Agenda

Review Quiz 7

Lecture Circular Functions

Project

Review Quiz 7

Solve for x.

$y_1 = 3^{2x} - 10\left(3^{x}\right) + 9 = 0 \rightarrow (3^x)^2 - 10(3^x) + 9 = 0$

$u = 3^x$

Round to the nearest hundredth if necessary.
If there is more than one solution, separate them with commas.

$(3^x)^2 = 3^{2x}$

$2, 0$

$u^2 - 10u + 9 = 0$

$(u-9)(u-1) = 0$

$u = 9 \quad u = 1$

$\boxed{3^x = 9} \quad \boxed{3^x = 1}$

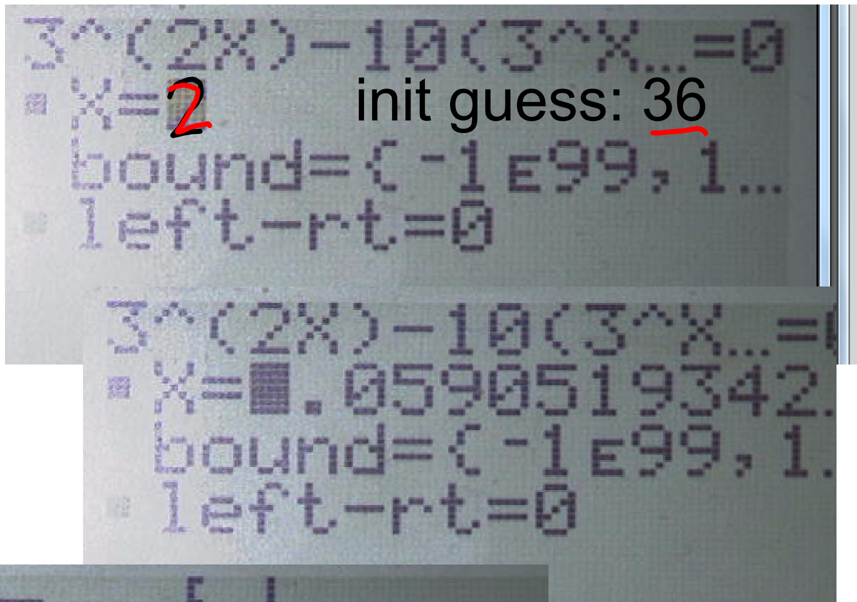
$3^2 = 9 \quad 3^0 = 1$

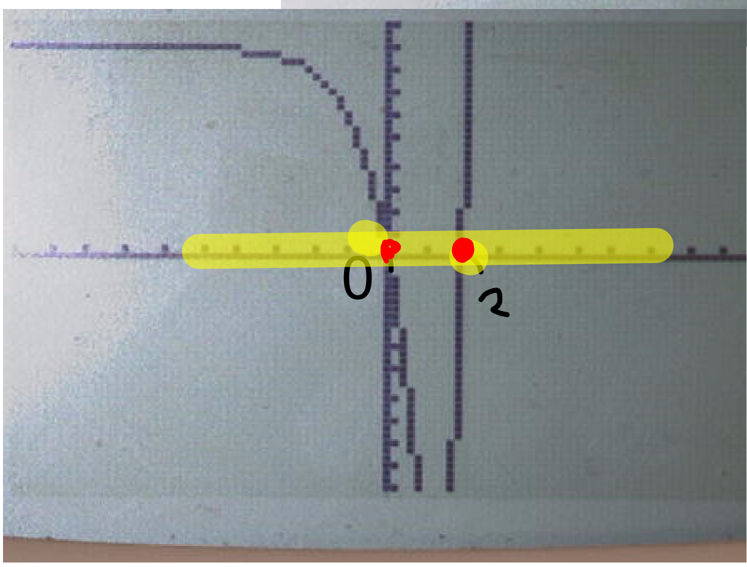
Review Quiz 7

Solve for x.

$$3^{2x} - 10\left(3^{x}\right) + 9 = 0$$

Round to the nearest hundredth if necessary.
If there is more than one solution, separate them with commas.





y= - - =0

$$3^{2x} - 10(3^{x}) + 9 = 0$$

$u = 3^x$

$u^2 = 3^{2x}$

$u^2 - 10u + 9 = 0$

$(u-9)(u-1)$

$u = 9, 1$

$3^x = 9 \qquad 3^x = 1$

$\Rightarrow x = \log_3 9$

$= \frac{\ln 9}{\ln 3}$

$= 2 \qquad = 0$

Prop 1 Definition
- Solve for exponent
- Get rid of log

$y = B^x \quad x = \log_B y$

Prop 2 Sum/Product

- $\log_B x + \log_B y = \log_B(xy)$

Prop 3 "Ladder"

$B^{\cancel{\log_A x}}$ B

Prop 4 Change of Base

$$\boxed{\log_B A = \frac{\log_x A}{\log_x B}}$$

Prop 5 Log of Both Sides

Solving an equation involving logarithms on both sides: Problem type 1

Solve for x.

$$-\log_4\overset{-2}{(x+3)} = 1 - \log_4\overset{-2}{(x+6)}$$

$-\log_4 1 = 1 - \log_4 4$

$0 \stackrel{?}{=} 1 - 1$

If there is more than one solution, separate them with commas. If there is no solution, click on "No solution".

$$\underbrace{\log_4(x+6) - \log_4(x+3)}_{P3} = 1$$

$$\underbrace{\log_4(x+6) + \log_4(x+3)^{-1}}_{P2} = 1$$

$$\underbrace{\log_4\left[(x+6)(x+3)^{-1}\right]}_{P1} = 1$$

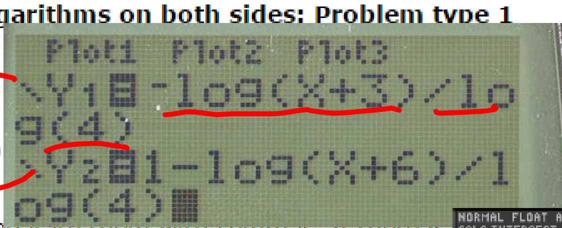
$$4^1 = \frac{x+6}{x+3}$$

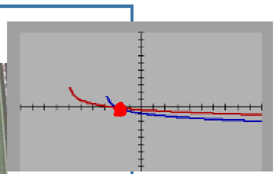
Algebra.

$$4x + 12 = x + 6$$

$$3x = -6 \qquad \boxed{x = -2}$$

Solving an equation involving logarithms on both sides: Problem type 1

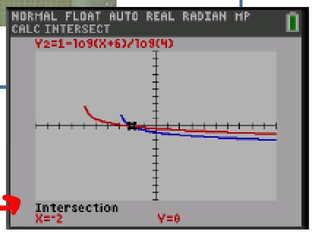




Solve for x.

$$-\log_4(x+3) = 1 - \log_4(x+6)$$

If there is more than one solution, separate them with commas. If there is
on "No solution".

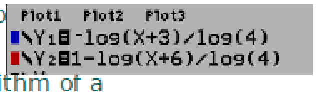


We can work as follows.

$$-\log_4(x+3) = 1 - \log_4(x+6)$$

Arith.

$$\log_4(x+6) - \log_4(x+3) = 1$$ P2 Moving the log terms to



$$\log_4\left(\frac{x+6}{x+3}\right) = 1$$ P1 Using the rule for logarithm of a quotient

$$\frac{x+6}{x+3} = 4^1$$ Writing in exponential form

Algebra

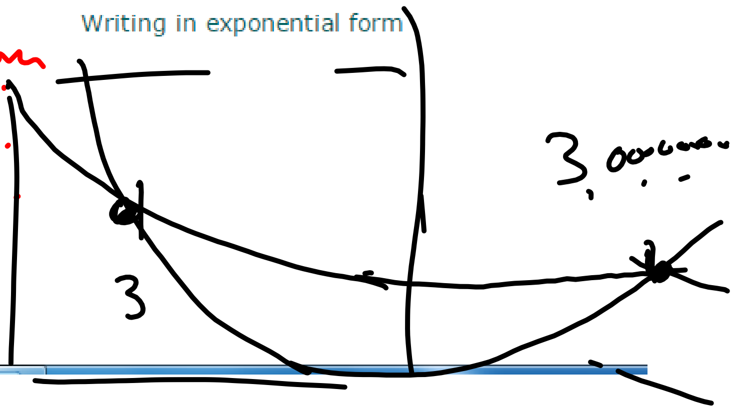
$$\frac{x+6}{x+3} = 4$$

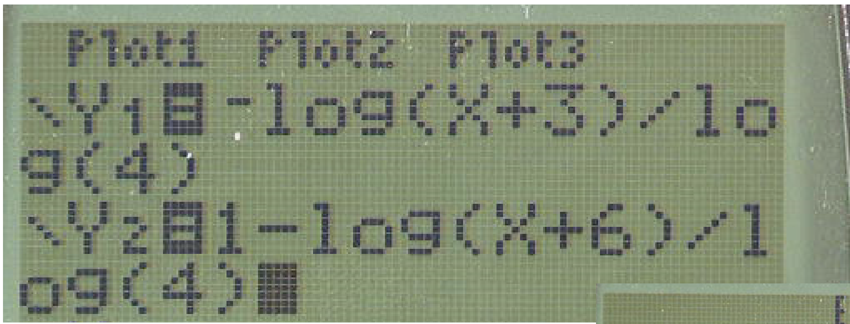
$$x+6 = 4(x+3)$$

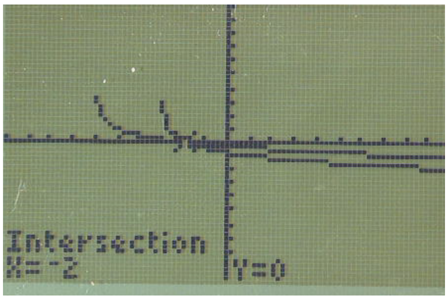
$$x+6 = 4x+12$$

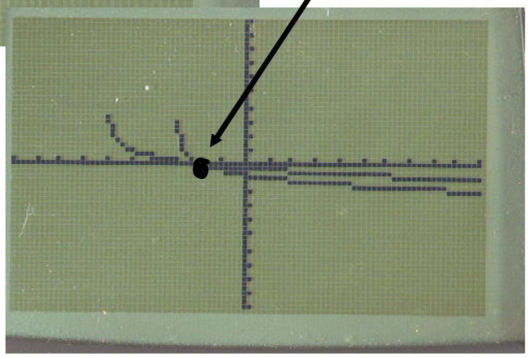
$$-3x = 6$$

$$x = -2$$







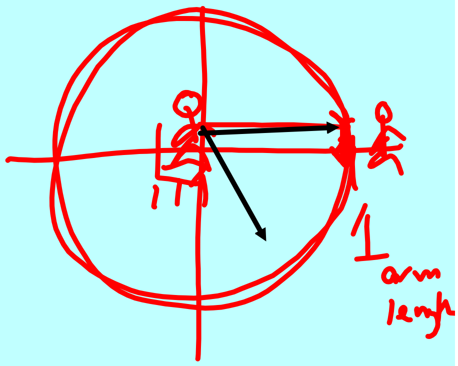


calc 5: in

enter ...

