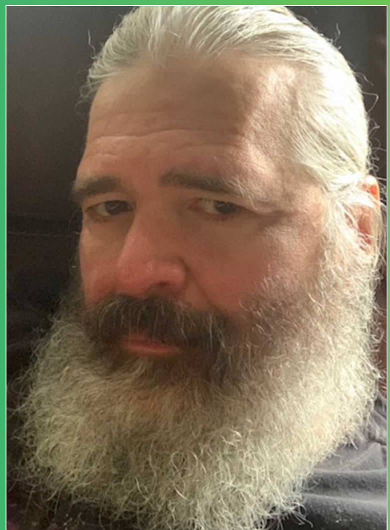
MAT 151 Calculus 1

Prof. Porter



Agenda

Homework Questiosn

Lecture:

Inverse Functions

Differentials

Groupwork

151d10

Homework Questions

This material is not on the derivatives test.

What is Math?

What is Precalculus?

What is Calculus?

What are the two rates of change?

How do we get from two points to one?

What is the meaning of velocity?

What applications have done with derivative?

Language, Study of Functions, Study of Change, Average and Instantaneous rates, Limits take average to instantaneous.

Velocity is

the Derivative is

the instantaneous rate of change is

the slope of the tangent line

Applications: Newtons Method for solving equations
Local Linear Approximations, Related Rates

Derivatives of Inverses

Application of Implicit Differentiation

$\left(\frac{1}{x^n}\right)' = -\frac{n}{x^{n+1}}$	$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$
$(\sqrt[m]{x})' = \frac{1}{m\sqrt[m]{x^{m-1}}}$	$(a^x)' = a^x \ln a$
$(e^x)' = e^x$	$(\log_a x)' = \frac{1}{x \ln a}$
$(\ln x)' = \frac{1}{x}$	$(\sin x)' = \cos x$
$(\cos x)' = -\sin x$	$(\tan x)' = \frac{1}{\cos^2 x} = \sec^2 x$
$(\cot x)' = -\frac{1}{\sin^2 x} = -\csc^2 x$	$(\sec x)' = \tan x \sec x$
$(\csc x)' = -\cot x \csc x$	$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$	$(\arctan x)' = \frac{1}{1+x^2}$
$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$	$(\operatorname{arcsec} x)' = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$(\operatorname{arccsc} x)' = -\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$(\sinh x)' = \cosh x$
$(\cosh x)' = \sinh x$	$(\tanh x)' = \operatorname{sech}^2 x$
$(\coth x)' = -\operatorname{csch}^2 x$	$(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$
$(\operatorname{csch} x)' = -\operatorname{csch} x \coth x$	$(\operatorname{arcsinh} x)' = \frac{1}{\sqrt{x^2+1}}$
$(\operatorname{arccosh} x)' = \frac{1}{\sqrt{x^2-1}}$	$(\operatorname{arctanh} x)' = \frac{1}{1-x^2}$

We know the derivative of sinx is cosx, but what about arcsinx?

Equation containing 'y'

So $\sin^{-1}(x) = y$ can be changed to $x = \sin(y)$

So use implicit differentiation:

$$\frac{d}{dx}x = \frac{d}{dx}\sin(y)$$

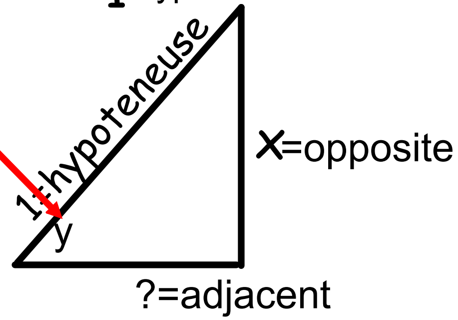
$$1 = \cos(y)\, y'$$

$$\text{or } y' = 1/\cos(y)$$

y is an angle

SOHCAHTOA

$\sin y = \dfrac{x}{1}$ opposite / hypoteneuse



Pythagorean Theorem says: $?^2+x^2=1^2$

So $? = \sqrt{(1-x^2)}$

$\cos y = \text{adj}/\text{hyp} = \sqrt{(1-x^2)}$

So $y' = 1/\cos(y) = 1/\sqrt{(1-x^2)}$

EX: Find the derivative $\arcsin(x^2+5)$

$$y = \sin^{-1}(x^2+5)$$

$$y' = \frac{1}{\sqrt{1-(x^2+5)^2}} \text{ times } (x^2+5)' \quad \text{Chain Rule}$$

$$\boxed{y' = \frac{2x}{\sqrt{1-(x^2+5)^2}}}$$

Differentials

In calculus, the **differential** represents the principal part of the change in a function $y = f(x)$ with respect to changes in the independent variable. The differential *dy* is defined by

{\displaystyle dy=f'(x)\,dx,}

where {\displaystyle f'(x)} is the derivative of *f* with respect to *x*, and *dx* is an additional real variable (so that *dy* is a function of *x* and *dx*). The notation is such that the equation

{\displaystyle dy={\frac {dy}{dx}}\,dx}

holds, where the derivative is represented in the Leibniz notation *dy*/*dx*, and this is consistent with regarding the derivative as the quotient of the differentials. One also writes

\displaystyle df(x)=f'(x)\,dx.}

Like taking the derivative with respect to NOTHING!

