Agenda

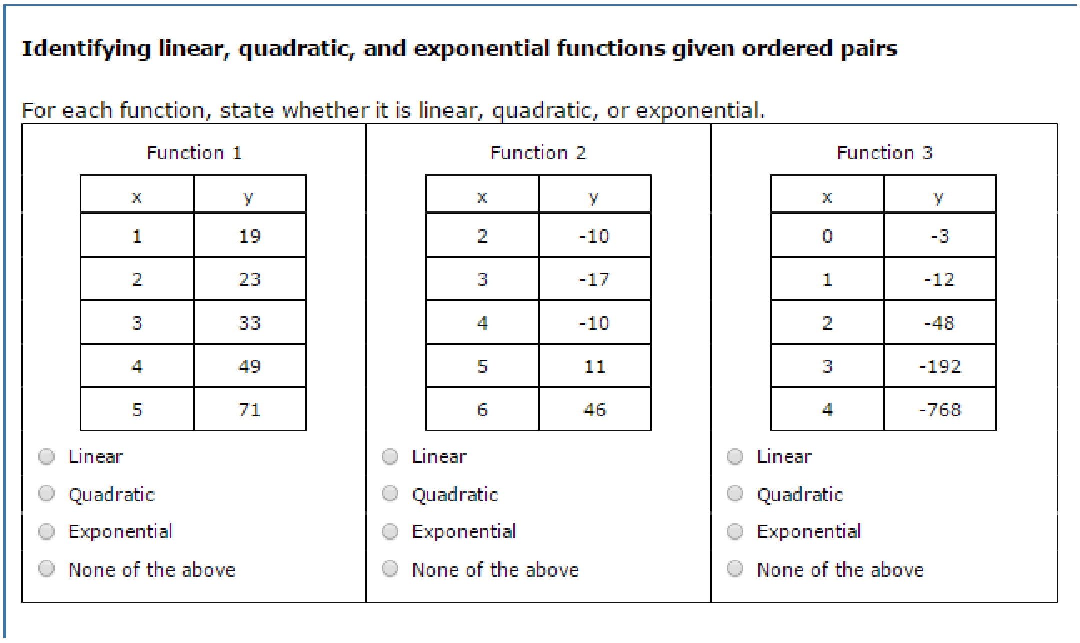
Review Quiz 6

Review Asymptotes

Lecture: Properties of Logs

Groupwork

Review Quiz 6



Identifying linear, quadratic, and exponential functions given ordered pairs

For each function, state whether it is linear, quadratic, or exponential.