Lecture Circular Functions

- Converting a decimal degree to degrees-minutes-seconds
- Converting between degree and radian measure: Problem type 2
- Coterminal angles
- Arc length and central angle measure
- Finding trigonometric ratios from a point on the unit circle
- Trigonometric functions and special angles: Problem type 3
- Evaluating expressions involving sine and cosine
- Even and odd properties of trigonometric functions
- Using a calculator to approximate cosecant, secant, and cotangent values
- Evaluating a sinusoidal function that models a real-world situation
- Sine, cosine, and tangent ratios: Numbers for side lengths
- Sine, cosine, and tangent ratios: Variables for side lengths
- Using the Pythagorean Theorem to find a trigonometric ratio
- Finding trigonometric ratios given a right triangle
- Using a trigonometric ratio to find a side length in a right triangle
- Using trigonometry to find a length in a word problem with one right triangle



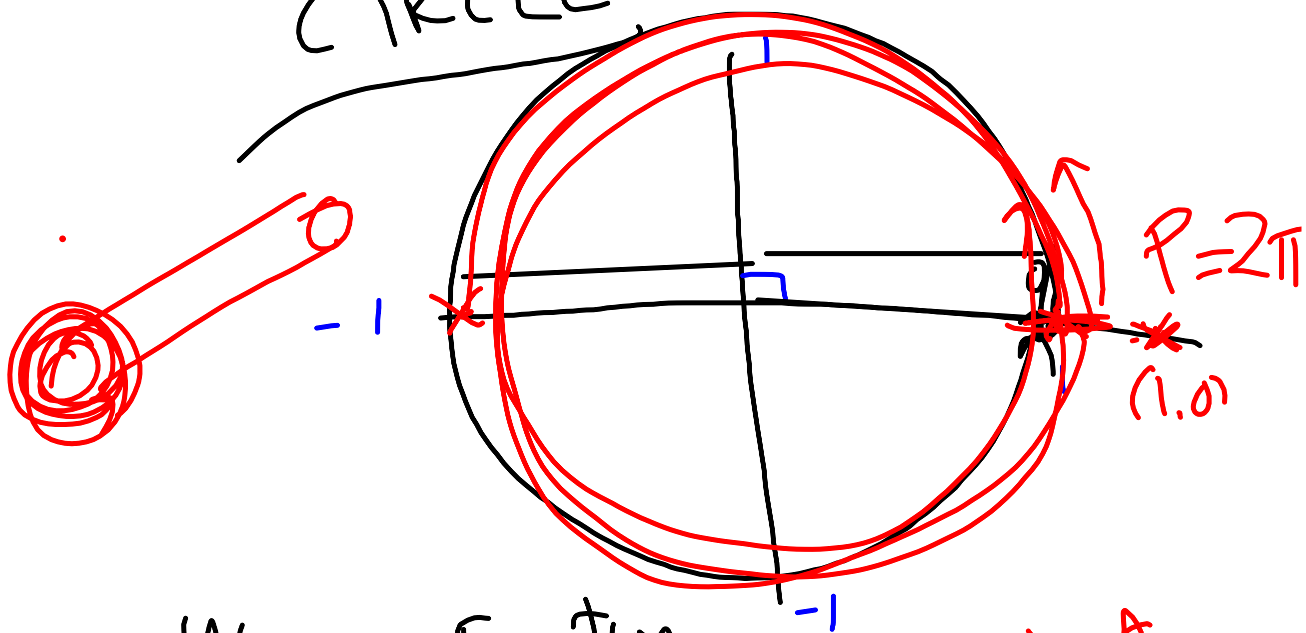
6.28 strides

Unit circle $r = 1$

Circum. $2\pi(1) = 2\pi$ strides

UNIT (1) CIRCLE

radius = 1



Wrapping Function.

input   output

$W(0) = (1, 0)$

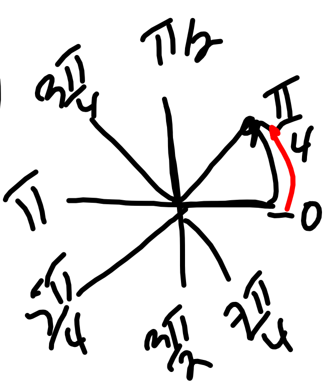
$W\left(\frac{\pi}{2}\right) = (0, 1)$

$W(\pi) = (-1, 0)$

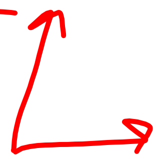
$W(2\pi) = (1, 0)$

$W\left(\frac{\pi}{4}\right) = (.707.., .707..)$

$\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$



Degrees (Angles) $\measuredangle$



$$W(0^\circ) = (1, 0)$$

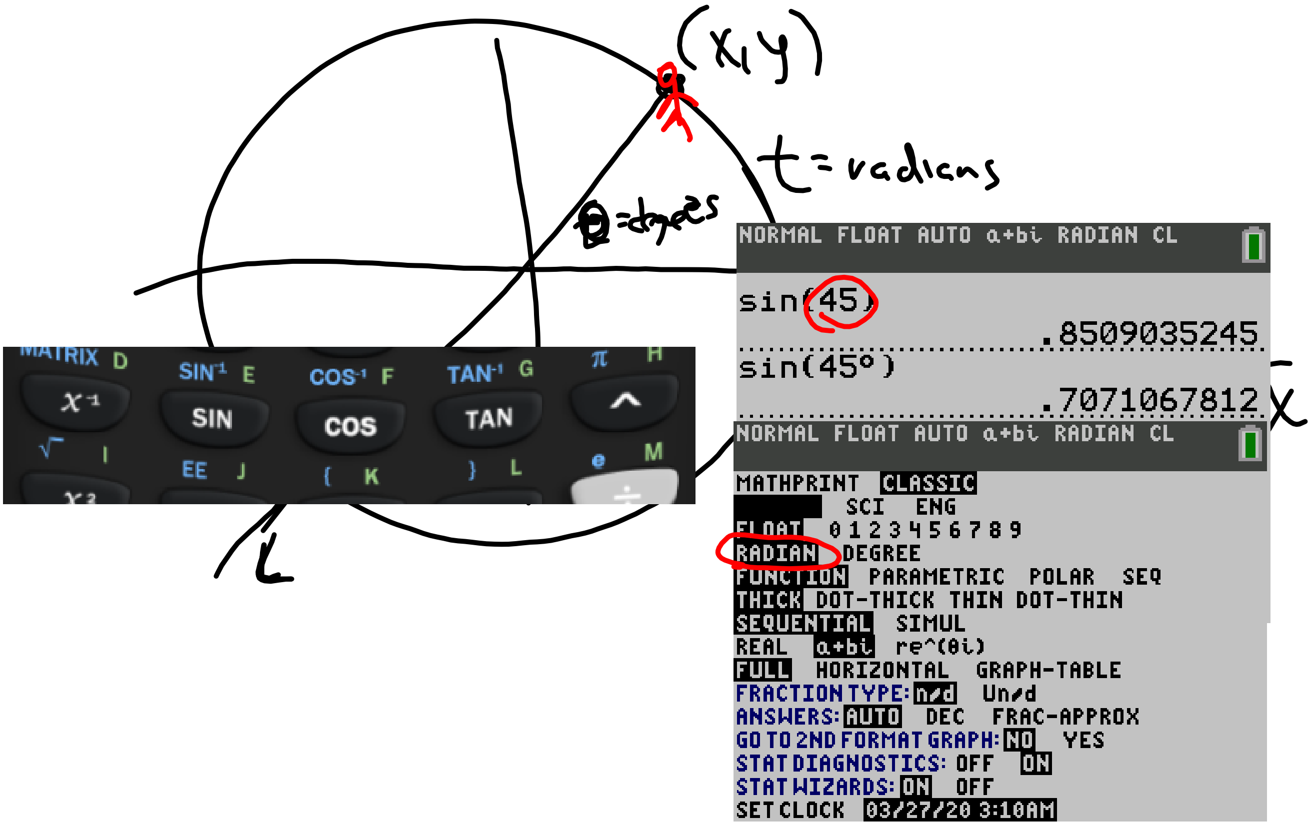
$$W(45^\circ) = (.707\ldots, .707\ldots)$$

$$W(90^\circ) = (0, 1)$$

$$W(90{,}000^\circ) = ?$$

$$(x, y) = (\cos t, \sin t)$$

"cosine" "sine"

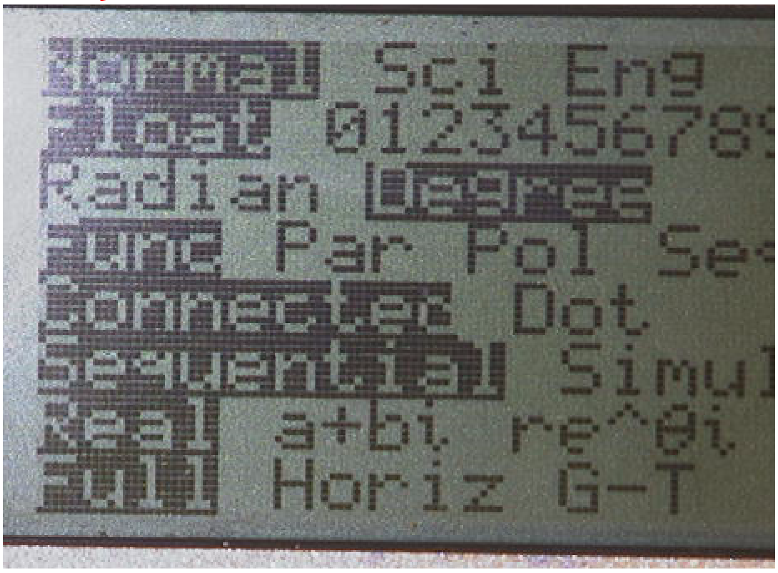


$x = \cos t$

$y_1 = \cos x$ (x input)

