Agenda

Review Quiz #6 / Midterm Review

Definite Integrals

Anti derivatives

Average Value

Review Quiz 6

Give your final answer as a reduced fraction.

Evaluate $\lim\limits_{x \to 81} \dfrac{4\sqrt{x} - 36}{x - 81}$. $= \dfrac{4 \cdot 9 - 36}{81 - 81} = \dfrac{0}{0}$

$1x$ L'HR.

$\lim\limits_{x \to 81} \dfrac{4 \cdot \frac{1}{2} x^{-1/2}}{1} = 2 \cdot \dfrac{1}{9} = \dfrac{2}{9}$

$.222$

$\lim\limits_{x \to 81} \dfrac{4\sqrt{x} - 36}{x - 81} =$ ☐

Give your final answer as a reduced fraction.

Evaluate $\lim_{x \to 81} \dfrac{4\sqrt{x} - 36}{x - 81}$.

$\lim_{x \to 81} \dfrac{4\sqrt{x} - 36}{x - 81} = \dfrac{0}{0}$

$\lim_{x \to 81} \dfrac{4 \cdot \frac{1}{2} x^{-1/2}}{1}$

$= 2 \cdot \dfrac{1}{\sqrt{81}} = \dfrac{2}{9}$

$\dfrac{4\sqrt{x} + 36}{4\sqrt{x} + 36} = \lim_{x \to 81} \dfrac{16x - 36^2}{(x - 81)(4\sqrt{x} + 36)}$

$= \lim_{x \to 81} \dfrac{16\cancel{(x - 81)}}{\cancel{(x - 81)}(4\sqrt{x} + 36)}$

$= \dfrac{16}{4 \cdot 9 + 36} = \dfrac{16}{72}$

Determine where the graph of $f(x) = 2x^3 + 27x^2 + 34x - 61$ is concave down.

$f(x)$ is concave down where x $< -\frac{54}{12}$.

$f' = 6x^2 + 54x + 34$

$f'' = 12x + 54$

$12x + 54 < 0$

$12x < -54$

$x < -\frac{54}{12}$

Determine where the graph of $f(x) = 2x^3 + 27x^2 + 34x - 61$ is concave down.

$f(x)$ is concave down where x ☐ .

$y' = 6x^2 + 54x + 34$

$y'' = 12x + 54 < 0$

$\frac{12x}{12} < \frac{-54}{12}$

$x < -54/12$

Give your final answer in interval notation.

Determine the intervals where the graph of $f(x) = 7x + 4/x$ is concave up and concave d

$f(x)$ is concave up on $(0,\infty)$ and concave down on $(-\infty,0)$

$y = 7x + 4x^{-1}$

$y' = 7 - 4x^{-2}$

$y'' = +8x^{-3}$

$-8x^{-3} < 0$

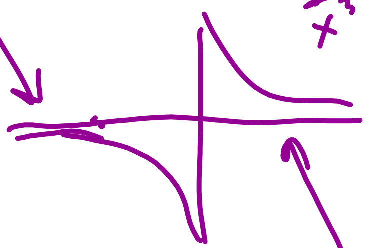
$y = \frac{+8}{x^3} < 0$

so

$x^3 < 0$

$x < 0$

$\frac{8}{x^3}$



$(0,\infty)$

Give your final answer in interval notation.

Determine the intervals where the graph of $f(x) = 7x + 4/x$ is concave up and concave d

$f(x)$ is concave up on $(0,\infty)$ **and concave down on** $(-\infty,0)$.

$y' = 7 + 4(-1)x^{-2}$

$y'' = 8x^{-3} = \frac{8}{x^3}$

Con up

$\frac{8}{x^3} > 0$

$x^3 > 0$

Con down

$\frac{8}{x^3} < 0$

$x^3 < 0$

Determine the intervals where the graph of the given function is concave up and concave down.

$f(x) = 19x^{4/3} + 8x^{1/3}$

Concave up: $x >$ # **and** $x <$ 0 **, concave down:** 0 $< x <$ #

$y' = 19 \cdot \frac{4}{3} x^{-1/3} + 8 \cdot \frac{1}{3} \cdot x^{-2/3}$

$y'' = 19 \cdot \frac{4}{3} \cdot -\frac{1}{3} x^{-2/3} + 8 \cdot \frac{1}{3} \cdot -\frac{2}{3} \cdot x^{-5/3}$

$x^{-5/3} \left(19 \cdot \frac{4}{3} - \frac{1}{3} x + 8 \cdot \frac{1}{3} \cdot -\frac{2}{3} \right) = 0$

$x \neq 0$ $\quad = 0$

#

Determine the intervals where the graph of the given function is concave up and concave down.

$f(x) = 19x^{4/3} + 8x^{1/3}$

$+ \quad 0 \quad - \quad 4/19 \quad +$

Concave up: $x >$ 4/19 **and** $x <$ 0 **, concave down:** 0 $< x <$ 4/19

$$y' = 19 \cdot \frac{4}{3} x^{1/3} + \frac{8}{3} x^{-2/3}$$

$$\frac{1}{3} - \frac{3}{3} = \frac{-2}{3}$$

$$y'' = \frac{19 \cdot 4}{9} x^{-2/3} + \frac{-16}{9} x^{-5/3} = 0$$

$$\frac{x^{-5/3}}{9}\left(19 \cdot 4\, x - 16\right) = 0$$

$x = 0$

$$x = \frac{16}{19 \cdot 4} = \frac{4}{19}$$

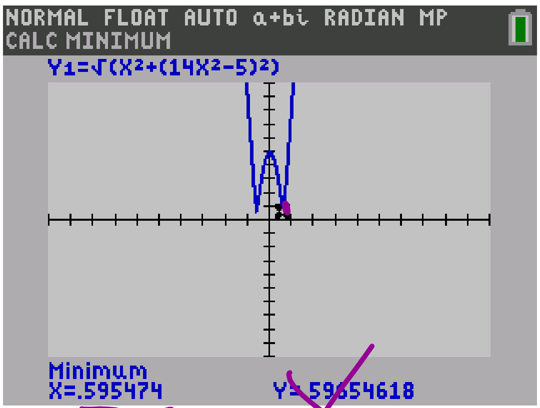
(x,y) distance (0,5)