How fuzzy a number is by itself.

If it depends on another variable, it represents how fuzzy it is on its own.

Derivative $\frac{d}{dx} x^2 = 2x$

Derivative w/ Chain Rule $\frac{d}{dx} y^2 = 2y \frac{dy}{dx}$

Related Rate $\frac{d}{dt} y^2 = 2y \frac{dy}{dt}$

Differential

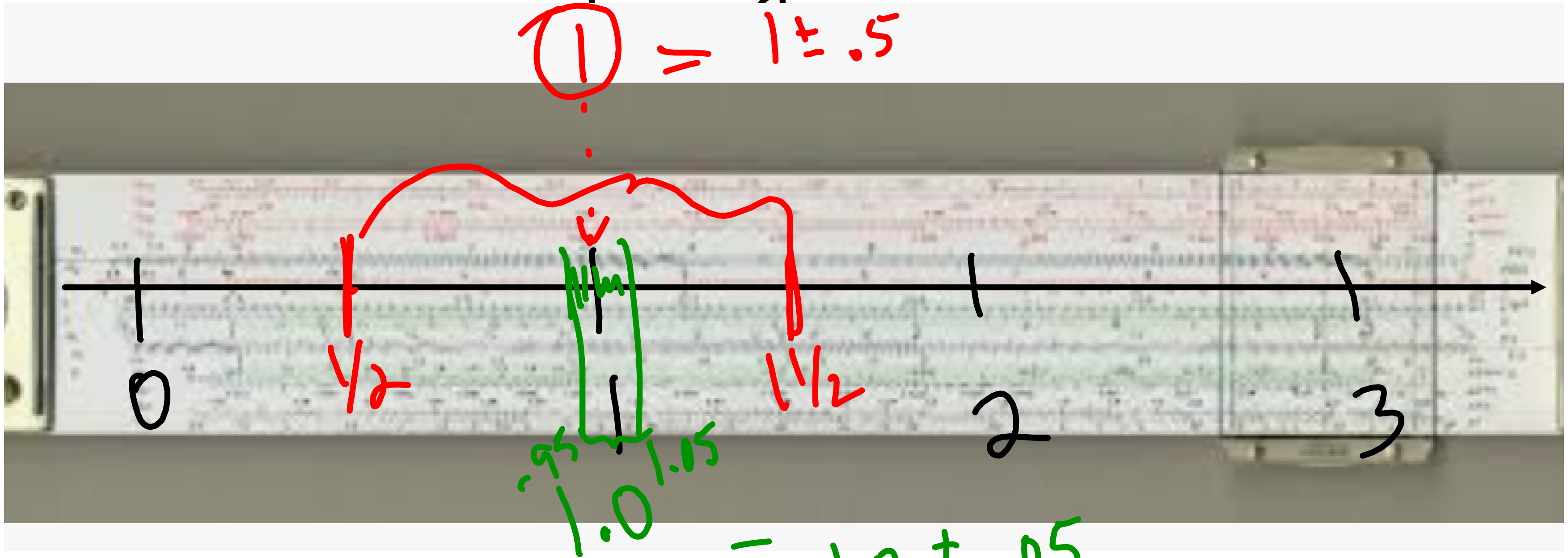
$d\, y^2 = 2y\, dy$

Differential

How is changing with respect to nothing

dy ← y is changing

Fuzzy Numbers



= 1.0 ± .05

1.00 ± .005 ← "fuzzy" Differential Error

Mercer

$4000 \rightarrow 4000 \pm 500$

$4100 \rightarrow 4100 \pm 50$

$4120 \rightarrow 4120 \pm 5$

$4121 \rightarrow 4121 \pm .5$

default

Differential

$$f'(x) \equiv \frac{d}{dx} f(x)$$

$$\frac{d}{dt} f(x) = f'(x) \cdot \frac{dx}{dt}$$

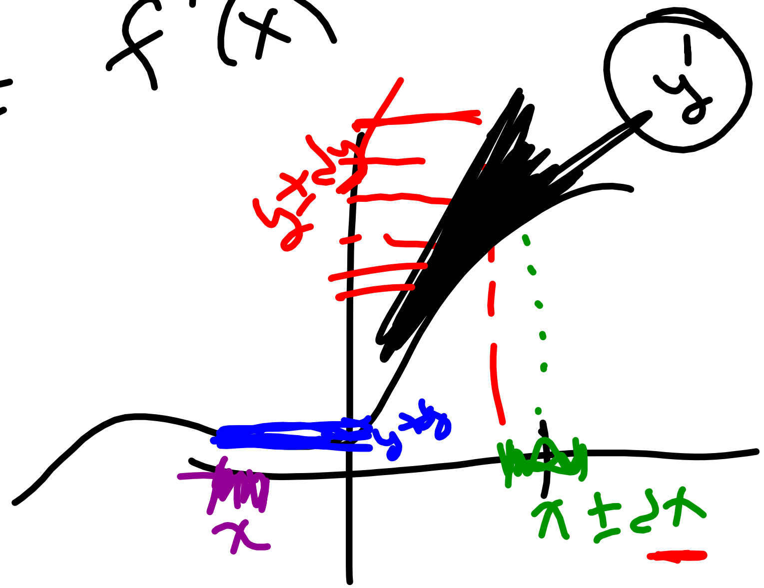
$$d f(x) = f'(x) \cdot dx$$

Differential

$$d f(x) = f'(x)\, dX$$

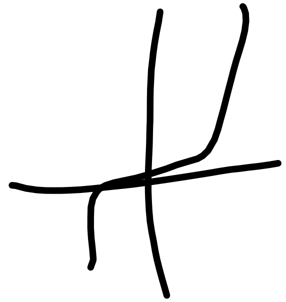
1 Take derivative w.r.t Nothing

$$\frac{dy}{dx} = f'(x)$$



Ex

$$\boxed{y = x^3}$$



$$\frac{dy}{dx} = 3x^2 \leftarrow \boxed{dy} = \boxed{3x^2}\,\boxed{dx}$$

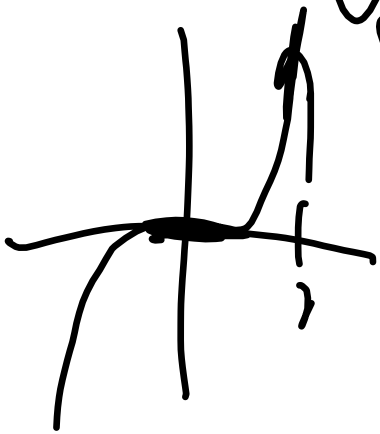
$$\Delta y \approx \boxed{dy} = f'(x)\,dx$$

Exact change in y

dx

$$\Delta y = y_2 - y_1$$

$$dy =$$



$y = x^3$

$\underline{x}\ 1 \rightarrow \underline{x}\ 2$

$y(1) = 1 \quad y(2) = 8$

$\Delta y = 8 - 1 = 7$

Approx.

$y' = 3x^2$

$y'(2) = 12 = dy$

ERROR

x – Number (dx)