$y = \sin t$

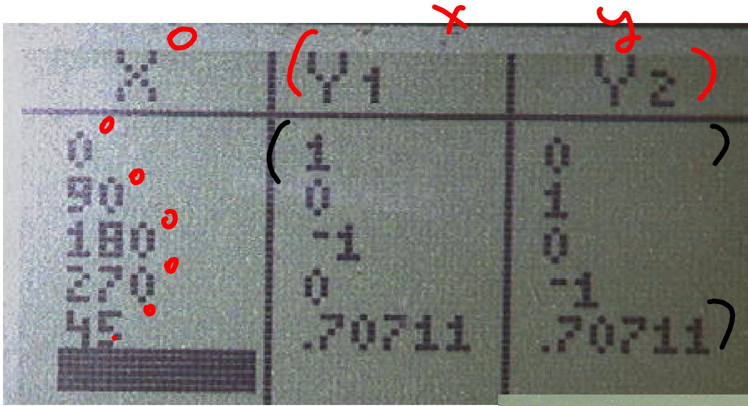
$y_2 = \sin x$



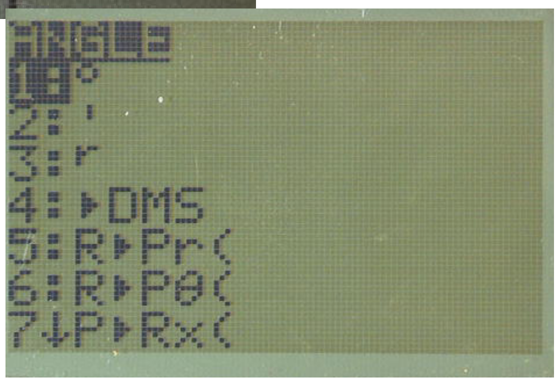
mode

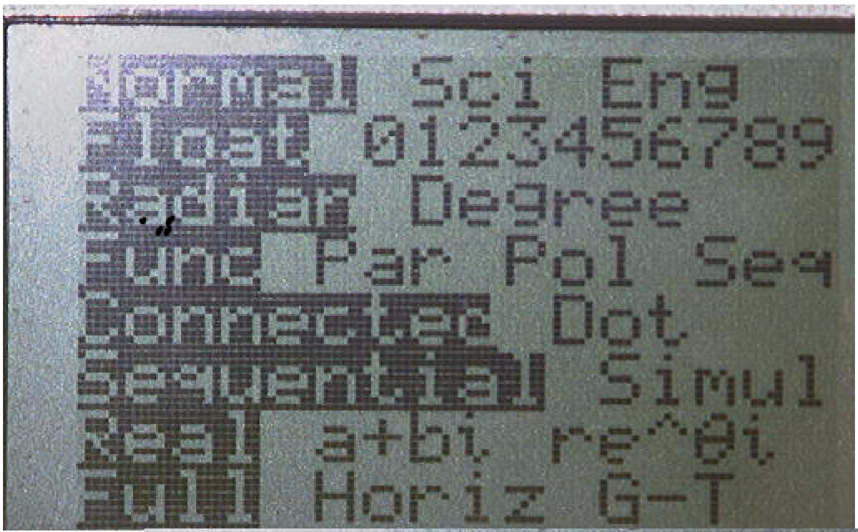
$y_1 = \cos X$ (x coord)
$y_2 = \sin X$ (y coord)

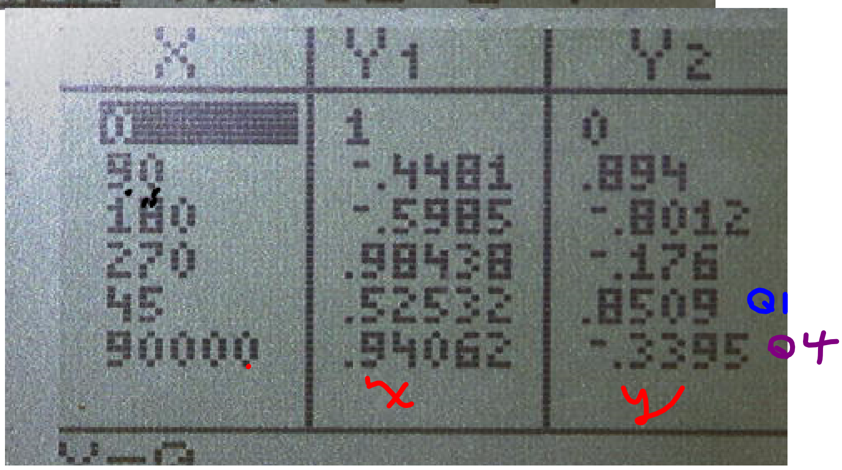
"θ" = theta

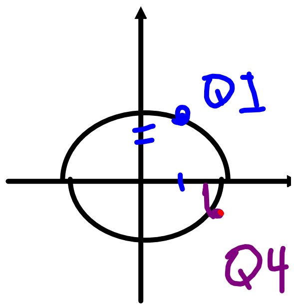


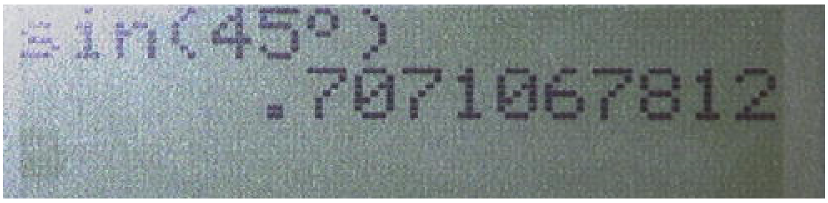
$W(0°) = (1, 0)$
$W(90°) = (0, 1)$
$W(45°) =$
$(.707.., .707..)$




Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T


X Y1 Y2
0 1 0
90 -.4481 .894
180 -.5985 -.8012
270 .98438 -.176
45 .52532 .8509
90000 .94062 -.3395
x
y
Q1
Q4


Q1
Q4



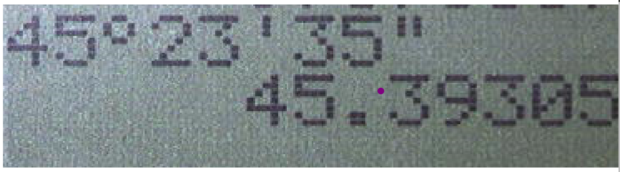
Degrees
60 Minutes in Degree
60 Seconds in Min
3600 seconds in Degree

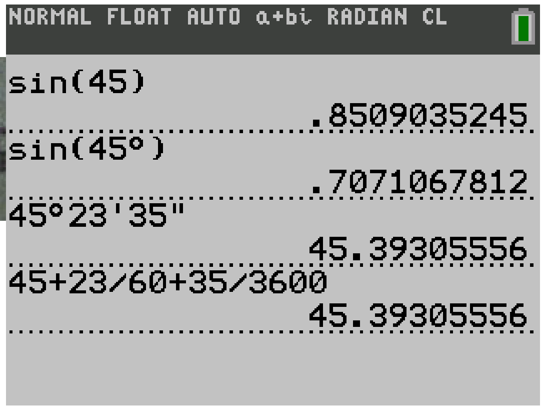
45° 23' 35"

$$45 + 23/60 + 35/3600$$

$$45 + .3833 + .009722$$

$$45.389722$$





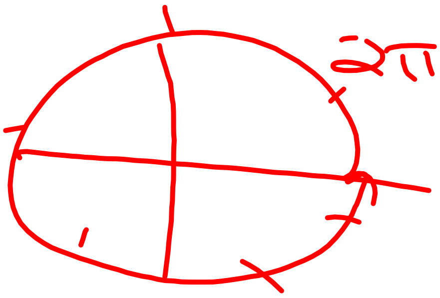
change 23.3455 degrees to DMS?

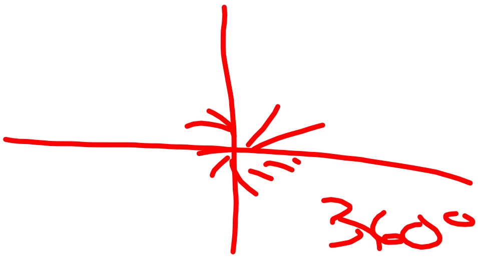
23 degrees

.3455*60 = 20.73 20 minutes

.73 * 60 = 43.8   44 seconds

23^{o} 20' 44"





$$36° \cdot \boxed{\frac{\pi}{180°}} = \frac{36\pi}{180} = \frac{2\pi}{10} = \pi/5$$

$$360° = 2\pi$$

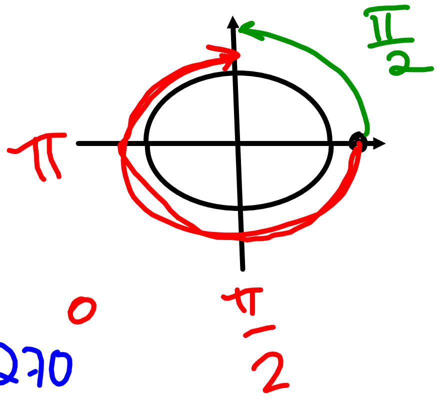
$$180° = \pi$$

$$\boxed{\frac{180°}{\pi}} = \boxed{\frac{\pi}{180°}} = \boxed{1}$$

$$\left(\frac{\cancel{\pi}}{6}\right)\cdot\left(\frac{180°}{\cancel{\pi}}\right) = \cancel{180}° \; 30°$$

$$\pi/6 = 30°$$

Convert $-\frac{3\pi}{2}$ radians to degree measure.