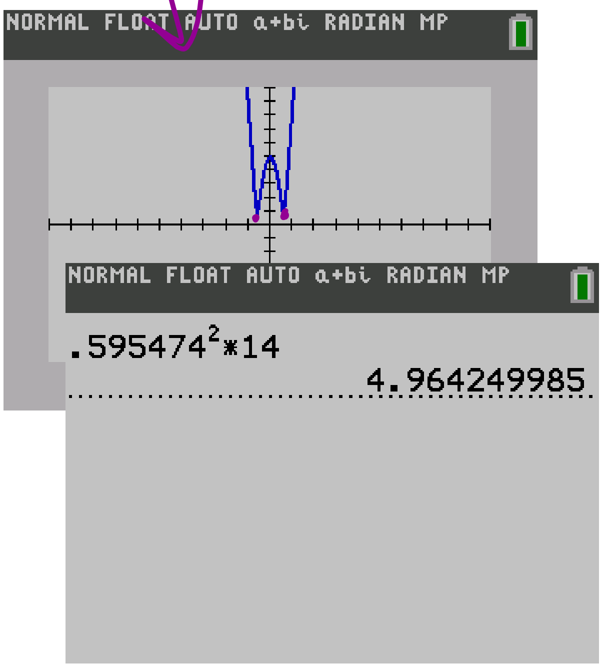
Find the points on the curve $\boxed{y = 14x^2}$ closest to the point (0, 5).

(-.5 ... , 4.95) and (.5 ... , 4.95)

$$d(x,y) = \sqrt{(x-0)^2 + (y-5)^2}$$

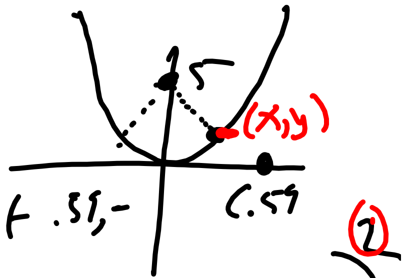
$$d(x) = \sqrt{x^2 + (14x^2 - 5)^2}$$





Find the points on the curve $y = 14x^2$ closest to the point (0, 5).

(☐ , ☐) and (☐ , ☐)



min distance^2 between two points

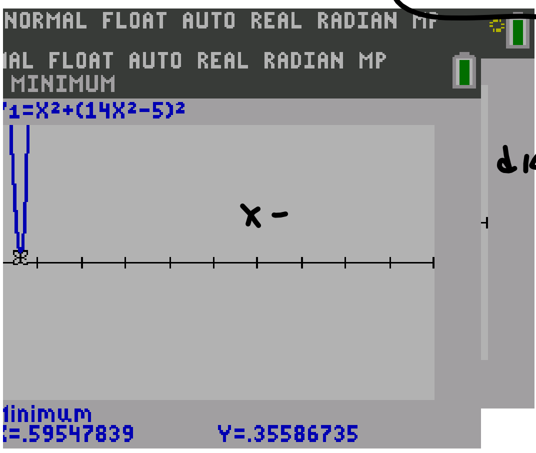
$$d^{②} = \left(\sqrt{(x-x_1)^2+(y-y_1)^2}\right)^{②}$$

$$d^2 = (x - \underset{0}{x_1})^2 + (y - \underset{5}{y_1})^2$$

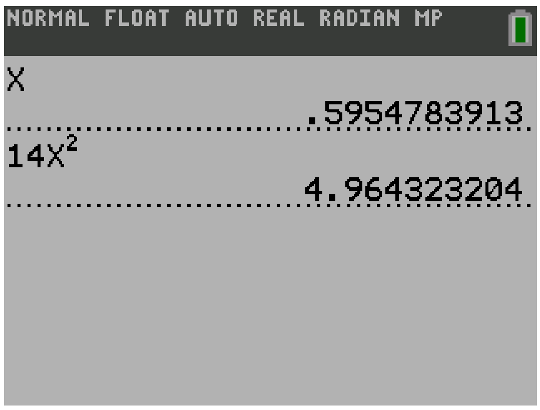
$$d^2 = (x)^2 + (14x^2 - 5)^2$$

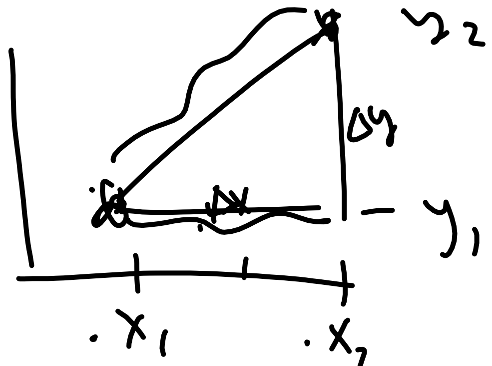
Constant " " "

$$d^2 = x^2 + (14x^2 - 5)^2$$



distance^2





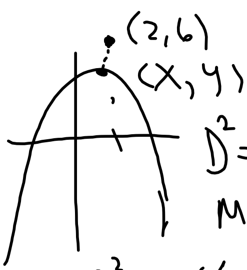
$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