Function 1

x	y
1	19
2	23
3	33
4	49
5	71

○ Linear
○ Quadratic
○ Exponential
○ None of the above

Function 2

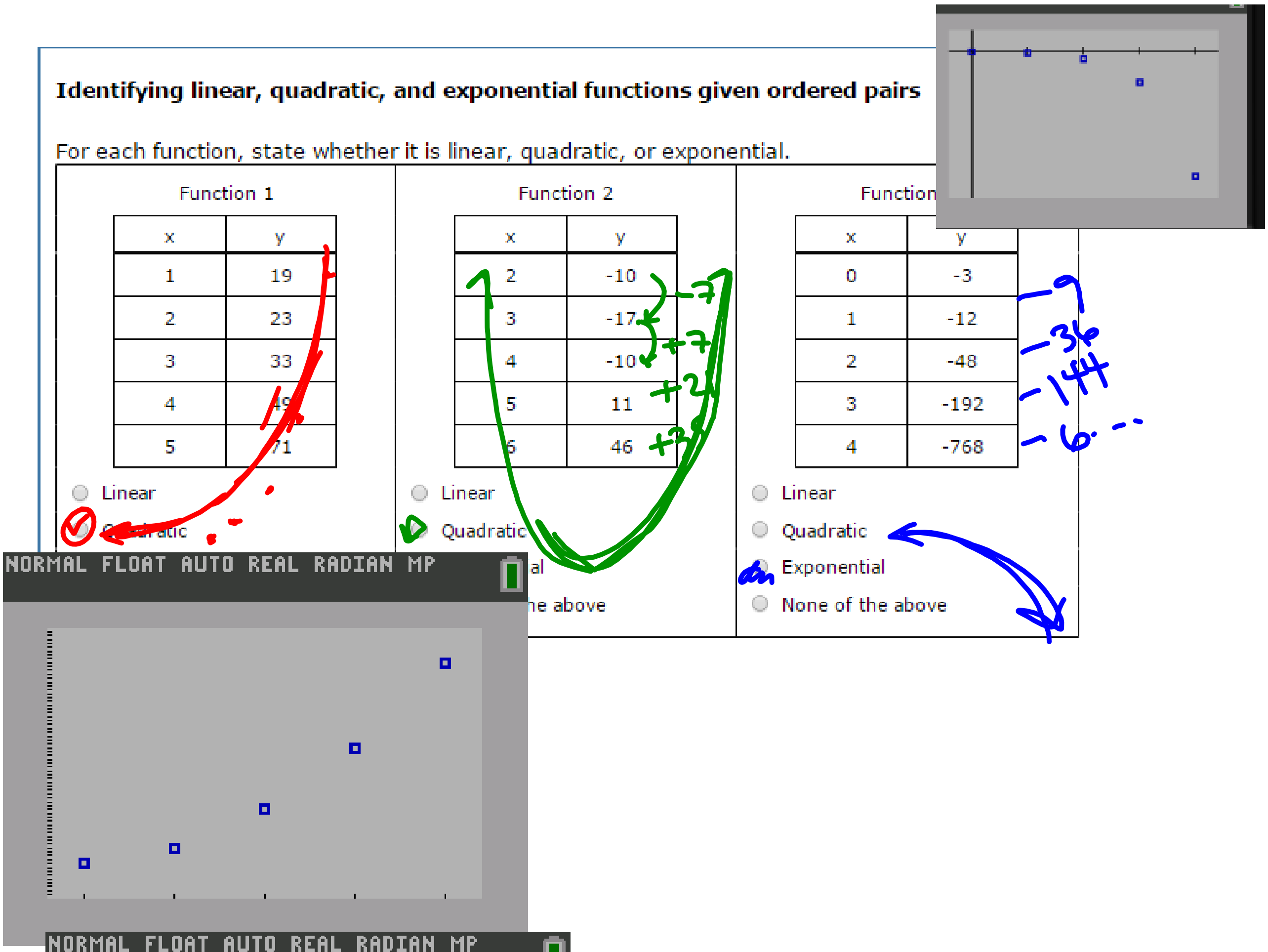
x	y
2	-10
3	-17
4	-10
5	11
6	46

○ Linear
○ Quadratic
○ Exponential
○ None of the above

Function 3

x	y
0	-3
1	-12
2	-48
3	-192
4	-768

○ Linear
○ Quadratic
○ Exponential
○ None of the above



Identifying linear, quadratic, and exponential functions given ordered pairs

For each function, state whether it is linear, quadratic, or exponential.

Function 1

x	y
1	19
2	23
3	33
4	49
5	71

- ○ Linear
- [✓ Quadratic]

Function 2

x	y
2	-10
3	-17
4	-10
5	11
6	46

-7
+7
+21
+3

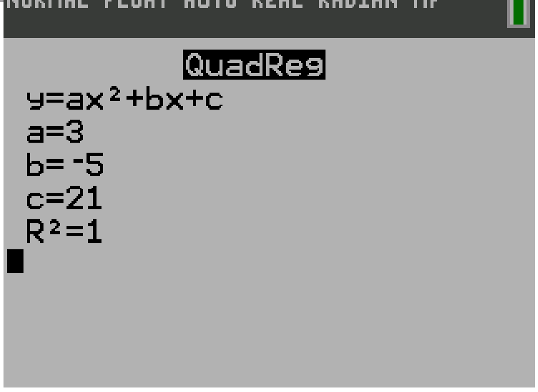
- ○ Linear
- [✓ Quadratic]
- al
- ne above

