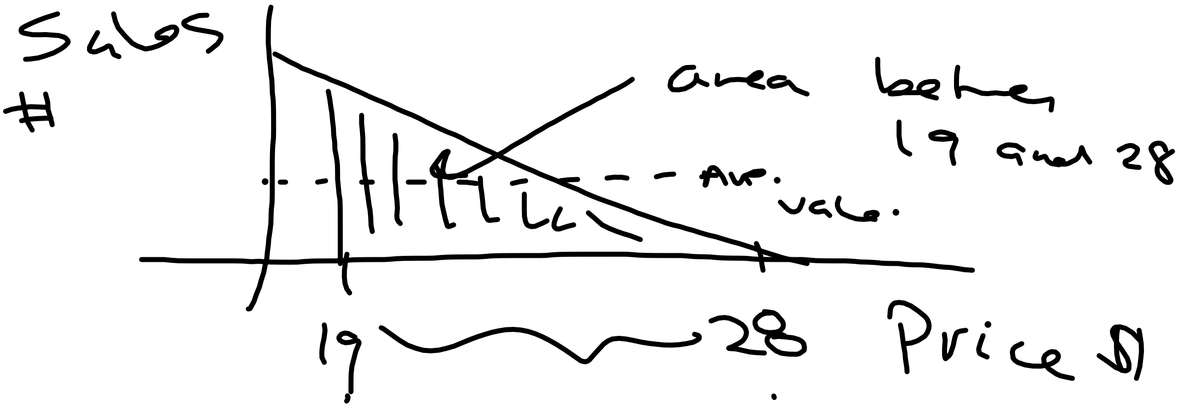
Answer: 480

$480/12 = 40$

Average Value

$$\frac{\text{area}}{\text{length}} \qquad \frac{\int_a^b f(x)\,dx}{b-a}$$

Project Review

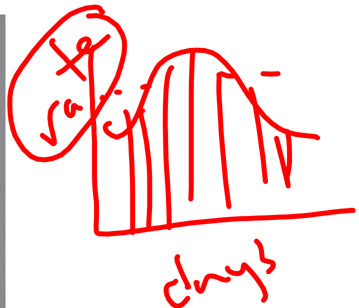


Area = 460 $·#

Area Sales $\frac{460\ \$\ \#}{(28-19)\ \$} = 40\ \#$

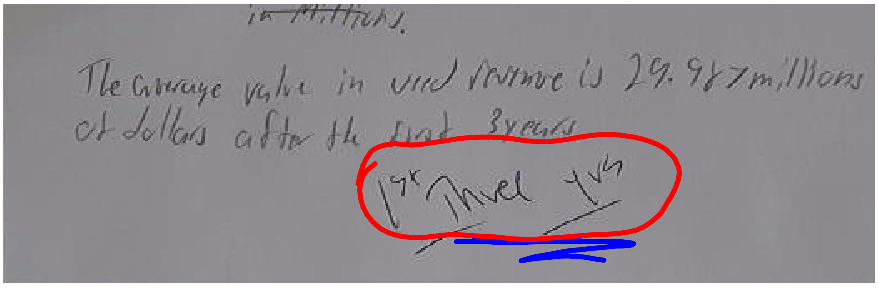
Conclusion in words:

According to the cubic regression for the population of NJ we determined the average ~~growth~~ of people per year was 7,279,715 People / yr.



average population of NJ between 1960 and 1990 was 7.2 million

$\int_1^3$

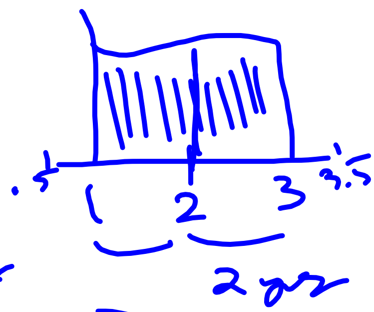

in millions.