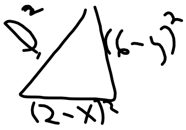
$$D^2 = (\quad)^2 + (\quad)^2$$

MAX

Distance^2 from Point

$(2, 6)$ To $y = 8 - x^2$





$$D^2 = (6-y)^2 + (2-x)^2$$

Mimize This Main Idea D^2

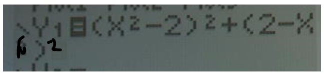
$$D^2_{(x,y)} = (6-y)^2 + (2-x)^2$$

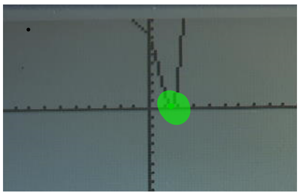
Constraint: $(6-(8-x^2))^2 + (2-x)^2$

$$D^2_{(x)} = (x^2-2)^2 + (2-x)^2$$

$$= x^4 - 4x^2 + 4 + 4 - 4x + x^2$$

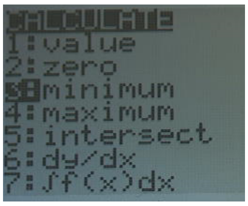
$$D^2 = x^4 - 3x^2 - 4x + 8$$

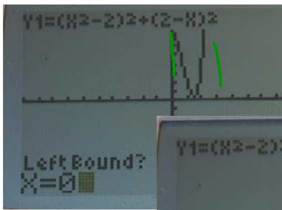


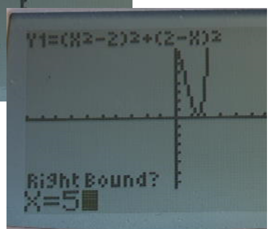


zoom 6:

(zoom 0: if this doesnt work



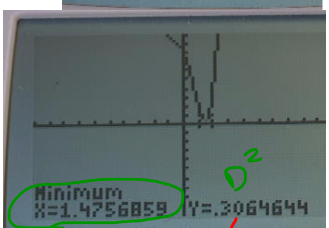




know x = 1.4757

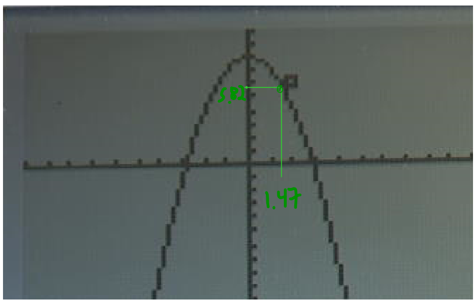
y = 8 - (1.4757)^2

= 5.8223



D^2= .3064644

D= .5536



Suppose that it costs a company $C(x) = 0.01x^2 + 32x + 6400$ dollars to manufacture x units of a product. For this cost function, the average cost function is $\overline{C}(x) = \dfrac{C(x)}{x}$.

Find the value of x that minimizes the average cost. The cost function can be related to the efficiency of the production process.

$x =$ 800

$\bar{C} = .01x + 32 + 6400 \cdot x^{-1}$

$C' = .01 - 6400x^{-2} = 0$

$6400x^{-2} = .01$

$x^{-2} = \frac{.01}{6400}$

$x^2 = \frac{6400}{.01}$

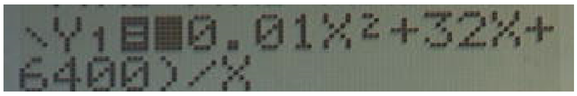
$x = 800$

Suppose that it costs a company $C(x) = 0.01x^2 + 32x + 6400$ dollars to manufacture x units of a product. For this cost function, the average cost function is $\overline{C}(x) = \frac{C(x)}{x}$.

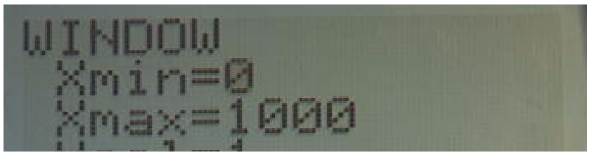
Find the value of x that minimizes the average cost. The cost function can be related to the efficiency of the production process.

$x =$ ______

$$= \frac{0.01x^2 + 32x + 6400}{x}$$

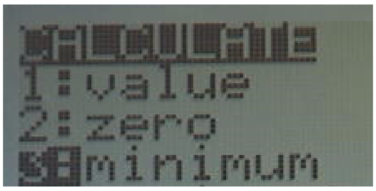


$y = .01x+32+6400x^{-1}$



$y'=.01-6400x^{-2}=0$

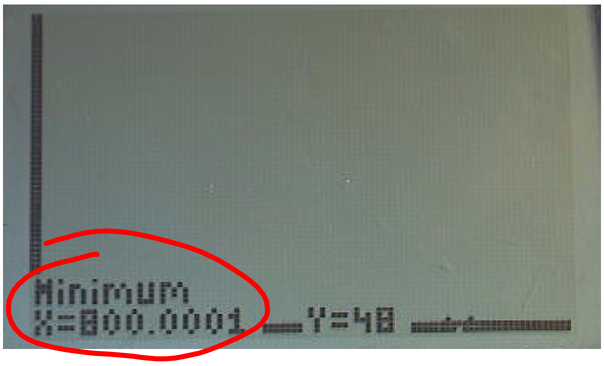
$.01=6400x^{-2}$



$.01/6400=x^{-2}$

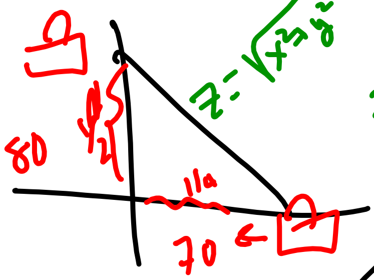
$x^2=6400/.01=640000$

$x=800$



A car is traveling at 80 mph due south at a point 1/2 mile north of an intersection. A police car is traveling at 70 mph due west at a point 1/4 mile due east of the same intersection. At that instant, the radar in the police car measures the rate at which the distance between the two cars is changing. What does the radar gun register? Round your answer to three decimal places.