1 → 1.5, .5

error ± .5 = dx

1995 → 1995.5, 1994.5

dx = ± .5

1.0 → 1.05, .95

± .05

$6523 < \begin{matrix} 6523.5 \\ 6522.5 \end{matrix}$ $\quad x = \pm\frac{1}{2}$

$\pm \frac{1}{2}(1)$

$7000 < \begin{matrix} 7500 \\ 6500 \end{matrix}$

$dx = \pm\frac{1}{2}(1000)$

$= \pm 500$

$6500 < \begin{matrix} 6550 \\ 6450 \end{matrix}$

$\pm\frac{1}{2}(100) = dx \pm 50$

6.500×10^3

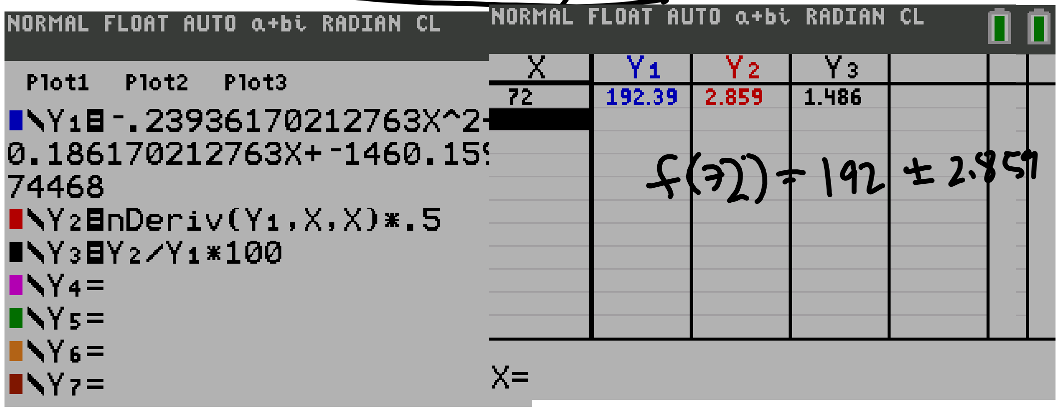
Project: Error

y1=regression

y' dx

y2=nderiv(y1, x, x) * 0.5 — error

y3= y2 / y1 * 100   % error



f(72) = 192 ± 2.859 lbs

At 72" a person should weigh 192 lbs +/- 2.9 lbs

which represent 1.4% error