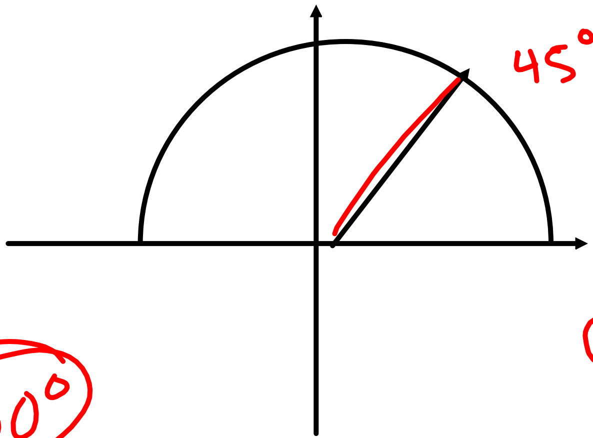
$$\frac{-3}{2}\cancel{\pi} \cdot \left(\frac{180^\circ}{\cancel{\pi}}\right) = -270^\circ$$

$$3' \cdot \frac{12''}{1'} = 36''$$

$$\pi = 180^\circ$$

coterminal

Coterminal



45° and

$45^\circ + 360^\circ$

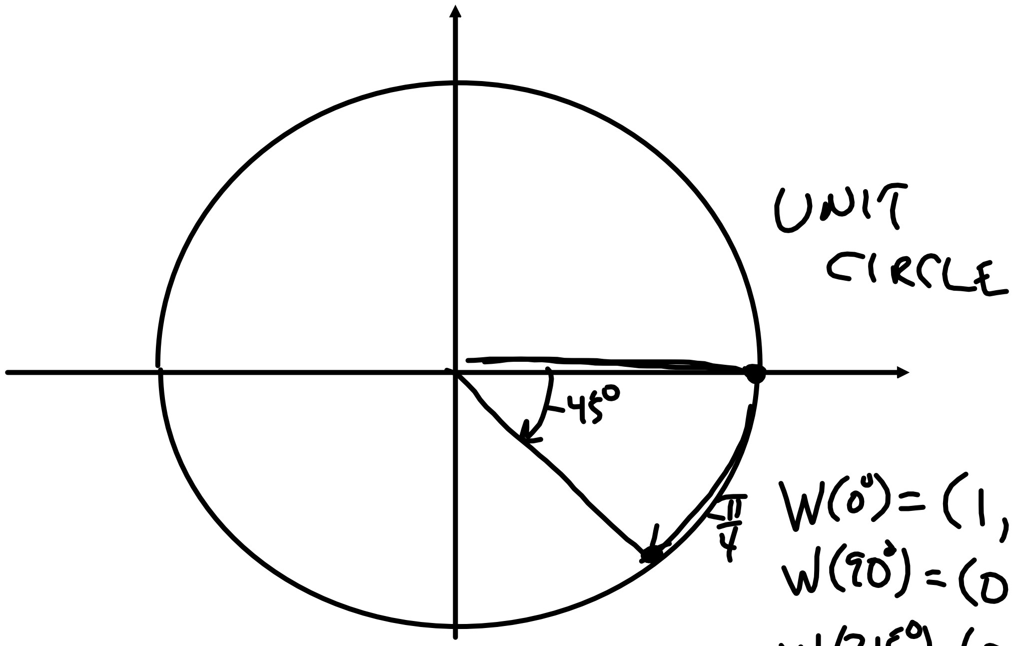
405°

are

coterminal

$$
\begin{array}{r}
780^\circ \\
-\ 360 \\
\hline
-\ 360 \\
\hline
60^\circ
\end{array}
$$

$0 < 60^\circ < 360$



$W(0^\circ) = (1, 0)$
$W(90^\circ) = (0, 1)$
$W(315^\circ) = (.7.., -.7..)$

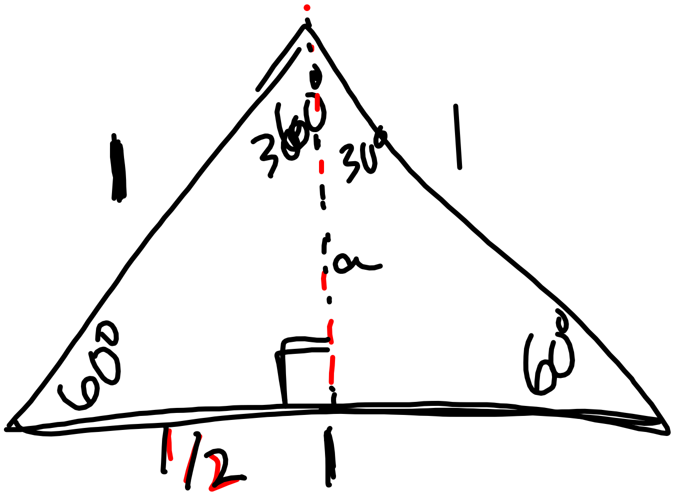
x-value · Cosine

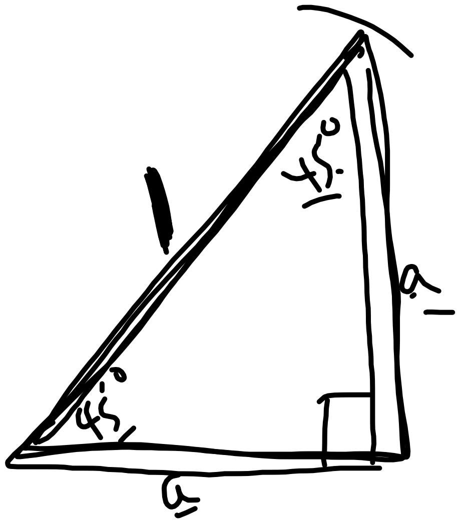
$\cos(0^\circ) = 1$
$\cos(90^\circ) = 0$
$\cos(315^\circ) = .7\ldots = \frac{\sqrt{2}}{2}$

y-vale Sine

$\sin(0^\circ) = 0$
$\sin(90^\circ) = 1$
$\sin(315^\circ) = -.7\ldots$

Special Angles





$$a^2 + (1/2)^2 = 1^2$$
$$a^2 + 1/4 = 1$$
$$a^2 = 3/4$$
$$a = \sqrt{3}/2$$
$$= .866\ldots$$

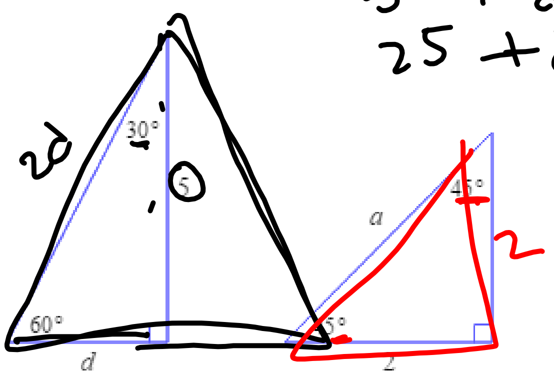
$$a^2 + a^2 = 1$$
$$2a^2 = 1$$
$$a^2 = 1/2$$
$$a = \sqrt{1/2}$$
$$= 1/\sqrt{2}$$
$$= \sqrt{2}/2$$
$$= .7070\ldots$$

	0°	30°	45°	60°	90°
$\cos(\theta)$	$\frac{\sqrt{4}}{2}$ 1	$\frac{\sqrt{3}}{2}$ .866	$\frac{\sqrt{2}}{2}$ .707	$\frac{\sqrt{1}}{2}$ .5	$\frac{\sqrt{0}}{2}$ 0
$\sin(\theta)$	$\frac{\sqrt{0}}{2}$ 0	$\frac{\sqrt{1}}{2}$ .5	$\frac{\sqrt{2}}{2}$ .707	$\frac{\sqrt{3}}{2}$ .866	$\frac{\sqrt{4}}{2}$ 1

Special right triangles: Exact answers

For the right triangles below, find the exact values of the side lengths d and a.

If necessary, write your responses in simplified radical form.



$5^2 + d^2 = (2d)^2$

$25 + d^2 = 4d^2$

$25 = 3d^2$

$\frac{25}{3} = d^2$

$d = \frac{5}{\sqrt{3}}$

$2^2 + 2^2 = a^2$

$8 = a^2 \quad a = \sqrt{8}$

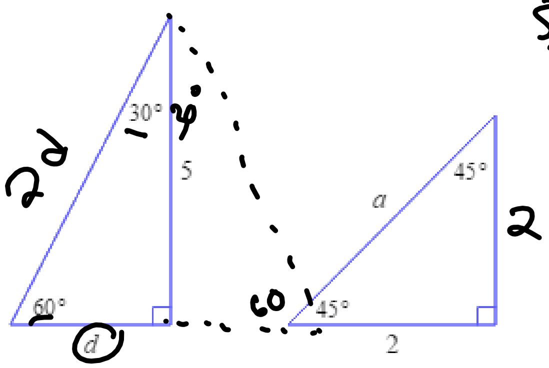
Special right triangles: Exact answers

For the right triangles below, find the exact values of the side lengths d and a.

If necessary, write your responses in simplified radical form.

$$d^2 + 5^2 = 4d^2$$

$$5^2 = 3d^2$$



$$d^2 = \frac{5^2}{3}$$

$$d = \frac{5}{\sqrt{3}} \qquad a = 2\sqrt{2}$$

Convert degrees to radians

\$3 won = \$ 1 gambled

$$\$20\text{ gambled}\cdot\frac{\$3\text{ won}}{\$1\text{ gambled}} = \$60\text{ won}$$

2pi = 360 degrees

$$45^{\circ}\cdot\left(\frac{\pi}{180^{\circ}}\right) = \frac{\pi}{4}\ \text{rad}$$

Arc length and central angle measure

A circular arc has measure 10 in and is intercepted by a central angle of $60°$. Find the radius r of the circle.