Your Answer: ______



$z = \sqrt{x^2+y^2}$

$x^2 + y^2 = z^2$

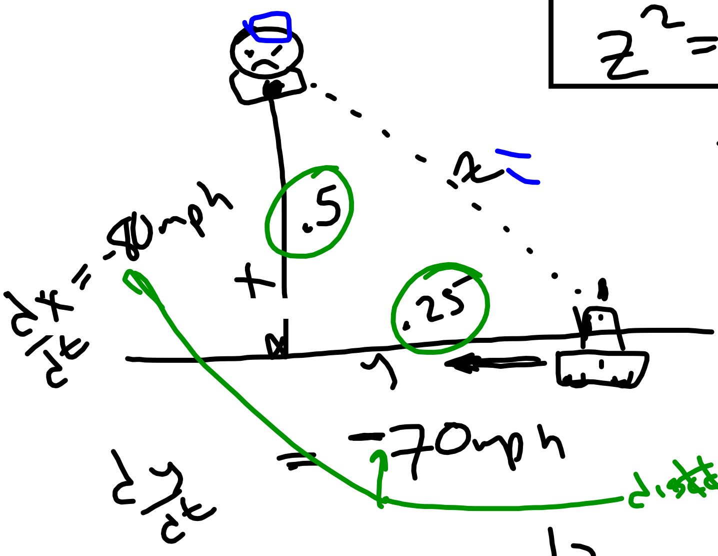
$2xx' + 2yy' = 2zz'$

$z' = \dfrac{xx' + yy'}{z}$

$$\boxed{\dfrac{x \cdot x' + yy'}{\sqrt{x^2+y^2}}} = \dfrac{\frac{1}{2}\cdot 80 + \frac{1}{4}\cdot 70}{\sqrt{(1/2)^2 + (1/4)^2}}$$

A car is traveling at 80 mph due south at a point 1/2 mile north of an intersection. A police car is traveling at 70 mph due west at a point 1/4 mile due east of the same intersection. At that instant, the radar in the police car measures the rate at which the distance between the two cars is changing. What does the radar gun register? Round your answer to three decimal places.

Your Answer: 102.86



$$\boxed{z^2 = x^2 + y^2}$$

$$z = \frac{\sqrt{5}}{4} = .559$$

$$\frac{dz}{dt} = ?$$

$$\frac{dx}{dt} = -80\text{mph}$$

$$\frac{dy}{dt} = -70\text{mph}$$

distance is closing

$$z\frac{dz}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$

$$\frac{(.55)\frac{dz}{dt}}{.55} = \frac{.5(-80) + (.25)(-70)}{.55}$$

$$= -102.86 \text{ mph}$$

A company's revenue for selling x (thousand) items is given by $R(x) = \dfrac{168x - x^2}{x^2 + 168}$.

Find the value of x that maximizes the revenue and find the maximum revenue.

$x =$ ____, maximum revenue is \$____

$y = (168x - x^2)/(x^2+168)$

A company's revenue for selling x (thousand) items is given by $R(x) = \dfrac{168x - x^2}{x^2 + 168}$.

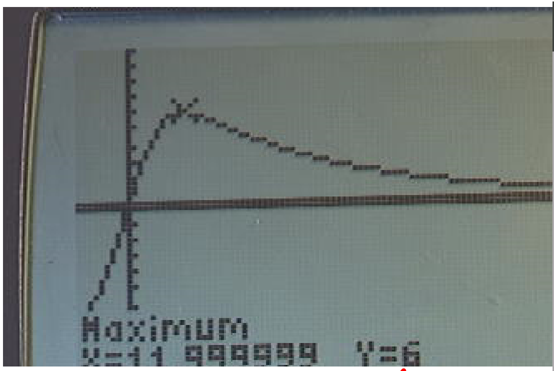
Find the value of x that maximizes the revenue and find the maximum revenue.

x = 12, maximum revenue is $ 6

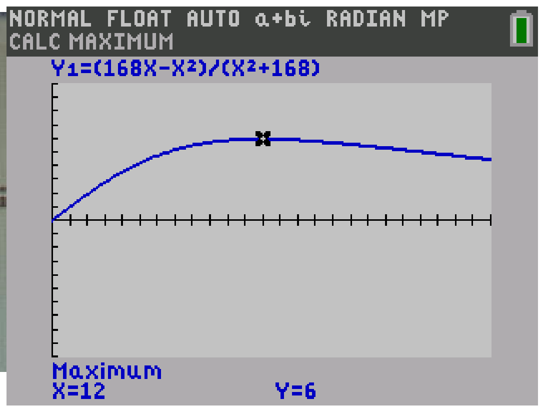
$$R' = (x^2+168)(168-2x) - (168x - x^2)(2x) = 0$$

$$\cancel{168x^2} \cancel{-2x^3} + \cancel{168^2} - 336x - \cancel{336x^2} + \cancel{2x^3} = 0$$

$$-\cancel{168x^2}$$



12 6



Find the general antiderivative. Use c as the constant of integration.

$$\int x^{1/4}(x^{5/4} - 32)\,dx$$

$$\int x^{6/4}\,dx - \int 32x^{1/4}\,dx$$

$$\frac{x^{6/4+4/4}}{5/2 = 10/4} - \frac{32\,x^{1/4+4/4}}{5/4} + C$$

$$\boxed{\frac{2}{5}x^{5/2} - \frac{128}{5}x^{5/4} + C}$$

Find the general antiderivative. Use c as the constant of integration.

$$\int x^{1/4}(x^{5/4} - 32)\,dx$$

$$\int x^{6/4} - 32x^{1/4}\,dx$$

$$\frac{x^{10/4}}{10/4} - 32\frac{x^{5/4}}{5/4} + C$$

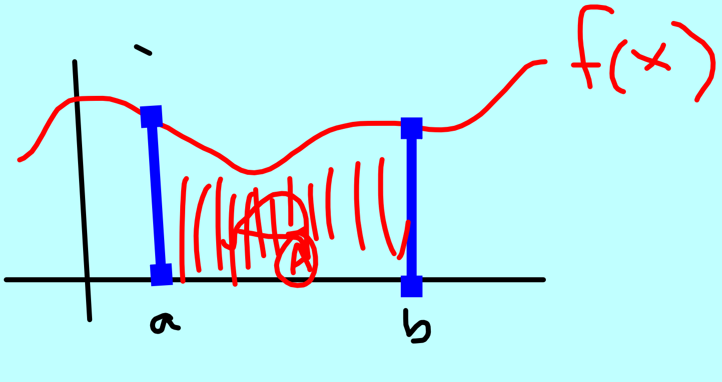
$$\frac{2}{5}x^{5/2} - \frac{128}{5}x^{5/4} + C$$