Function

x	y
0	-3
1	-12
2	-48
3	-192
4	-768

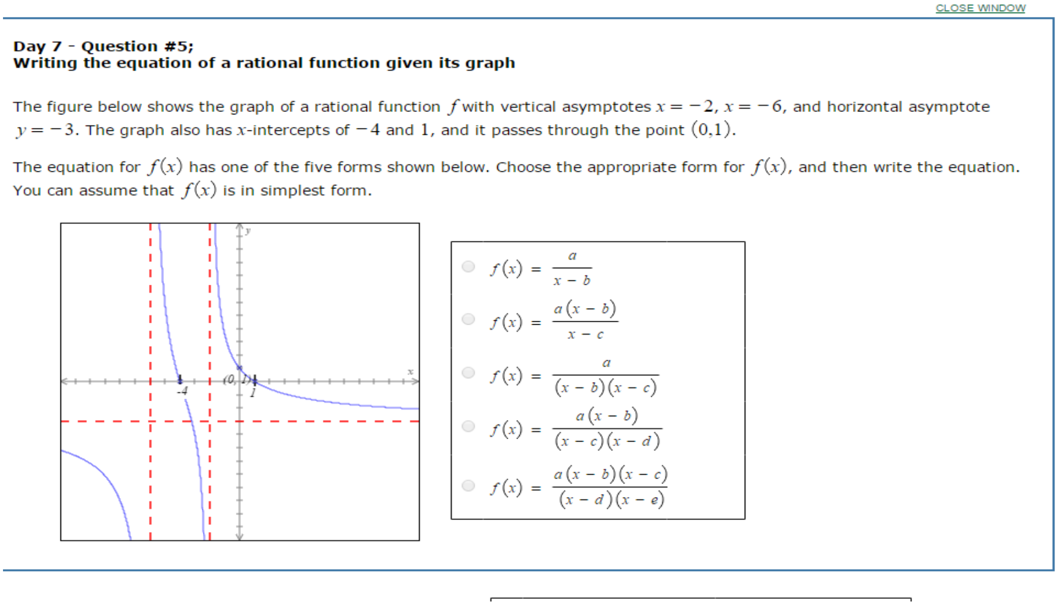
-9
-36
-144
-6...

- ○ Linear
- ○ Quadratic
- [✓ Exponential]
- ○ None of the above



```
NORMAL FLOAT AUTO REAL RADIAN MP
        QuadReg
 y=ax²+bx+c
 a=3
 b=-5
 c=21
 R²=1
```

Review Asymptotes



Day 7 - Question #5;
Writing the equation of a rational function given its graph

The figure below shows the graph of a rational function f with vertical asymptotes $x = -2$, $x = -6$, and horizontal asymptote $y = -3$. The graph also has x-intercepts of -4 and 1, and it passes through the point $(0,1)$.

The equation for $f(x)$ has one of the five forms shown below. Choose the appropriate form for $f(x)$, and then write the equation. You can assume that $f(x)$ is in simplest form.