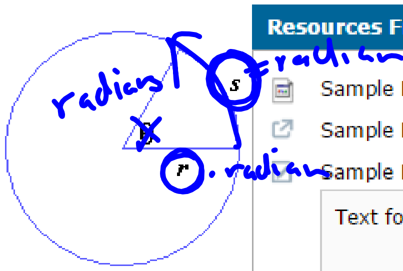
Do not round any intermediate computations, and round your answer to the nearest tenth.

Suppose that a central angle of θ radians intercepts an arc of length s in a circle of radius r.

Then the length s of the arc is given by the following.

$$s = r\theta$$

Deriving this formula



We can solve this equation for r.

$$r = \frac{s}{\theta}$$

In order to use this equation, we must first convert the given angle to radian measure.

$$\theta = 60° = 60\cdot\left(\frac{\pi}{\ }\right) \text{radians} = \frac{\pi}{\ } \text{radians}$$

Your resources will appear in the box below.

Resources From Your Instructor

Sample File

Sample Link

Sample Note

Text for students to read.

Additional Resources

Precalculus, 2nd Ed.
Coburn

$S = r \cdot \theta$

Angle in radians

45°

$\pi/4$

$8'' \cdot \pi/4 = 2\pi'' \approx 6$

CLOSE WINDOW

Arc length and central angle measure

A circular arc has measure 10 in and is intercepted by a central angle of 60° . Find the radius r of the circle.

Do not round any intermediate computations, and round

60degrees= .866

$s=r\theta$ = 10(.86

NORMAL FLOAT AUTO a+bi RADIAN CL

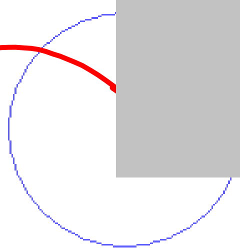
60*π/180

1.047197551

Suppose that a central angle of θ radians intercepts an arc in a circle of radius r.

Then the length s of the arc is given by the following.

θ



$s = r\theta$

Deriving this formula

$10 = r \cdot 1.047..$

Text for students to read.

We can solve this equation for r.

$$r = \frac{s}{\theta}$$

s= length of pizza bone

In order to use this equation, we must first convert the given angle to radian measure.

Additional Resources

Precalculus, 2nd Ed.
Coburn

$\theta = 60° = 60 \cdot \left(\frac{\pi}{\ }\right)$ radians $= \frac{\pi}{\ }$ radians

90degrees = pi/2 radians

8 inch slice (radius)

pizza bone (2 slices) = 8" times pi/2= 4pi"

Coterminal angles

Answer the following.

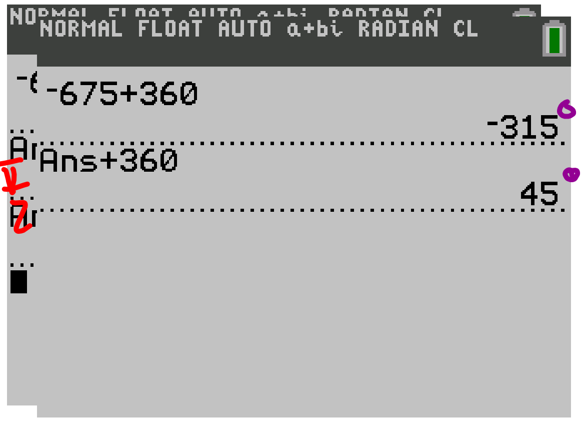
(a) Find an angle between 0 and 2π that is coterminal with $\frac{13\pi}{2}$. = 1

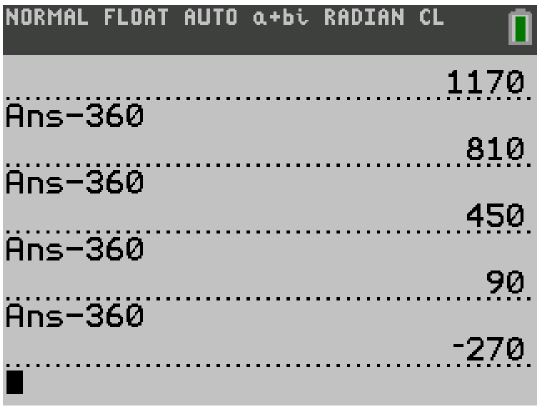
(b) Find an angle between $0°$ and $360°$ that is coterminal with $-675°$. = 45°

Give exact values for your answers.

$\frac{13\pi}{2} \cdot \frac{180}{\pi} = 1170 = 90°$

$90° \cdot \frac{\pi}{180} = \frac{\pi}{2}$





Coterminal angles

Answer the following.

(a) Find an angle between 0 and 2π that is coterminal with $\frac{13\pi}{2}$. $\frac{180}{\pi}$ 1170

(b) Find an angle between 0° and 360° that is coterminal with −675°. ⇔ 45°

Give exact values for your answers.

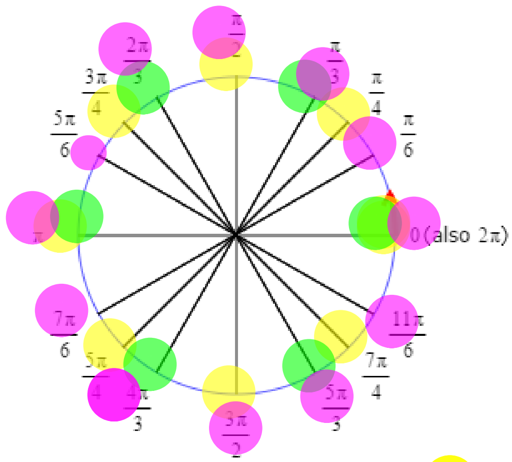
rad → degree

$$1170 - 360 = 810$$
$$810 - 360 = 450$$
$$450 - 360 = 90°$$

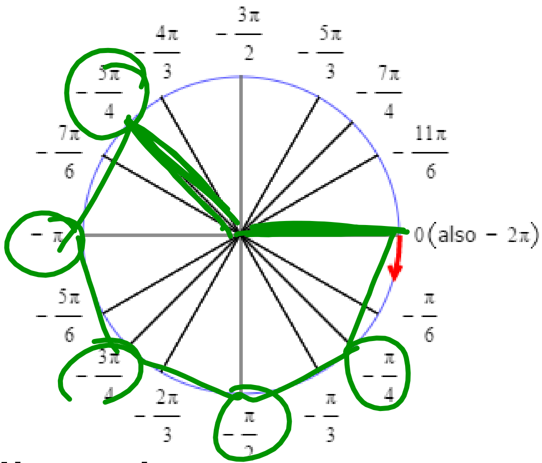
$$-675 + 360 = -315$$
$$-315 + 360 = 45°$$

$$\frac{\pi}{180°} \rightarrow \frac{\pi}{2}$$

Positive angles



Negative angles



8 slice pizza

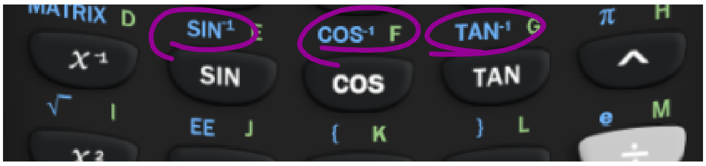
12 slice pizza

6 slice pizza

The current problem:

We must sketch $\theta = -\frac{5\pi}{4}$ in standard position.

$\sin(t)$



$\cos(t)$

$\tan(t)$

$$2^{-1} = \frac{1}{2}$$

inverse $f^{-1}(x) \leftarrow \sin^{-1}(t)$

$$\frac{1}{\sin(t)} = \underline{\csc(t)}$$ cosecant $\left(\frac{1}{y}\right)$

$$\frac{1}{\cos(t)} = \underline{\sec(t)}$$ secant $\left(\frac{1}{x}\right)$

$$\frac{1}{\tan(t)} = \underline{\cot(t)}$$ cotangent $\left(\frac{x}{y}\right)$

Finding trigonometric ratios from a point on the unit circle

Suppose that θ is an angle in standard position whose terminal side intersects the unit circle at $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

x y

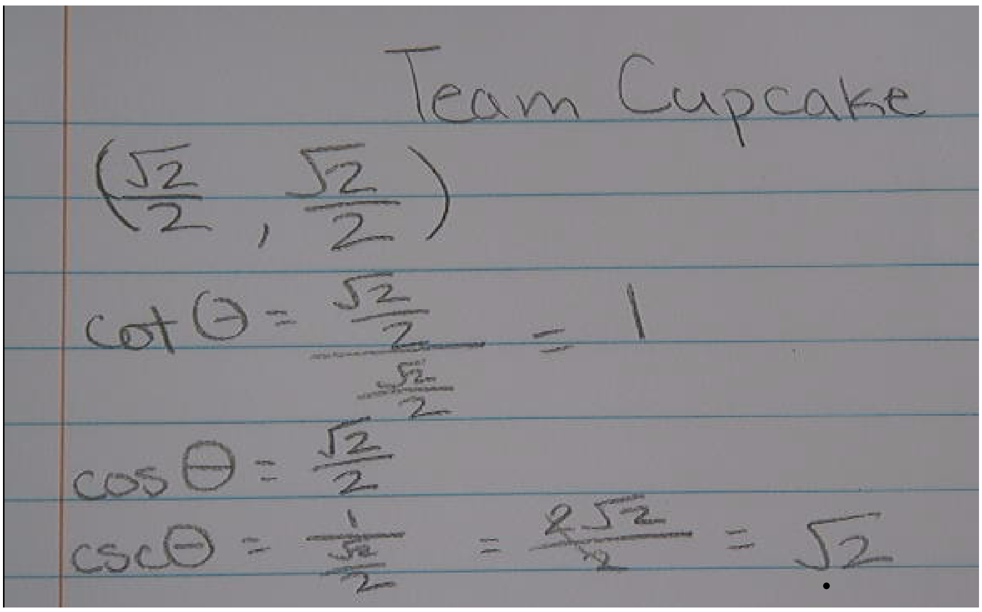
x/y x 1/y

Find the exact values of $\cot\theta$, $\cos\theta$, and $\csc\theta$.