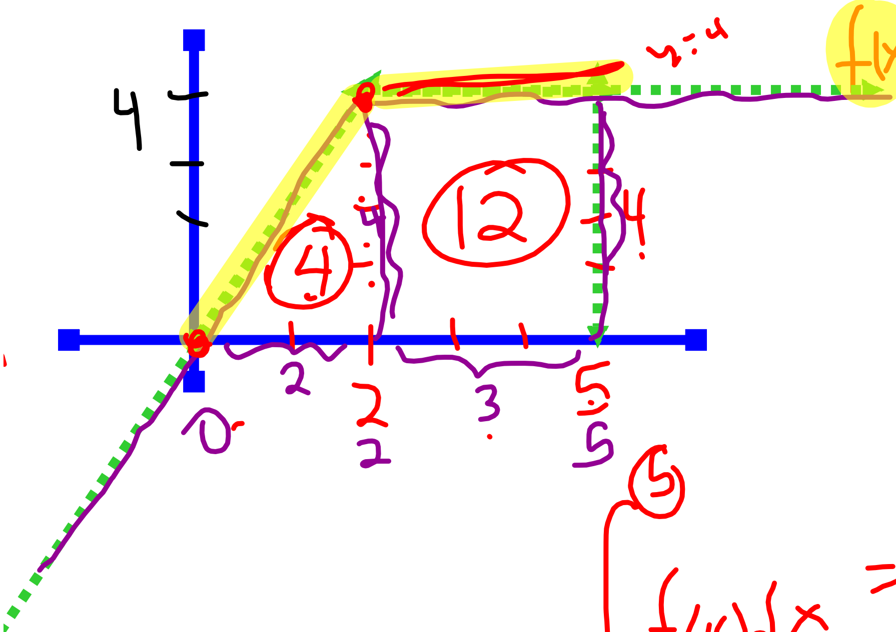
Definite Integrals = Area under curve

notation

$$A = \int_{x=a}^{b} f(x)\, dx$$

means: Area under curve f(x) from x=a to x=b





$$f(x) = \begin{cases} 2x & x < 2 \\ 4 & x > 2 \end{cases}$$

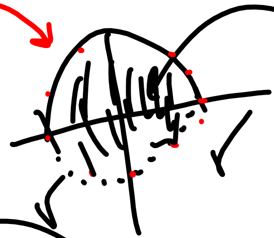
$$\int_0^5 f(x)\,dx = 12 + 4 = 16$$

area

Circle

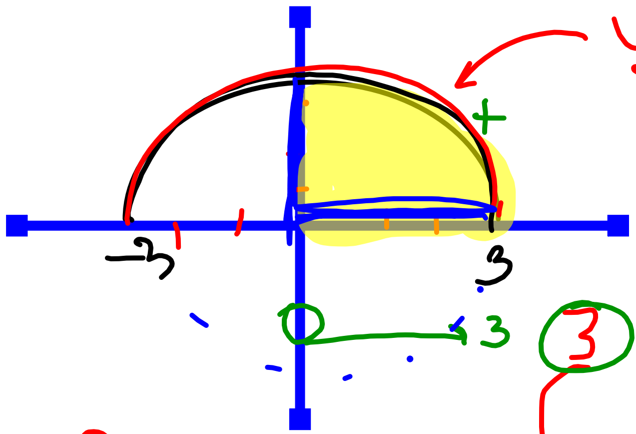
$$x^2 + y^2 = r^2$$

$$y = \pm\sqrt{r^2 - x^2}$$



πr^2

$$\int_{-r}^{r} \sqrt{r^2 - x^2}\, dx = \frac{1}{2}\pi r^2$$



$$y = \pm\sqrt{9 - x^2}$$

$$y^2 = 9 - x^2$$

$$x^2 + y^2 = 9$$

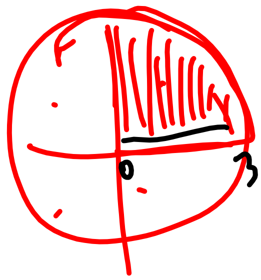
Circle of radius 3.

Area is $\pi r^2 = 9\pi$

$$\frac{9\pi}{4} = \int_0^3 \sqrt{9 - x^2}\, dx$$

Top of circle of radius 3

$$\int_0^3 \sqrt{9-x^2}\,dx = \frac{9\pi}{4}$$



$A = \pi r^2 \quad r=3$

$A = 9\pi$

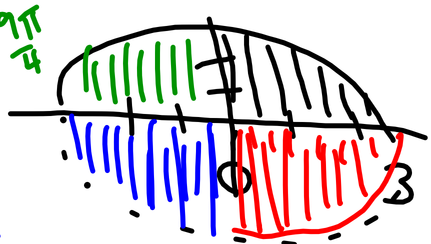
$$\int_0^3 \sqrt{9-x^2}\,dx = \frac{9\pi}{4}$$

$$y = \sqrt{9-x^2}$$

Area circle of $r=3$

$\pi r^2 = 9\pi$

$$\int_{-3}^{0} = \frac{9\pi}{4}$$

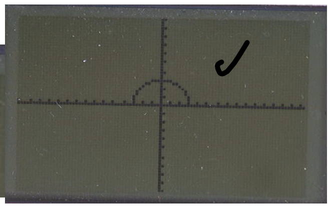


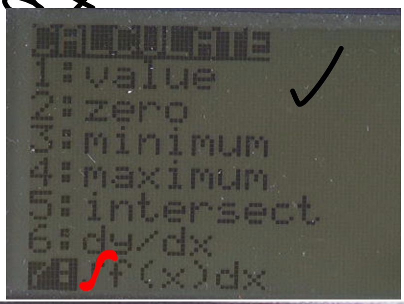
$$\int_{-3}^{0} -\sqrt{9-x^2}\,dx = -\frac{9\pi}{4}$$