according to the quad regression

$$y = f(x)$$

$$dy = f'(x) \cdot dx$$

$$= f'(x)\, dx$$

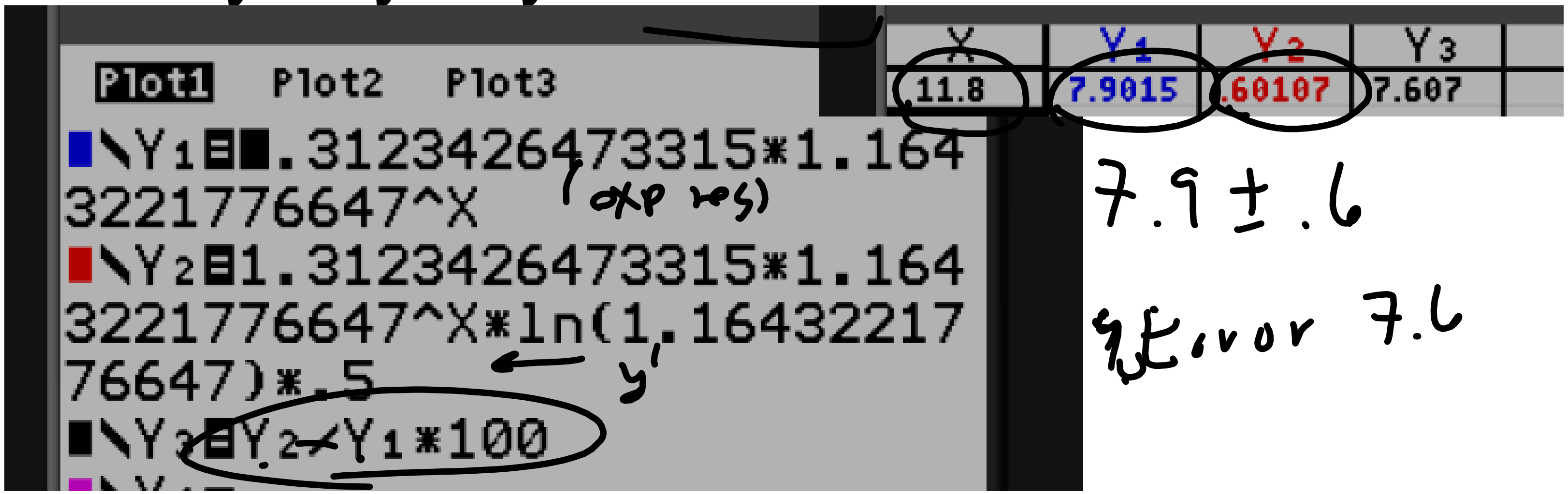
error in

derivative number

y = y1=regression $f'(x)\,dx = dy$

dy = y2=nderiv(y1, x, x) * __0.5______ error

y3= y2 / y1 * 100 % error



(exp res)

y'

$7.9 \pm .6$

% error 7.6

write $r(95) = 10$

$r'(95) * .5 = .5$

$r(95) / r'(95) \cdot 100 = .005$

	y1=regress	y2=error	y3=%error
1995	10	±.5	.005

speaker

In 95, LG is worth 10mil +/- .5mil with an %error of .005%

According to — regression

$Y_1 =$ regression

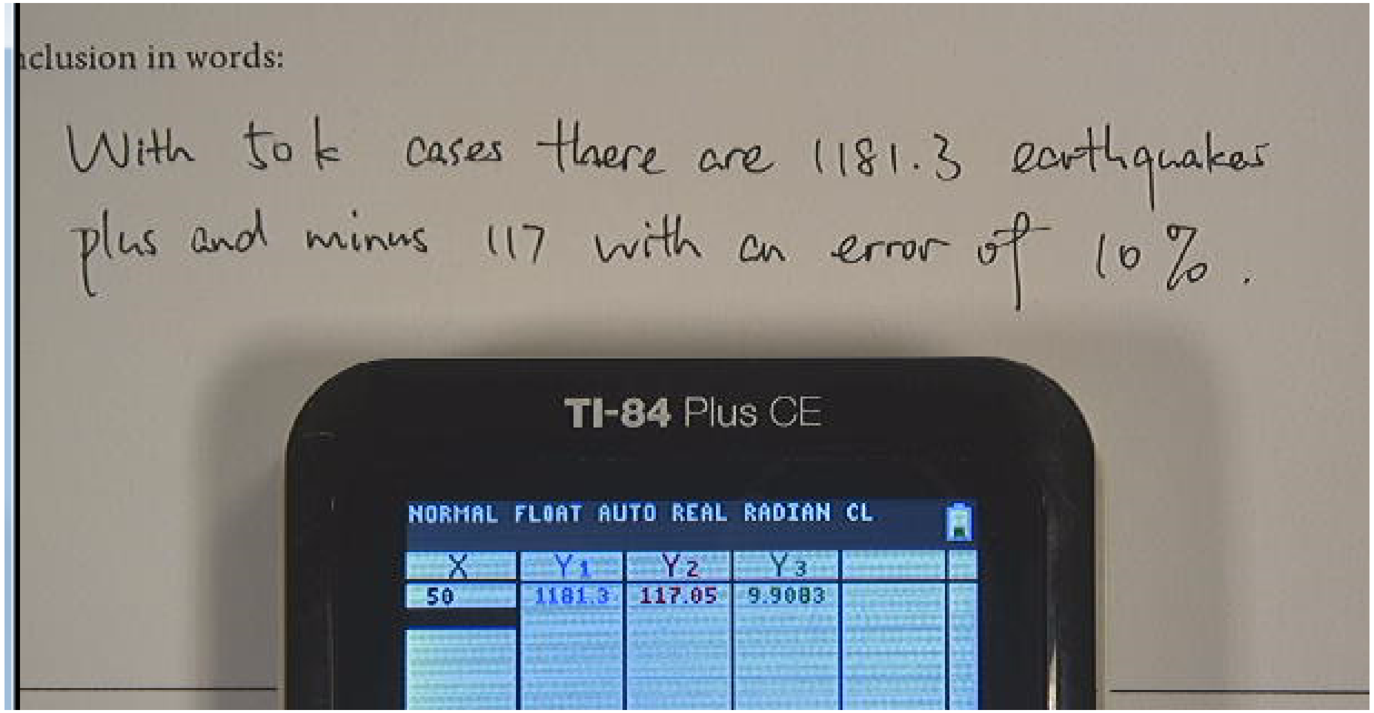
X	Y	Error	% error
1995	worth ±	Error	.005
	y	dy	