sint = y $\frac{\sqrt{2}}{2}$ csct= 1/y $2/\sqrt{2}$ co secant

cost= x $\frac{\sqrt{2}}{2}$ sect= 1/x $2/\sqrt{2}$ secant

tant= y/x 1 cot t= x/y 1 cotangent



$f^{-1}(x) \rightarrow$ inverse

$\sin^{-1}(x) \rightarrow$ inverse Sine

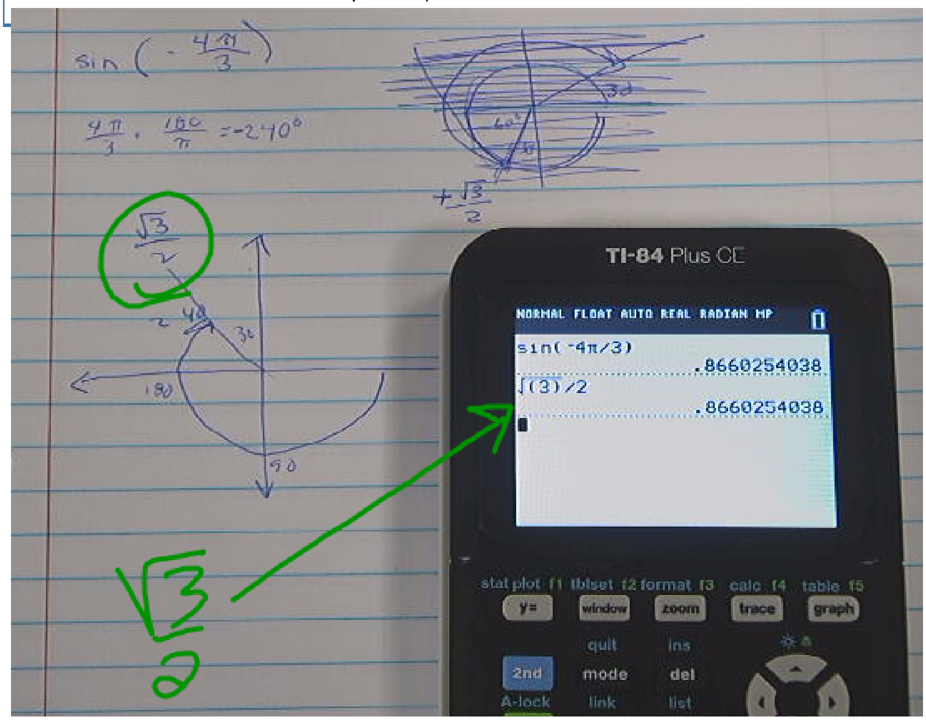
$$\frac{1}{\sin(x)} = \left(\sin(x)\right)^{-1} = \csc x$$

$$\frac{1}{\cos(x)} = \qquad = \sec x'$$

Examples

Trigonometric functions and special angles: Problem type 1

Find the exact value of $\sin\left(-\frac{4\pi}{3}\right)$.



$.707 = \frac{\sqrt{2}}{2}$

$.866 = \frac{\sqrt{3}}{2}$

$1.73 = \sqrt{3}$

$.57 = \frac{\sqrt{3}}{3}$

$1.414 = \sqrt{2}$

Trigonometric functions and special angles: Problem type 2

Find the exact values below. If applicable, click on "Undefined".

$\tan 0$

$\csc 0$

```
1/tan(-2π/3)
                .5773502692
√3/3
                .5773502692
```

Trigonometric functions and special angles: Problem type 2

Find the exact values below. If applicable, click on "Undefined".

$\cot\left(-\frac{2\pi}{3}\right) = \sqrt{3}/3$ $\qquad 1/\tan(-2\pi/3) =$

$\sec\left(-\frac{2\pi}{3}\right) = -2$ $\qquad 1/\cos(-2\pi/3) = -2$

	0°	30°	45°	60°	90°
$\cos(\theta)$	$\frac{\sqrt{4}}{2}$ 1	$\frac{\sqrt{3}}{2}$ .866	$\frac{\sqrt{2}}{2}$ .707	$\frac{\sqrt{1}}{2}$ .5	$\frac{\sqrt{0}}{2}$ 0
$\sin(\theta)$	$\frac{\sqrt{0}}{2}$ 0	$\frac{\sqrt{1}}{2}$ .5	$\frac{\sqrt{2}}{2}$ .707	$\frac{\sqrt{3}}{2}$ .866	$\frac{\sqrt{4}}{2}$ 1

$1/\cos$

$\sin/\cos$

$\frac{2\sqrt{3}}{3} \approx 1.155 \qquad \sqrt{2} \approx 1.414$

$\sqrt{3} \approx 1.73\ldots \qquad \frac{\sqrt{3}}{3} \approx .577$

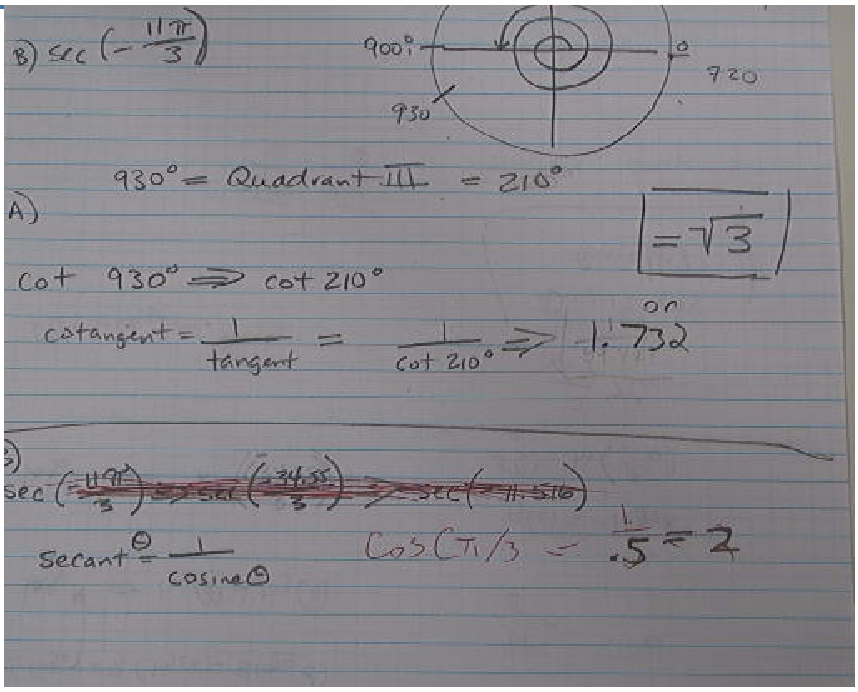
```
1/tan(-2π/3)
                .5773502692
1/cos(-2π/3)
                -2
```

Trigonometric functions and special angles: Problem type 3

Find the exact values below. If applicable, click on "Undefined."

$\cot 930°$

$\sec\left(-\dfrac{11\pi}{3}\right)$



Area of a sector of a circle

The area of a sector of a circle with a central angle of $\frac{7\pi}{6}$ radians is $23\mathrm{cm}^2$. Find the radius of the circle.

Do not round any intermediate computations. Round your answer to the nearest tenth.

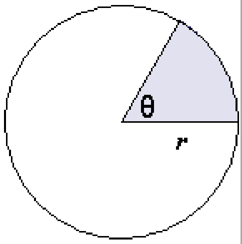
The area A of a sector of a circle with radius r and central angle θ is as follows.

$$A = \frac{1}{2}r^2\theta$$

Deriving the formula

Note that for this formula, θ *must* be in radians.

For our problem, the area is $A = 23\mathrm{cm}^2$ and the central angle is $\theta = \frac{7\pi}{6}$ radians.



Your resources will appear in the box below.

Resources From Your Instructor

- Sample File
- Sample Link
- Sample Note

Text for students to read.

Victoria
Justin.