The average value in used revenue is 29.987 millions of dollars after the first 3 years

1st Three yrs

Three years

$$\frac{\int_1^3 f(x)\,dx}{(3-1)} =$$



$$\frac{\int_{.5}^{3.5} f(x)\,dx}{3}$$

Preview

Summation Notation

Approx Area with Rectangles

Limits with Area

Fundamental Theorem of Calculus

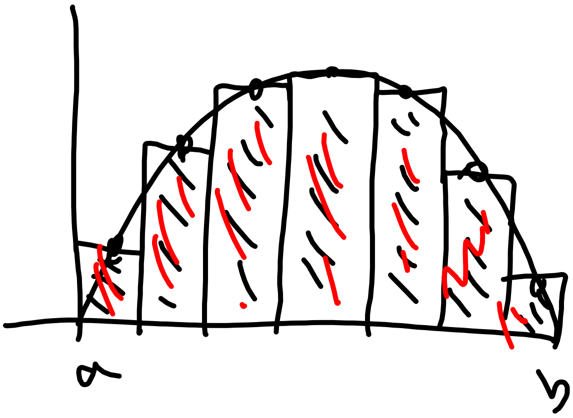
Integration with Substitution

$$\boxed{\Sigma}$$

Sigma Notation

$$\sum_{\underset{\text{index}}{i=1}}^{3} \underbrace{2i^2+7} = \underbrace{\boxed{2(1)^2+7}}_{i=1} + \underbrace{\boxed{2(2)^2+7}}_{i=2} + \underbrace{\boxed{2(3)^2+1}}_{i=3}$$

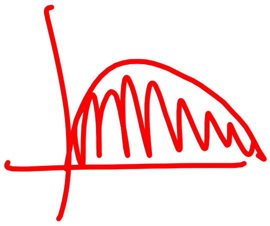
$$\boxed{\text{sum}(\text{seq}(2x+7, x, 1, 3, 1)) = 45}$$



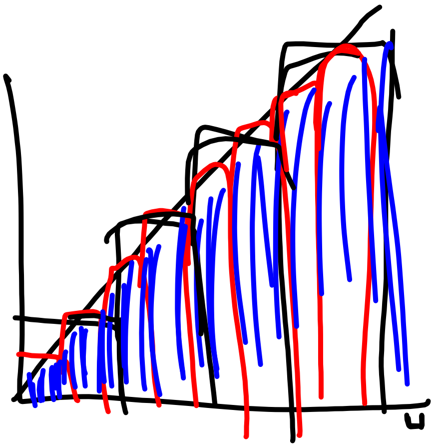
Approximate

Area $\int_a^b f(x)dx$

with rectangles



Limits



$\cancel{A}A(n)$

$$\lim_{n \to \infty} AA(n) = \int_a^b f(x)\,dx$$

$\infty \cdot 0 =$ area under curve

Fundamental Theorem

$$\boxed{\int_a^b \underline{f(x)}\,dx = F(\underline{b}) - F(\underline{a})}$$

Area under curve

Anti derivative

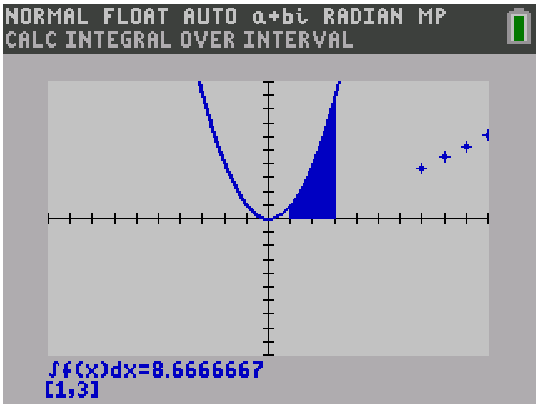
$\int f(x)\,dx$

Ex

$$\int_1^3 \boxed{x^2}\,dx$$

$$= \left.\boxed{\frac{x^3}{3}}\right|_1^3 = \frac{\boxed{3}^3 - \boxed{1}^3}{3} = 27-1$$

$$= \frac{26}{3}$$



$8^2/_3$

Integration w/ substitution

$$\frac{d}{dx}\sin(e^x) = \cos(e^x)\cdot e^x$$

$$\int \cos(e^x)\, e^x\, dx = \sin(e^x) + C$$

$u = e^x$

$\int \cos u\, du$ $\quad du = e^x dx$

$\sin u + C$

$\sin e^x$