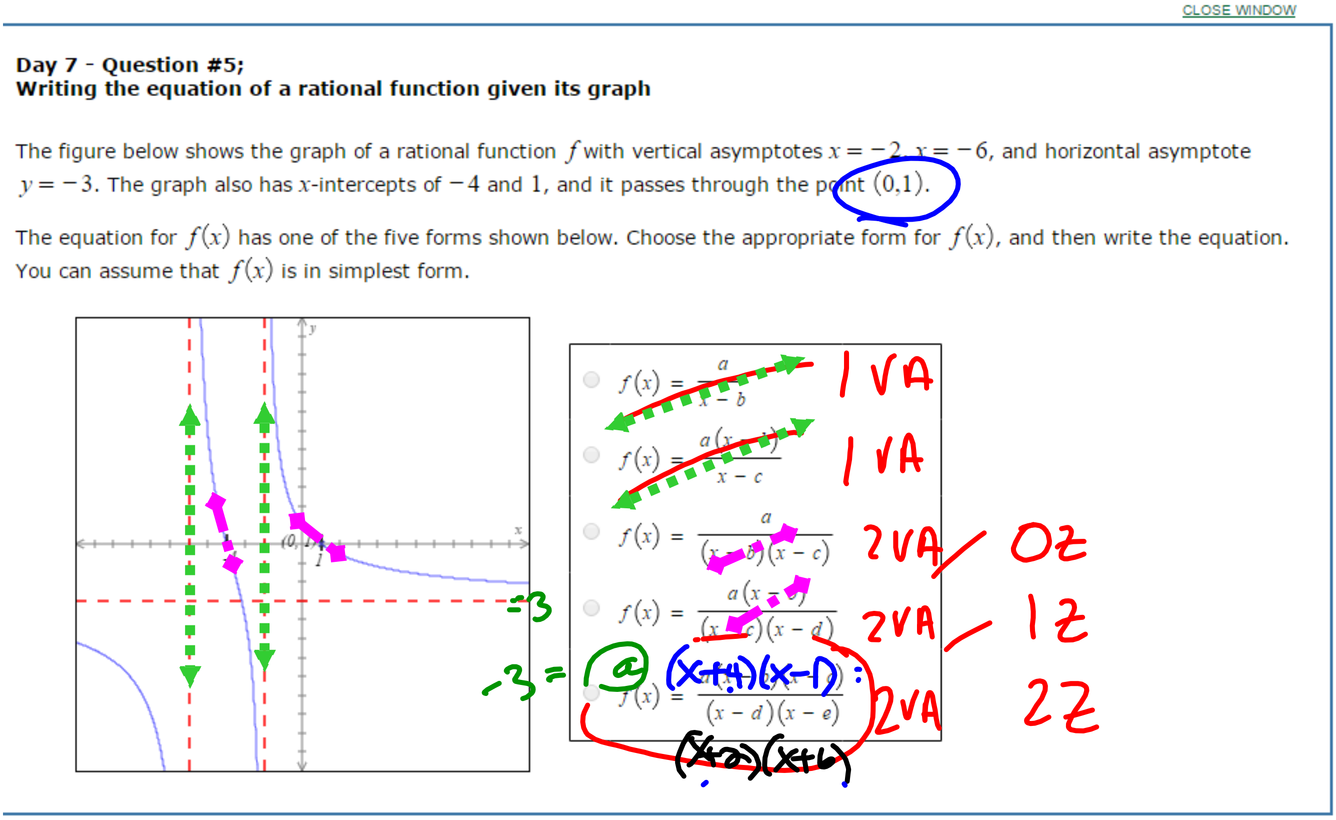
○ $f(x) = \dfrac{a}{x-b}$

○ $f(x) = \dfrac{a(x-b)}{x-c}$

○ $f(x) = \dfrac{a}{(x-b)(x-c)}$

○ $f(x) = \dfrac{a(x-b)}{(x-c)(x-d)}$

○ $f(x) = \dfrac{a(x-b)(x-c)}{(x-d)(x-e)}$



Day 7 - Question #5;
Writing the equation of a rational function given its graph

The figure below shows the graph of a rational function f with vertical asymptotes $x=-2$, $x=-6$, and horizontal asymptote $y=-3$. The graph also has x-intercepts of -4 and 1, and it passes through the point $(0,1)$.

The equation for $f(x)$ has one of the five forms shown below. Choose the appropriate form for $f(x)$, and then write the equation. You can assume that $f(x)$ is in simplest form.