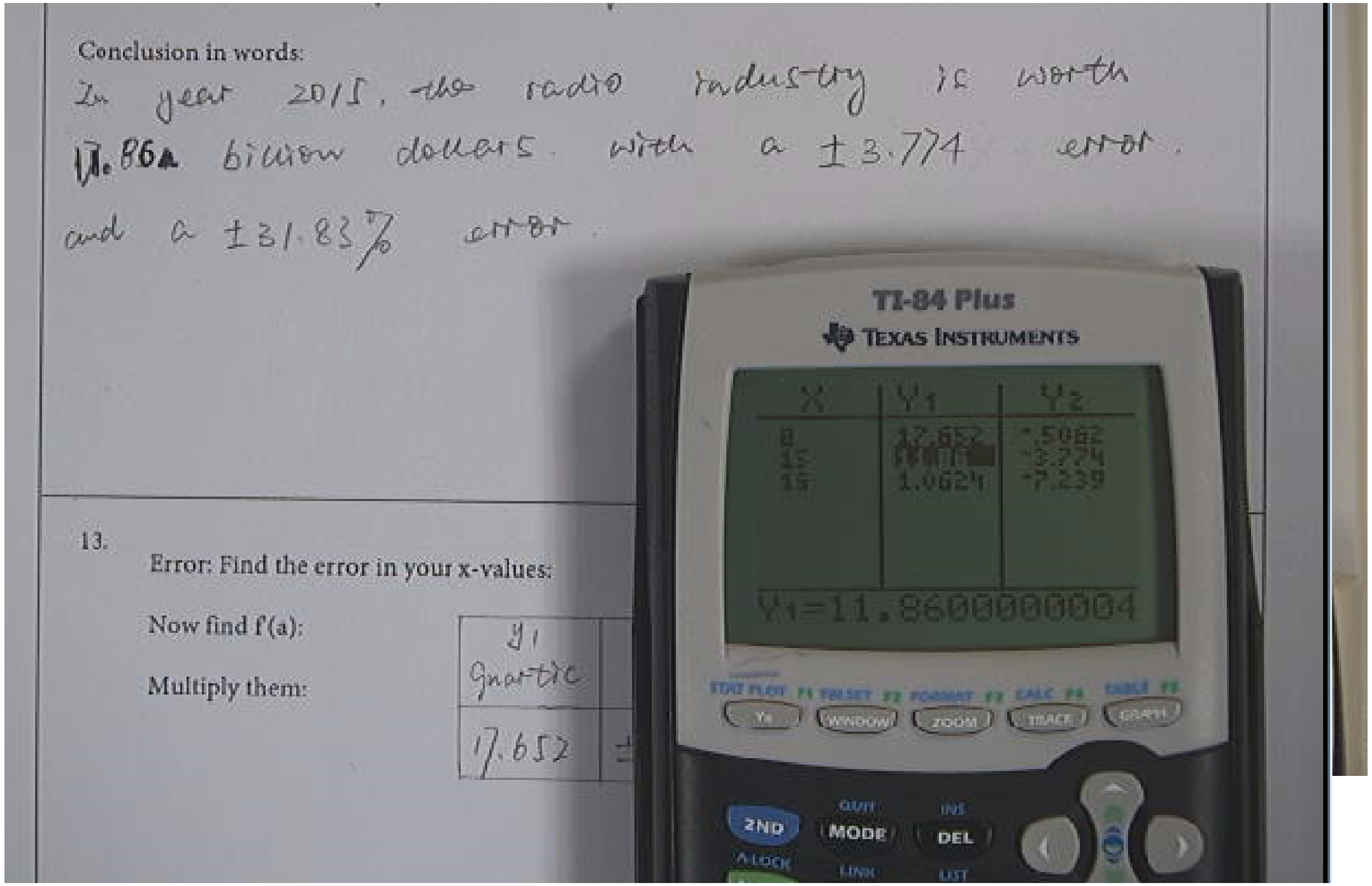
$f'(1995) \cdot dx$

$Y_2 = \text{nder}(Y_1, X, Y) \cdot .5$

$Y_3 = Y_2 / Y_1 \times 100 = \%$ error

In 95, LG is worth 10mil +/- .5mil with an error of .005%


clusion in words:
With 50k cases there are 1181.3 earthquakes
plus and minus 117 with an error of 10%.
TI-84 Plus CE
NORMAL FLOAT AUTO REAL RADIAN CL
X
Y1
Y2
Y3
50
1181.3
117.05
9.9083