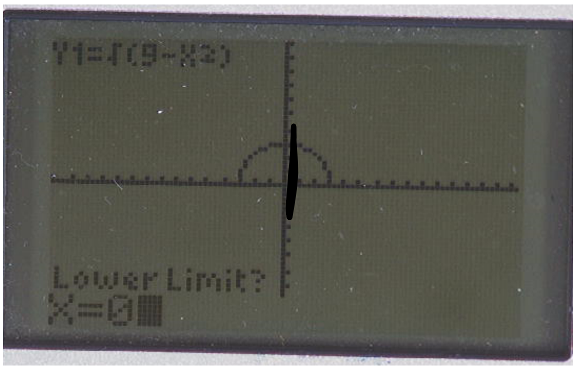
$$\int_0^3 -\sqrt{9-x^2}\,dx = -\frac{9\pi}{4}$$

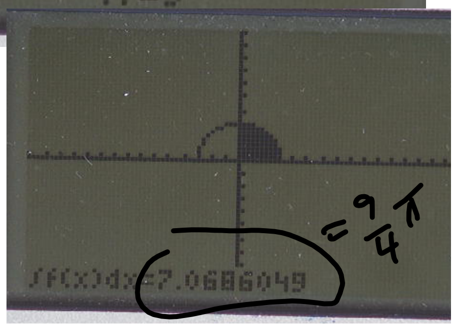
$$\rightarrow Y_1 = \sqrt{(9 - x^2)}$$

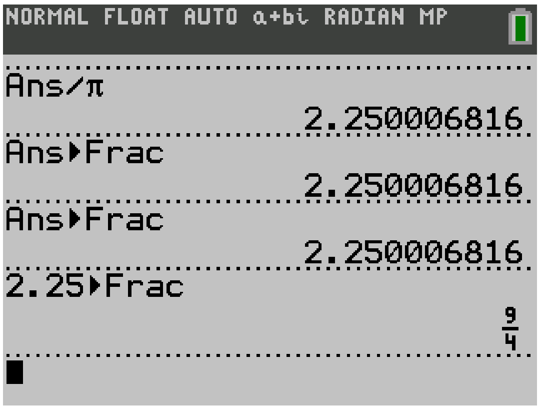
Calc 7: $\int f(x)\,dx$











Notation

ANTI-DERIVATIVE

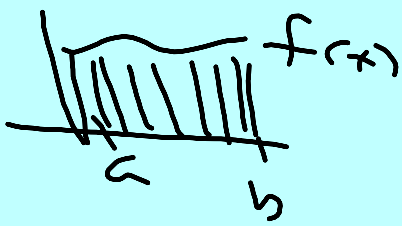
$$\int f(x)\,dx = F(x) + C$$

OR. Indefinite Integral

$$F'(x) = f(x)$$

Definite Integral

$$\int_a^b f(x)\,dx$$



Area under f(x) between a & b

Evaluate $\int 4x^3\, dx$. $= 4\int x^3 dx$

$= \cancel{4} \cdot \frac{x^4}{\cancel{4}} + C$

$= x^4 + C$

$$\frac{d}{dx}\Big(\frac{x^n}{n} = \frac{n\,x^{n-1}}{n}$$

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1} \right) = x^n$$

$$\frac{x^{n+1}}{n+1} + C = \int x^n \, dx$$

Power Rule

$$\boxed{\int x^n \, dx = \frac{x^{n+1}}{n+1} + C}$$

"Product" Rule.

$$\boxed{\int c \cdot f(x)\,dx = c\int f(x)\,dx}$$

Sum Rule

$$\boxed{\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx}$$

Evaluate $\int 4t^{10}\,dt$.

$$\frac{4t^{11}}{11}+C$$

Evaluate $\int \frac{1}{x^7}\,dx$.

$$x^{-7+1} \to \frac{x^{-6}}{-6}+C$$

Evaluate $\int 4t^{10}\, dt$.

$$4 \int t^{10}\, dt \Rightarrow 4\, \frac{t^{10+1}}{10+1} + C$$

Power. Rule.

$$= 4\, \frac{t^{11}}{11} + C$$

Evaluate $\int \frac{1}{x^7}\,dx$.

$$= \int x^{-7}\,dx$$

$$= \frac{x^{-7+1}}{-7+1} + c = \frac{x^{-6}}{-6} + c$$

$$\int e^x \, dx = e^x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1}(x) + c$$

Special Rule

$$\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + c$$

$$\int \frac{\sec^2 x}{\tan x}\,dx = \ln|\tan x| + C$$

$f'(x)$

$= f(x)$

$$\boxed{\int \frac{f'(x)}{f(x)} = \ln|\tan(x)| + C}$$

$$\int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{f'(x)}{f(x)}\,dx$$

$f'(x)$

$f(x)$

$$= -\ln|\cos(x)| + C$$

Evaluate $\int (5\cos x + 9x^9)\,dx$. Use c as the constant.

$$\int 5\cos x\,dx + \int 9x^9\,dx$$

$$5\int \cos x\,dx + 9\int x^9\,dx$$

$$5\sin x + 9\frac{x^{10}}{10} + c$$

$-\int \cot(x)\,dx$

f'

f

Evaluate $\displaystyle\int -\frac{\cancel{\csc x}\,\cot x}{\cancel{\csc x}}\,dx.$

$\_\_\_\_\_\_ = \ln|\csc x| + C$

because its special $\left(\frac{f'}{f}\right)$

Evaluate the given integral.

$$\int e^{14x}\,dx$$

$$= \frac{e^{14x}}{14} + C$$

$$\int \frac{d}{dx} \frac{e^{Ax}}{A} = \int \frac{e^{Ax} \cdot \cancel{A}}{\cancel{A}}$$

$$\frac{e^{Ax}}{A} = \int e^{Ax}\,dx$$

$$A = N$$

Evaluate the given integral.

$$\int e^{14x}\,dx = \frac{e^{14x}}{14} + C$$

$$\frac{d}{dx}\frac{e^{14x}}{14} = \frac{e^{14x}\cdot 14}{14}$$

Find the general antiderivative.

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}(x)$$

$$\int \frac{4x}{x^2+1}\,dx = 2\int \frac{2x}{x^2+1}\,dx$$

$f(x)$

$2x = f'(x)$

$$2\ln|x^2+1| + C$$

Find the general antiderivative.

$$\int \frac{4x}{x^2+1} = 2\int \frac{2x}{x^2+1}\,dx$$

$$= 2\,\boxed{\int \frac{f'}{f}\,dx \quad \text{special}}$$

$$= 2\ln|x^2+1| + C$$

Write the given (total) area as an integral or sum of integrals.