○ $f(x)=\frac{a}{x-b}$ 1 VA

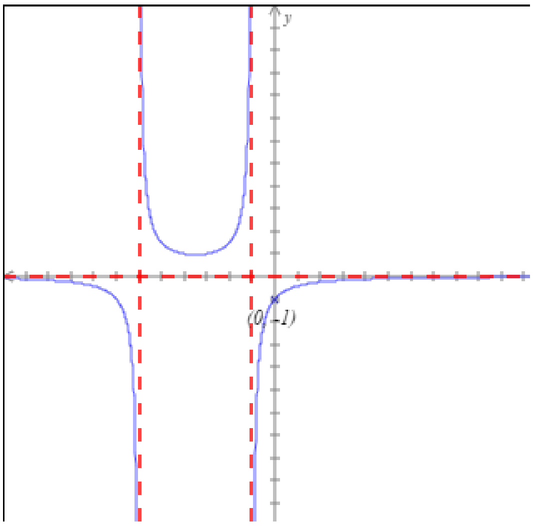
○ $f(x)=\frac{a(x-b)}{x-c}$ 1 VA

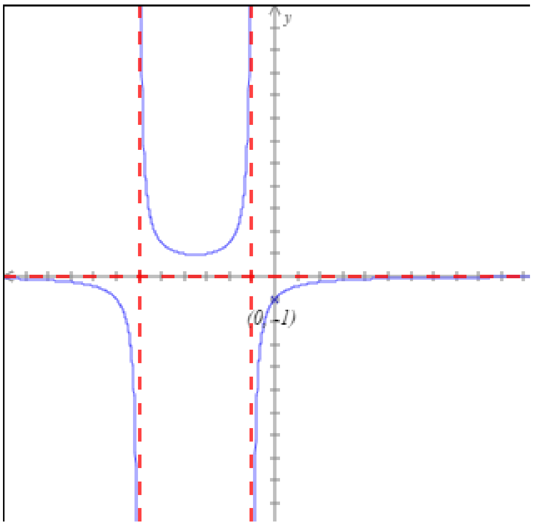
○ $f(x)=\frac{a}{(x-b)(x-c)}$ 2VA / 0Z

○ $f(x)=\frac{a(x-b)}{(x-c)(x-d)}$ 2VA / 1Z

○ $f(x)=\frac{a(x-b)(x-c)}{(x-d)(x-e)}$ 2VA 2Z

$$-3=a\ \frac{(x+4)(x-1)}{(x+2)(x+6)}$$


y
(0, -1)