Conclusion in words:

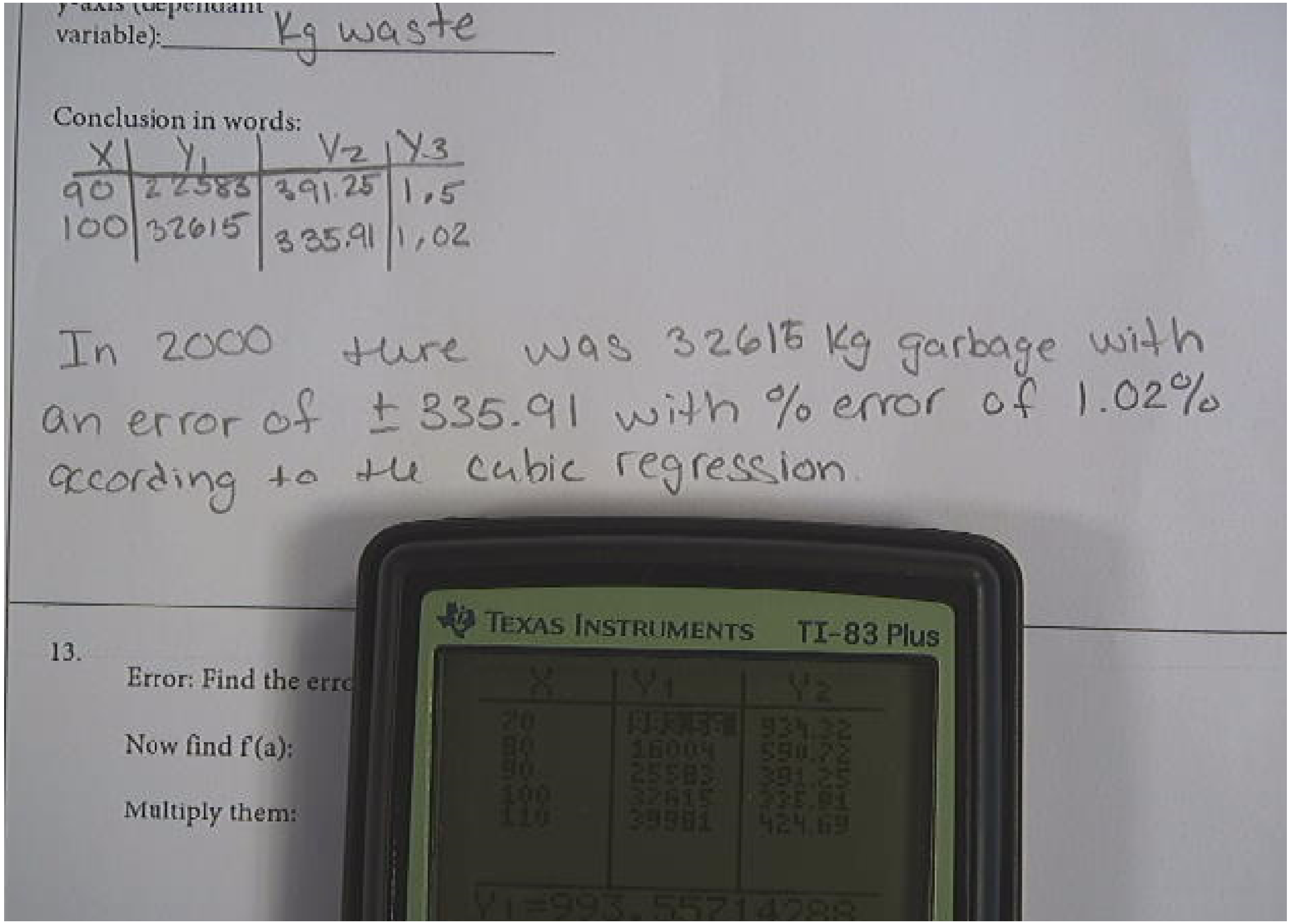
In year 2015, the radio industry is worth 11.86 billion dollars with a ±3.774 error. and a ±31.83% error.

13.

Error: Find the error in your x-values:

Now find f'(a):

Multiply them:



y-axis (dependant variable): Kg waste

Conclusion in words:

X	Y_1	Y_2	Y_3
90	22583	391.25	1,5
100	32615	335.91	1,02

In 2000 there was 32615 Kg garbage with an error of ± 335.91 with % error of 1.02% according to the cubic regression.

13.

Error: Find the erro

Now find f'(a):

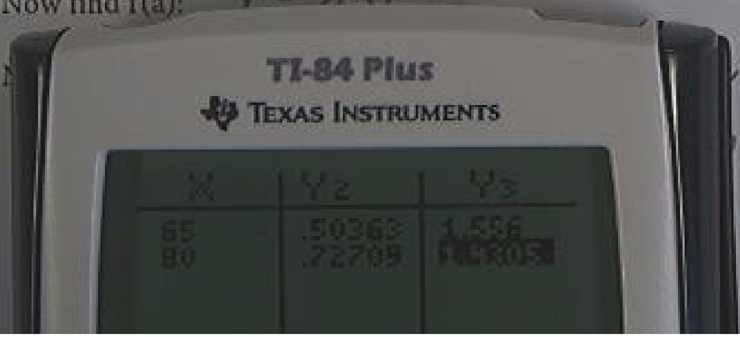
Multiply them:

lusion in words:

At 80mil Spent they will win 51 games with an error of ± 1 game with a percent error of 1.5.