The area above the x−axis and below $y = 9 - x^2$ ≤ 0

A. $\displaystyle\int_0^6 -2x \; dx$

B. $\displaystyle\int_0^6 (9 - x^2) \; dx$

✓ C. $\displaystyle\int_{-3}^{3} (9 - x^2) \; dx$

D. $\displaystyle\int_{-3}^{3} -2x \; dx$

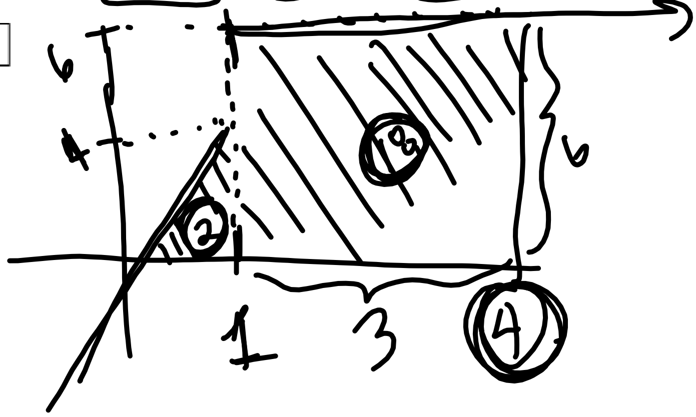
Area under $9 - x^2$

1 out of 3 atte

Write your answer in decimal form.

Compute the area of $\int_0^4 f(x)\ dx$ for $f(x) = 4x$ if $x < 1$, and $f(x) = 6$ if $x \geq 1$.

Your Answer: 20



1 out of 3 atte

Write your answer in decimal form.

Compute the area of $\int_0^4 f(x)\ dx$ for $f(x) = 4x$ if $x < 1$, and $f(x) = 6$ if $x \geq 1$.

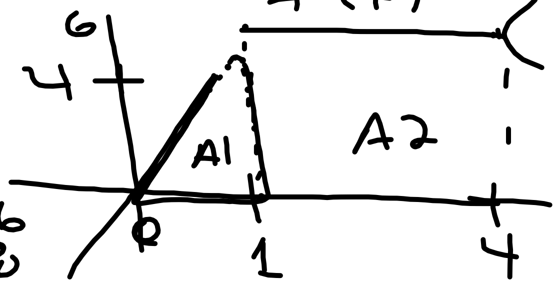
Your Answer: $\boxed{20}$ (2+18)

$$f(x) = \begin{cases} 4x & x < 1 \\ 6 & x \geq 1 \end{cases}$$

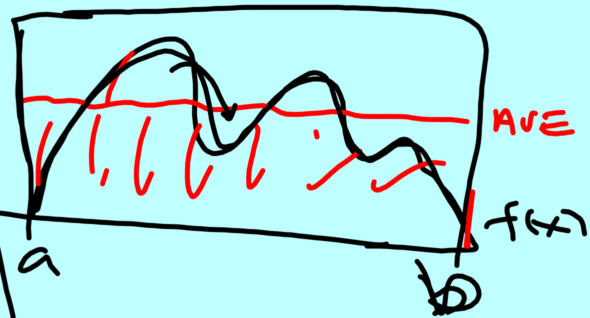
A1= 2

A2=3.6

18

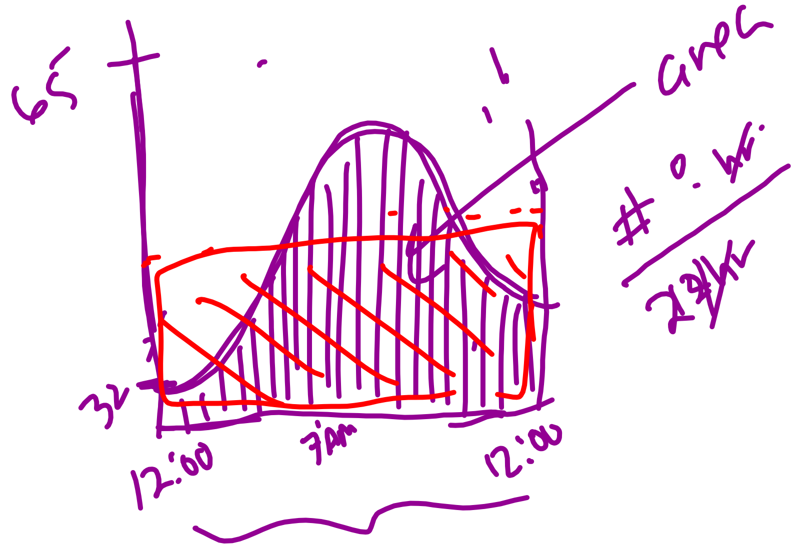


Lecture Average Value



$$\frac{\text{Area } \int_a^b f(x)\,dx}{\text{length } \; b-a} = \text{ave value}$$

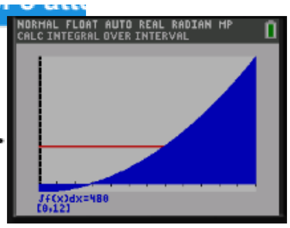
Average Temp (in March)



Round your answer to the nearest whole number.

Compute the average value of the function $f(x) = x^2 - 8$ on the interval [0,12].