$A = \frac{1}{2}r^2\theta.$

$A = 23\,cm^2. \qquad \theta = \frac{7\pi}{6}.$

$23 = \frac{1}{2}r^2 \cdot \frac{7\pi}{6}.$

$\frac{7\pi}{6} \cdot r^2 = 46.$

$7\pi \cdot r^2 = 276.$

$\pi \cdot r^2 = \frac{276}{7}$

$r^2 = \frac{276}{7\pi}$

$r > 0.$

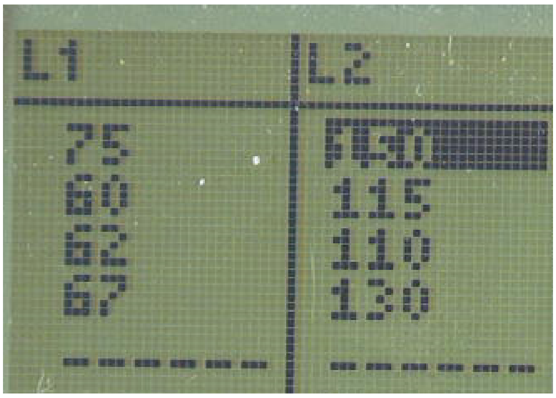
$r = \sqrt{\frac{276}{7\pi}}.$

$r \approx 3.5.$

Project Overview

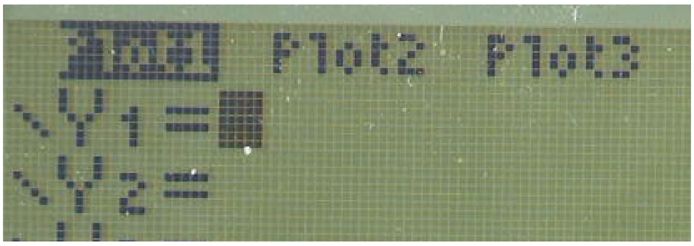
Enter Data

stat edit



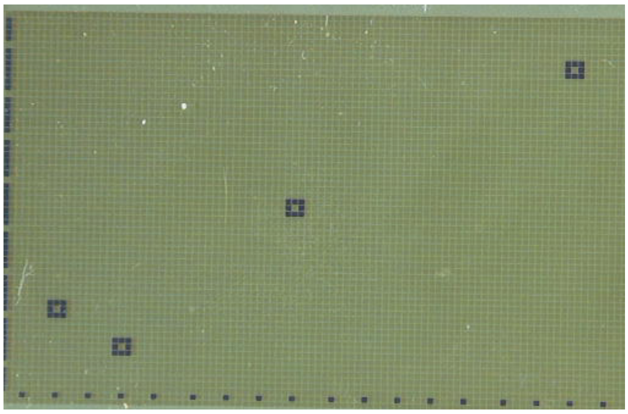
Plot

y= ^ enter



turns on plots

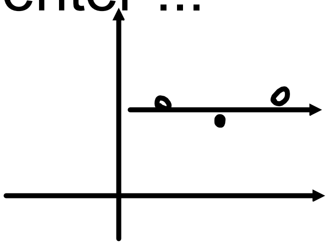
zoom 9 : zoomstat

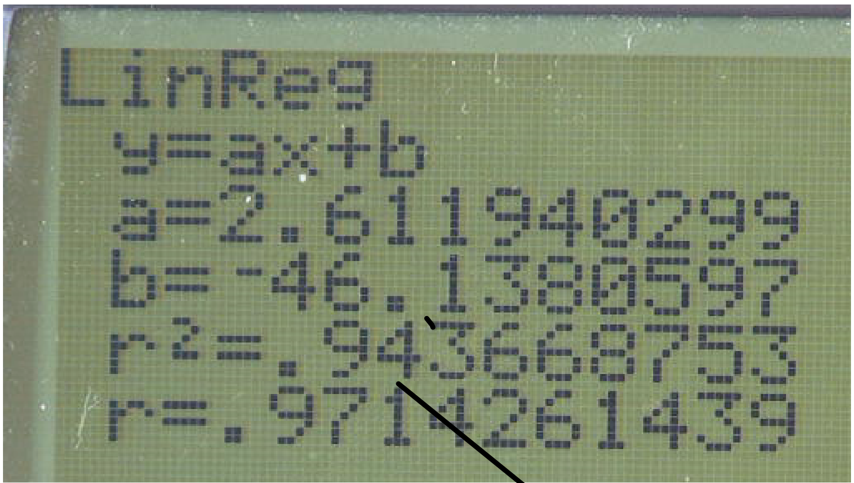


Finding Regressions

stat > calc 4: Linear

enter ...





To get r =

catalog (2nd 0) arrow down to "diagnosticON"

94%

corr

enter ..

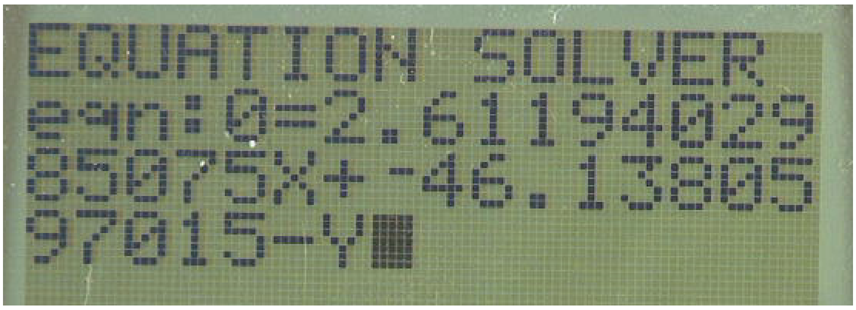
Evaluate and Solve

Y(72)=calc 1:, vars, trace, table

220= Y(x) solver, intersection, hand

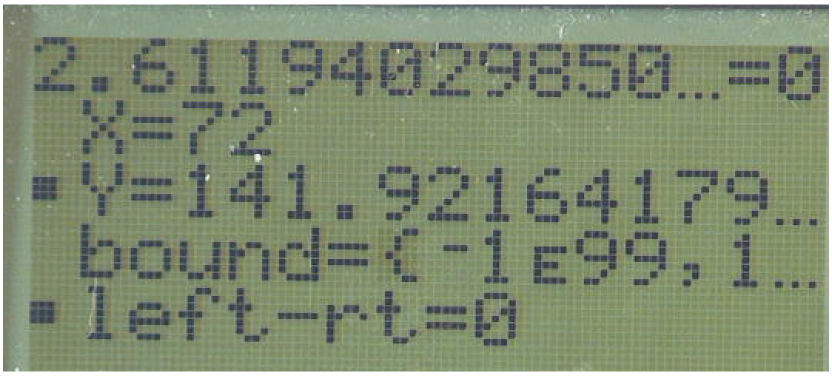
both? Go to solver   Math 0: (B)

var 5: > > 1: regeq (copies in last regression) minus Y (alpha 1)



x= 72

y=0 (alpha enter)

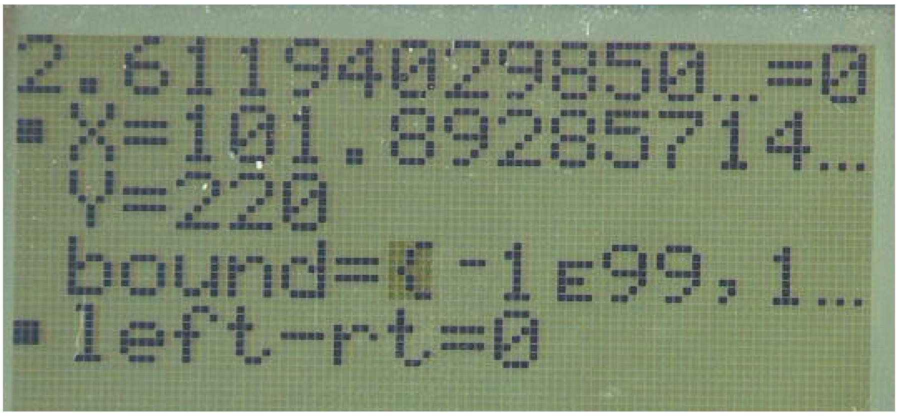


According to the linear regression, a person of 72 inches should weigh 141 lbs.

Solve,

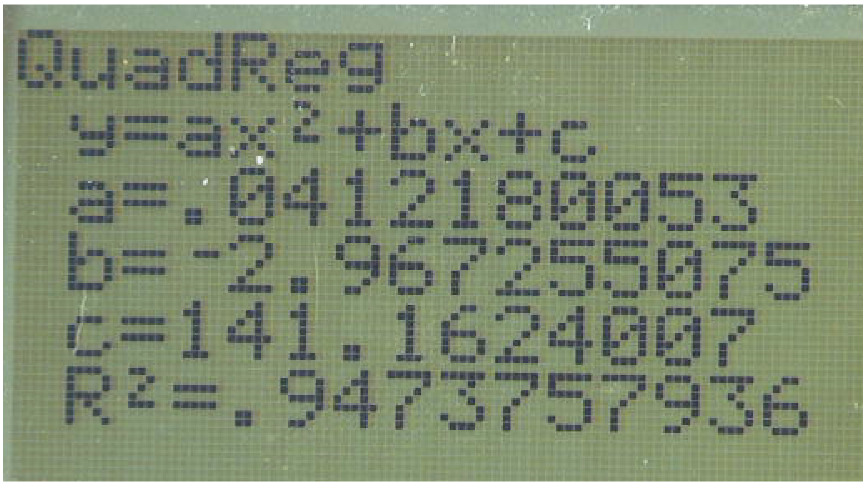
Y= 220lbs

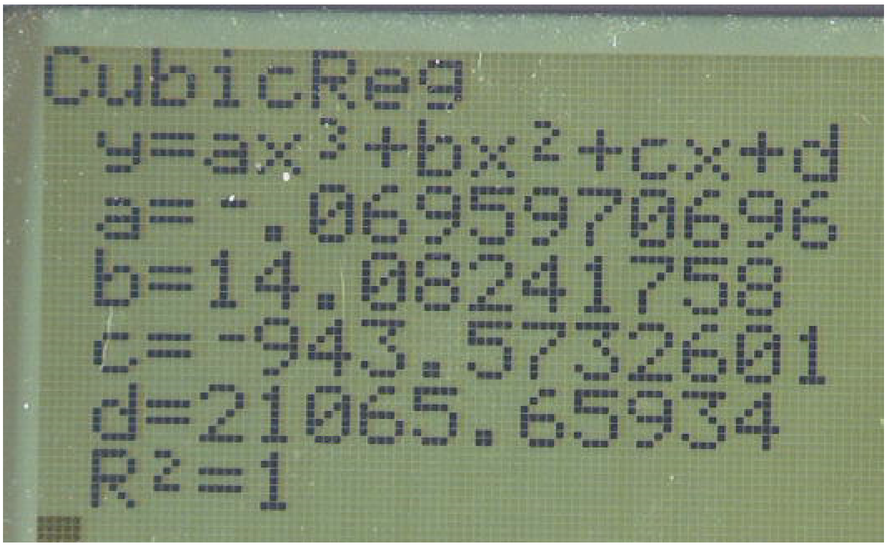
x= 72 (alpha) (enter)



A person of 220 lbs should be over 8 foot (according to the linear regression)

Repeat for all the regressions!