Error: Find the error in your x-values: Dx = .5

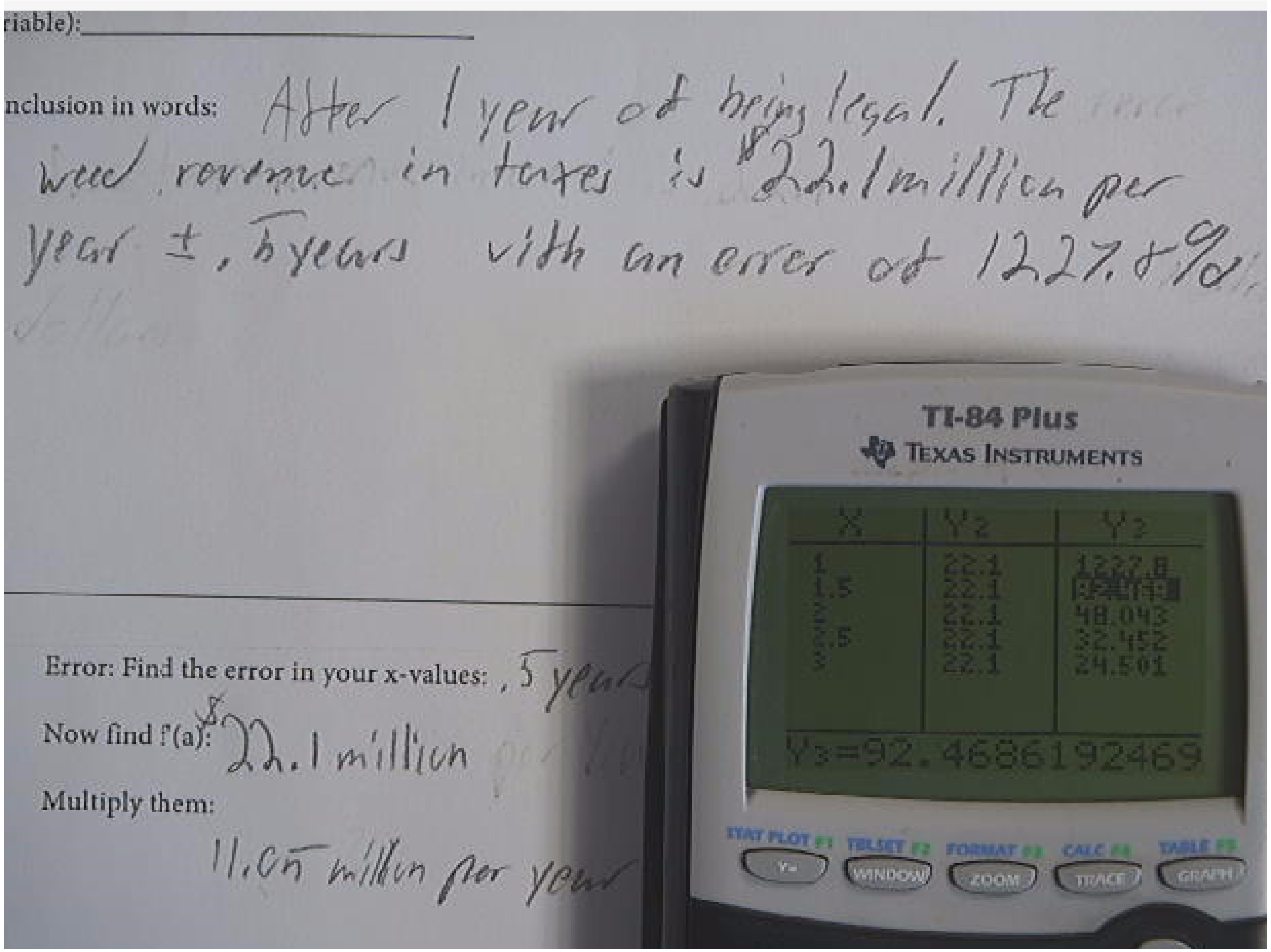
Now find f(a): y = f(a)