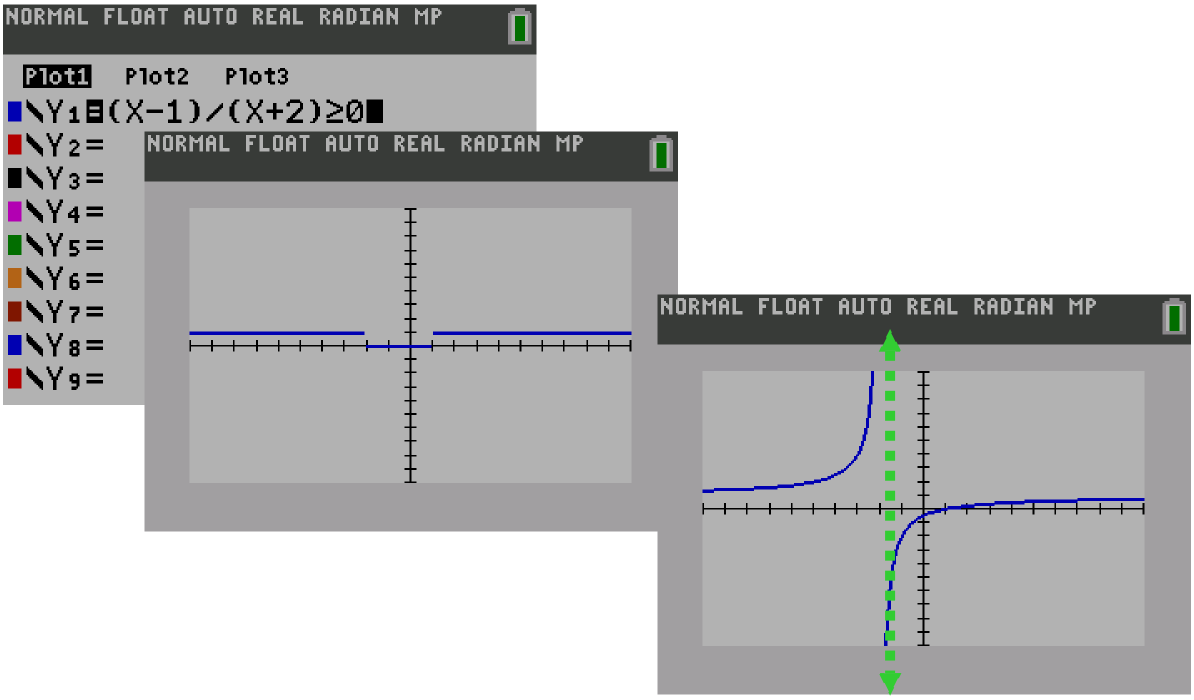
$$y = \frac{a}{(x+1)(x+6)}$$

$$-1 = \frac{a}{6}$$

$$a = -6$$

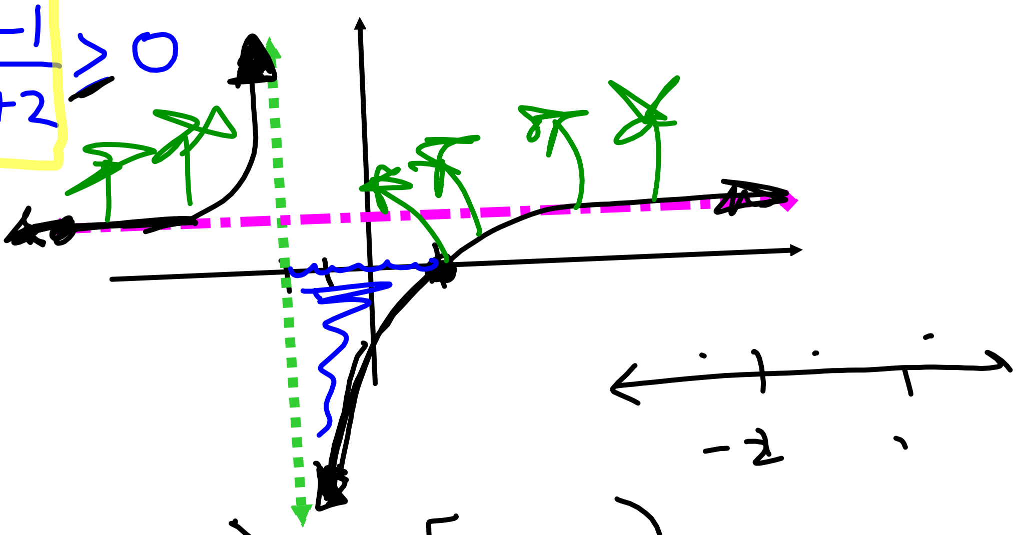
Ex

$$\frac{x-1}{x+2} \geq 0$$



Y1=(X−1)/(X+2)≥0
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=
Y9=

$$\boxed{y=\frac{x-1}{x+2}}\geq 0$$