Your Answer: 40



Use calculat

$Y_1 = x^2 - 8$

Calc 7: $\int f(x)dx$

Lower: 0

Upper: 12

Answer: 480

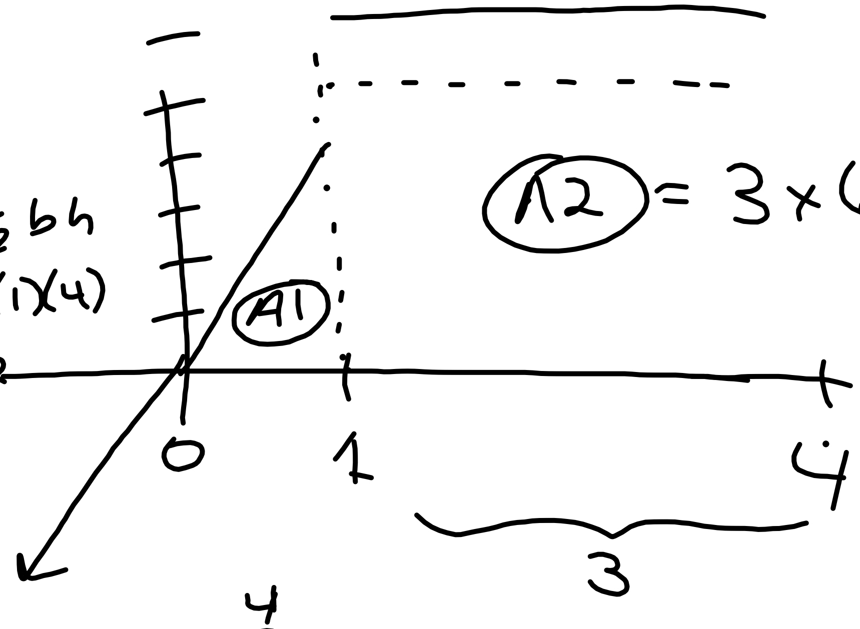
$480/12 = 40$

Average Value

$$\frac{\text{area}}{\text{length}} \qquad \frac{\int_a^b f(x)\,dx}{b-a}$$

$A1 = \frac{1}{2}bh$
$= \frac{1}{2}(1)(4)$
$= 2$

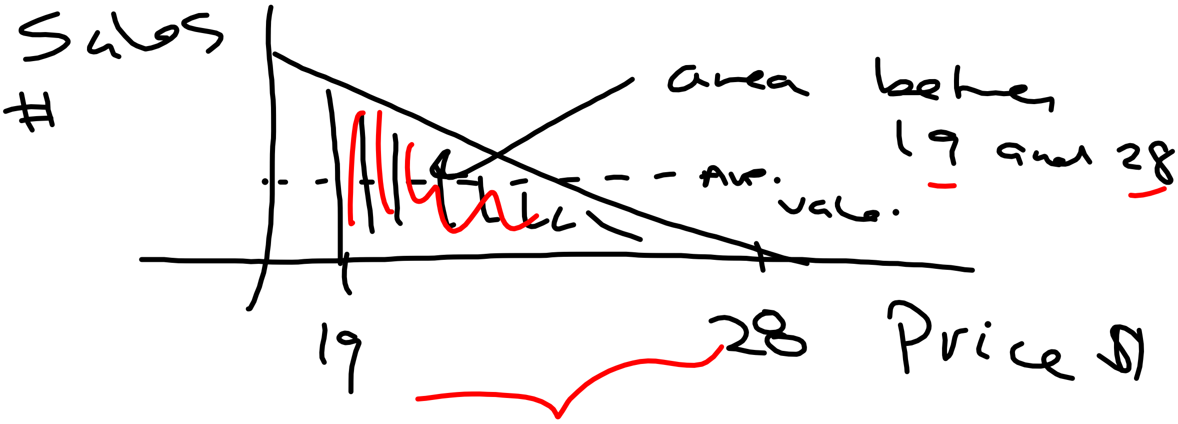
$A2 = 3 \times 6 = 18$



$$\int_0^4 f(x)\,dx = 20$$

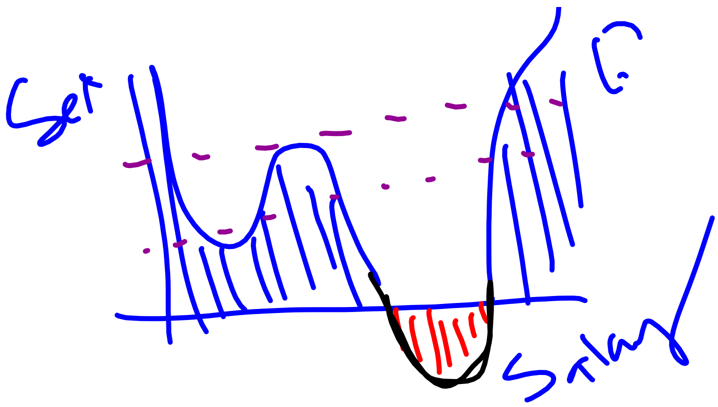
Ave value $= \frac{20}{4-0} = 5$

Project Review



Area = 460 $\$\cdot\#$

Area Sales $\dfrac{460\ \$\cdot\#}{(28-19)\ \$} = 50^{+}\ \#$



$$= \frac{\int_{25}^{250} P4(x)\,dx}{250 - 25} = \frac{\text{sex} \cdot \text{salary}}{\text{salary}} = \text{sex}$$

Conclusion in words:

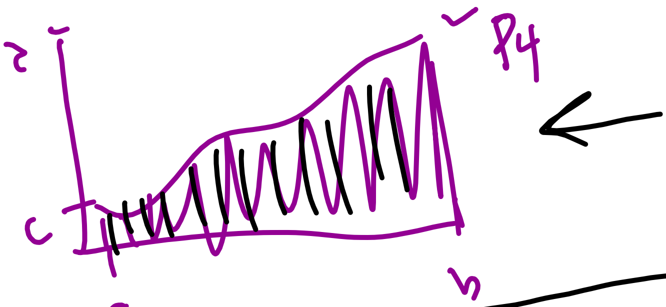
According to the cubic regression for the population of NJ we determined the average ~~growth~~ of people per year was 7,279,715 ~~People~~/yr.

average population of NJ between 1960 and 1990 was 7.2 million

Conclusion in words:

According to quad regression from the years 2004 to 2015 the average amount of face book users would be 771 thousand users per year.

Picture



Math

Area value = $\int_a^b P_4(x)\,dx$ = $\frac{}{b-a}$ units

Word. Thm