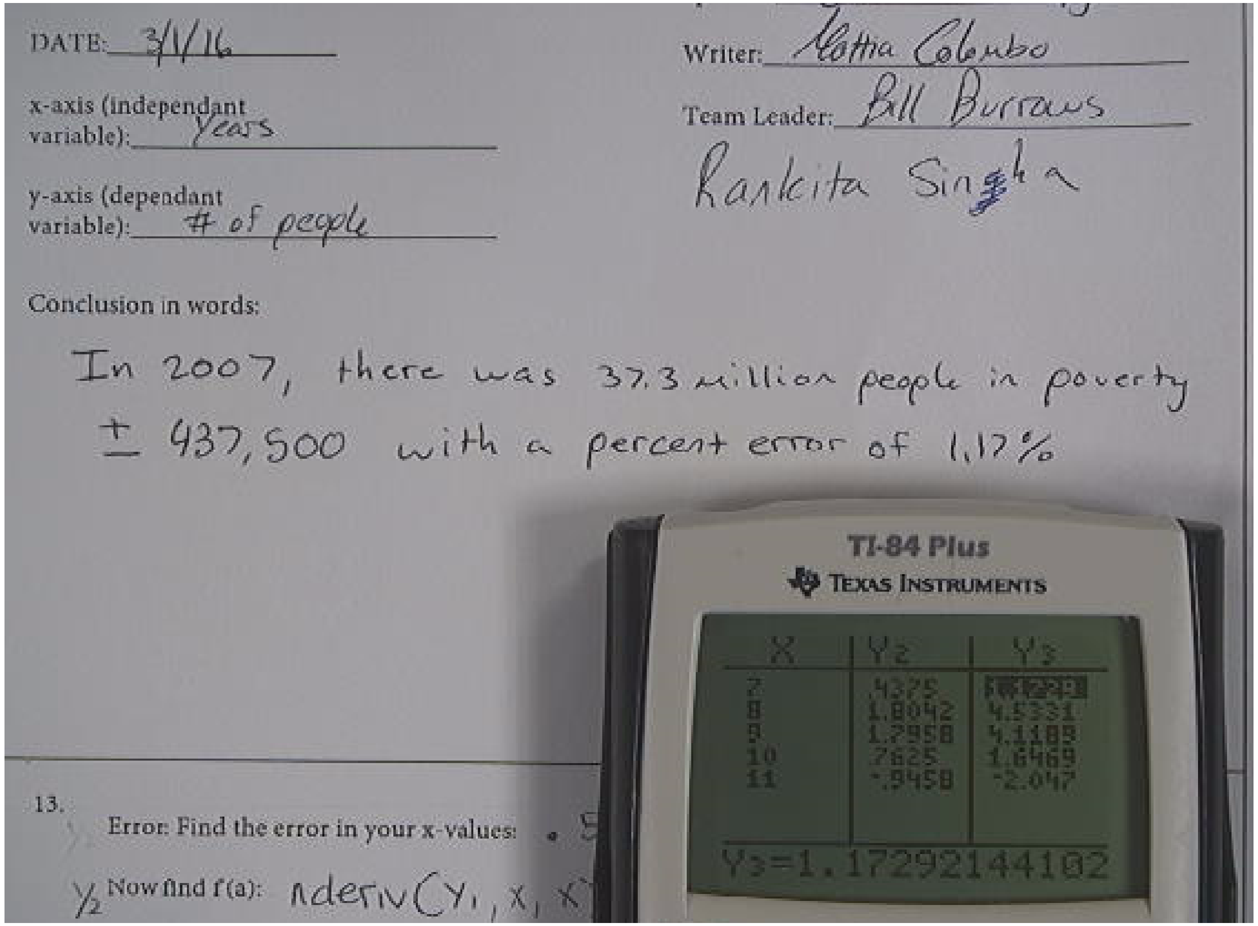


Y,x,x) · .5 = .712

Y1 x 100 = 1.5

= 1.


riable):
nclusion in words: After 1 year of being legal. The weed revenue in taxes is $22.1 million per year ±.5 years with an error of 1227.8% dollars
Error: Find the error in your x-values: .5 years
Now find f(a): $22.1 million
Multiply them:
11.05 million per year
TI-84 Plus
TEXAS INSTRUMENTS
X | Y2 | Y3
1 | 22.1 | 1227.8
1.5 | 22.1 |
2 | 22.1 | 48.043
2.5 | 22.1 | 32.452
3 | 22.1 | 24.501
Y3=92.4686192469
STAT PLOT F1
TBLSET F2
FORMAT F3
CALC F4
TABLE F5
Y=
WINDOW
ZOOM
TRACE
GRAPH



DATE: 3/1/16
Writer: Mattia Colombo
x-axis (independant variable): Years
Team Leader: Bill Burraws
y-axis (dependant variable): # of people
Rankita Singha
Conclusion in words:
In 2007, there was 37.3 million people in poverty ± 437,500 with a percent error of 1.17%
13.
Error: Find the error in your x-values:
Now find f(a): nderiv(Y1, X,
TI-84 Plus
Texas Instruments
X
Y2
Y3
7
8
9
10
11
.4375
1.8042
1.7958
.7625
-.9458
4.5331
4.1189
1.6469
-2.047
Y3=1.17292144102

Group Name: Team Xiao

DATE: 3/1/16

x-axis (independant variable): years dx = 0.5 yr

y-axis (dependant variable): people

Speaker:

Writer: Mariam Shah

Team Leader: Yvette

Conclusion in words:

In 2016 according to the exponential regression the population of NJ will be 1.1×10^7 people with an error of ± 62344 & a percent error of 0.57%

13. Error: Find the error in your x-values: dx = 0.5 years

Now find f'(a): exponential regression

Multiply them: y = a·b^x

STOP

This material is for the midterm