2 points = line

3 points= quartic

4 points cubic

5 points = quartic

```
ExpReg
 y=a*b^x
 a=33.2852456
 b=1.020290709
 r²=.9315020862
 r=.9651435573
```

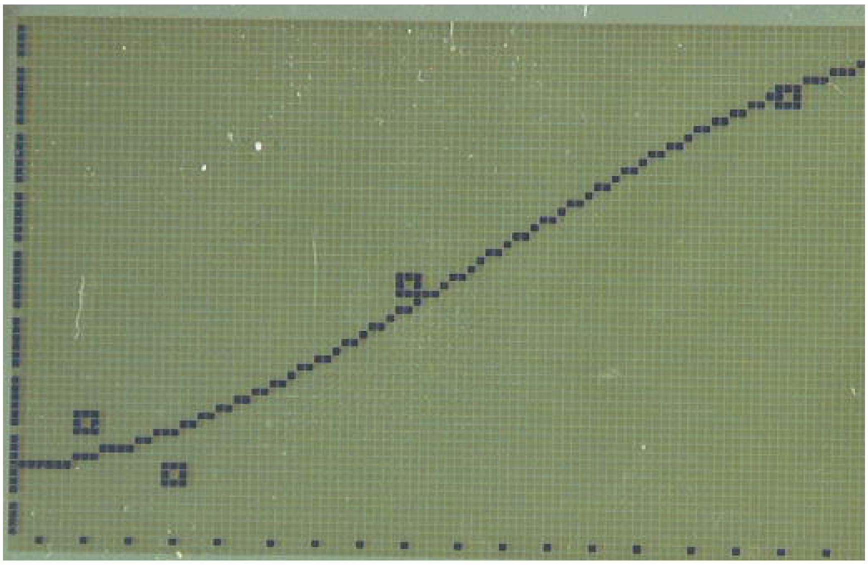
```
LnReg
 y=a+blnx
 a=⁻608.7483142
 b=175.5884504
 r²=.9390026143
 r=.9690214726
```

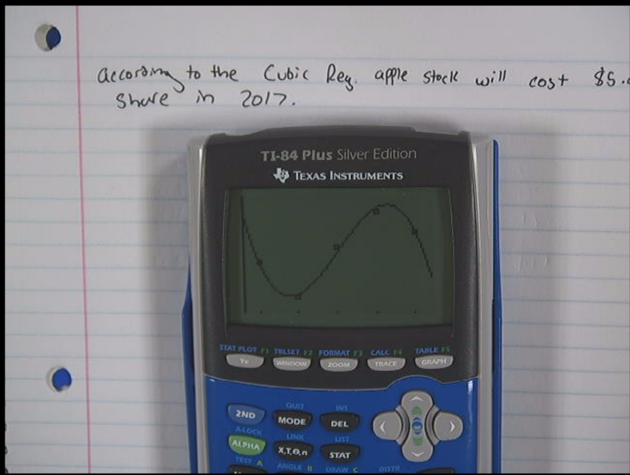
```
SinReg
 y=a*sin(bx+c)+d
 a=22.62225067
 b=.1396263402
 c=2.987897982
 d=132.8745623
```

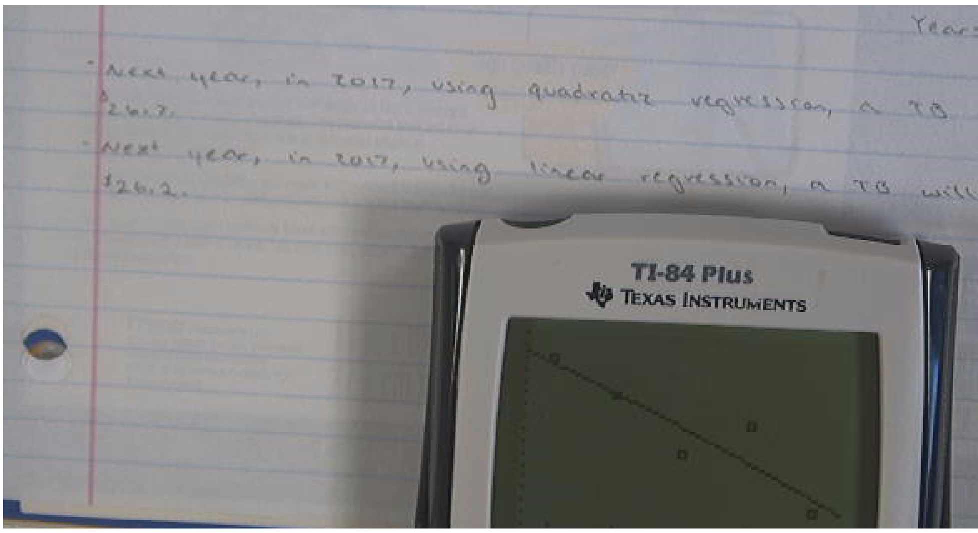
Graph the regressions

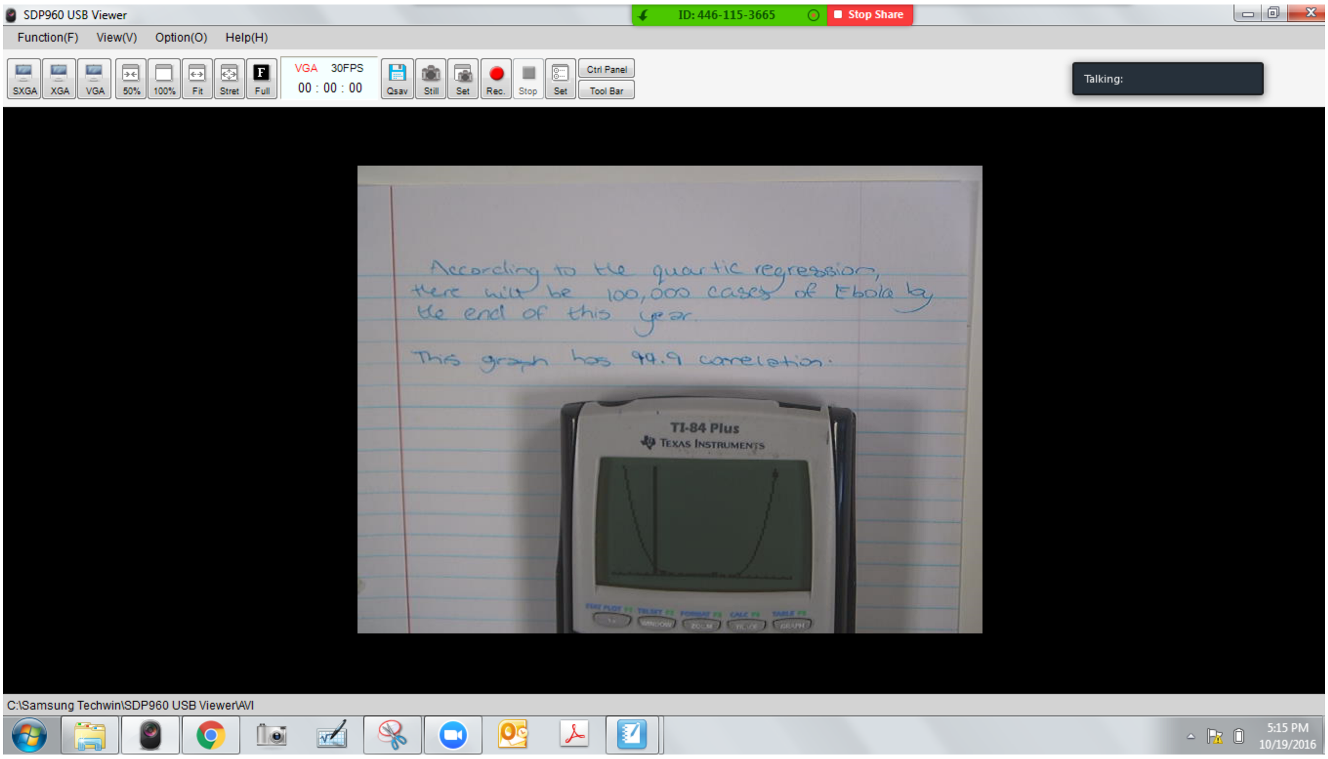
y= var 5 >> 1

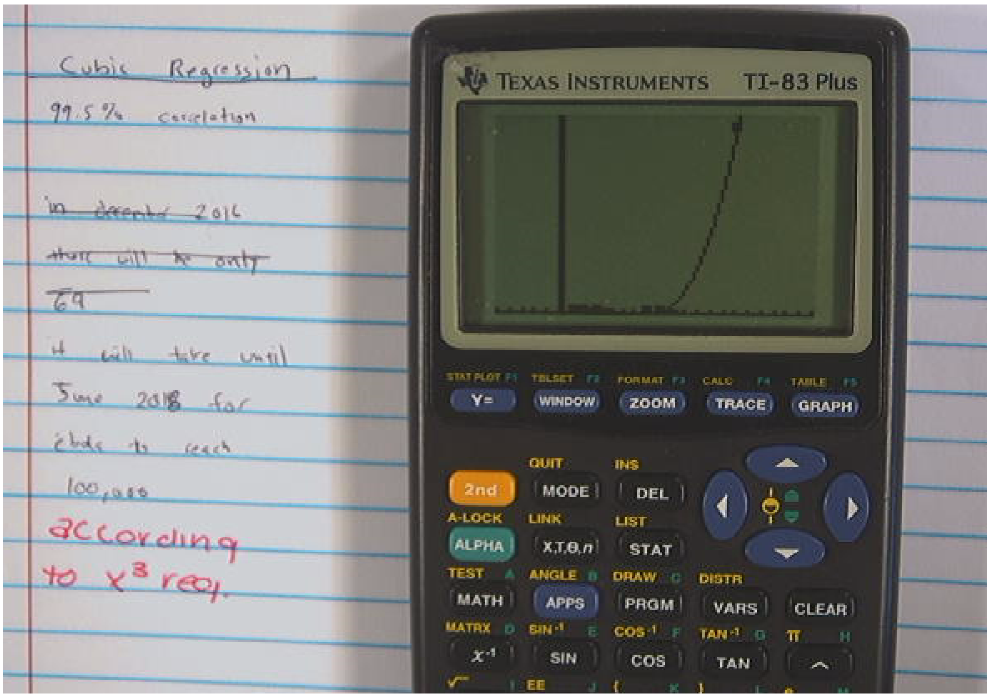
graph




according to the Cubic Reg. apple stock will cost
share in 2017.
TI-84 Plus Silver Edition
Texas Instruments


Next year, in 2017, using quadratic regression, a TB
26.7.
Next year, in 2017, using linear regression, a TB will
$26.2.
TI-84 Plus
Texas Instruments


SDP960 USB Viewer
ID: 446-115-3665
Stop Share
Function(F) View(V) Option(O) Help(H)
SXGA XGA VGA 50% 100% Fit Stret Full
VGA 30FPS
00 : 00 : 00
Qsav Still Set Rec. Stop Set
Ctrl Panel
Tool Bar
Talking:
According to the quartic regression, there will be 100,000 cases of Ebola by the end of this year.
This graph has 99.9 correlation.
TI-84 Plus
Texas Instruments
C:\Samsung Techwin\SDP960 USB Viewer\AVI
5:15 PM
10/19/2016


Cubic Regression
99.5% correlation
~~in december 2016~~
~~there will be only~~
~~69~~
it will take until
June 2018 for
clubs to reach
100,000
according
to x3 reg.
TEXAS INSTRUMENTS
TI-83 Plus