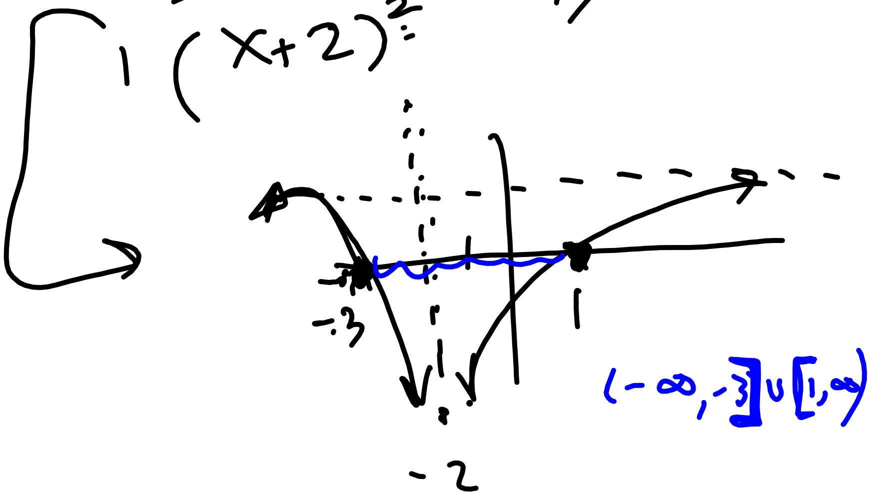


$$(-\infty,-2)\cup[1,\infty)$$

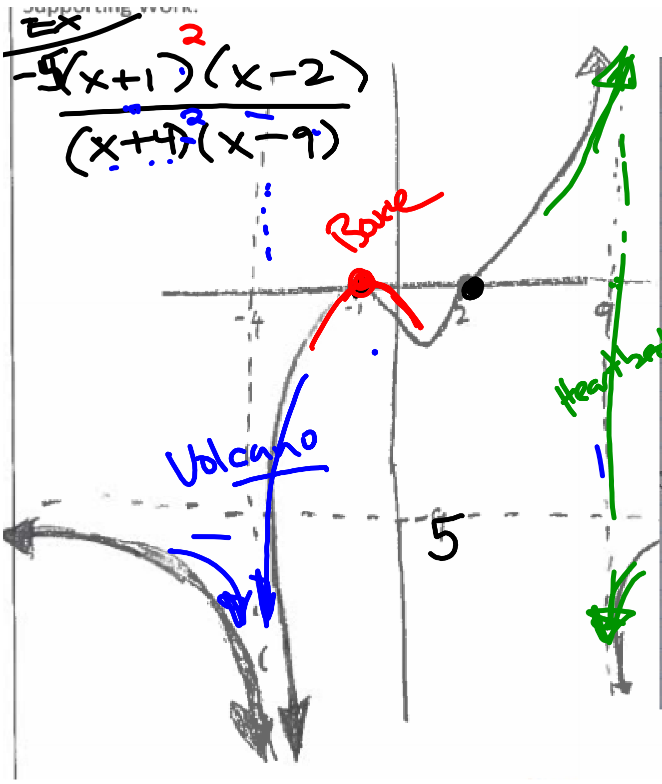
Ex $\frac{(x-1)(x+3)}{(x+2)^2} \geq 0$

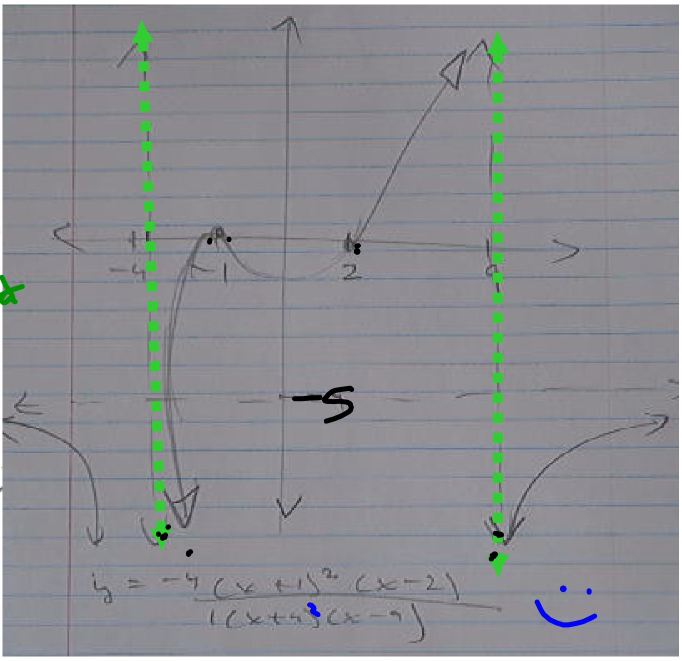
Ex

$$\frac{1\,(x-1)(x+3)}{1\,(x+2)^2} \geq 0$$



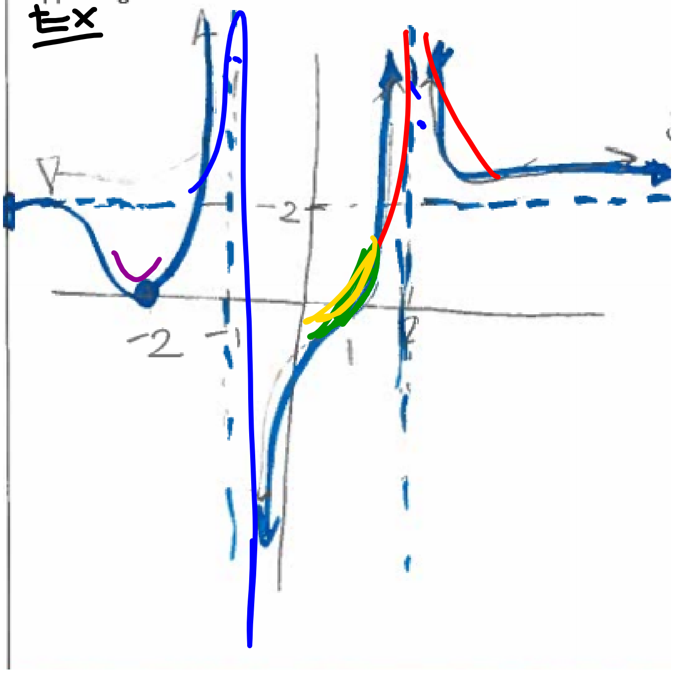
$(-\infty,-3] \cup [1,\infty)$


Supporting Work:
Ex
-5(x+1)^2(x-2)
(x+4)^3(x-9)
Bounce
Volcano
Heartbeat
-4
-1
2
9
5


-4
-1
2
9
-5
y = -4 (x+1)^2 (x-2)
(x+4)^3 (x-9)

Supporting Work:

Ex



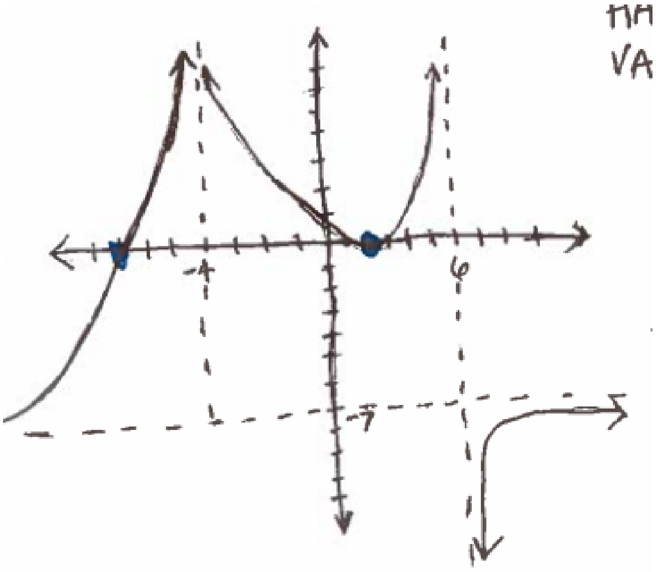
Zeros: -2, 1

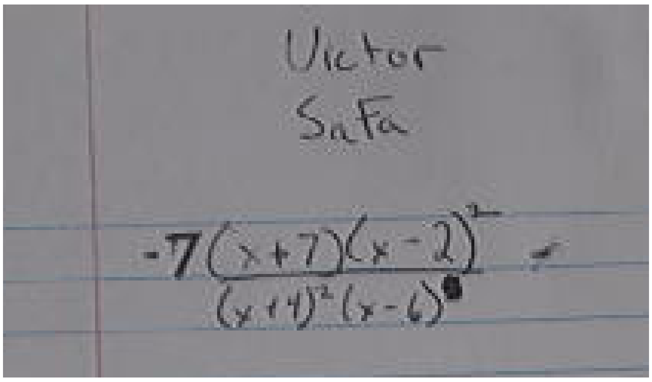
VA: -1, 2

$$y = \frac{)^{2}(x-1)^{3}}{+1)^{3}(x-2)^{2}}$$

= 5

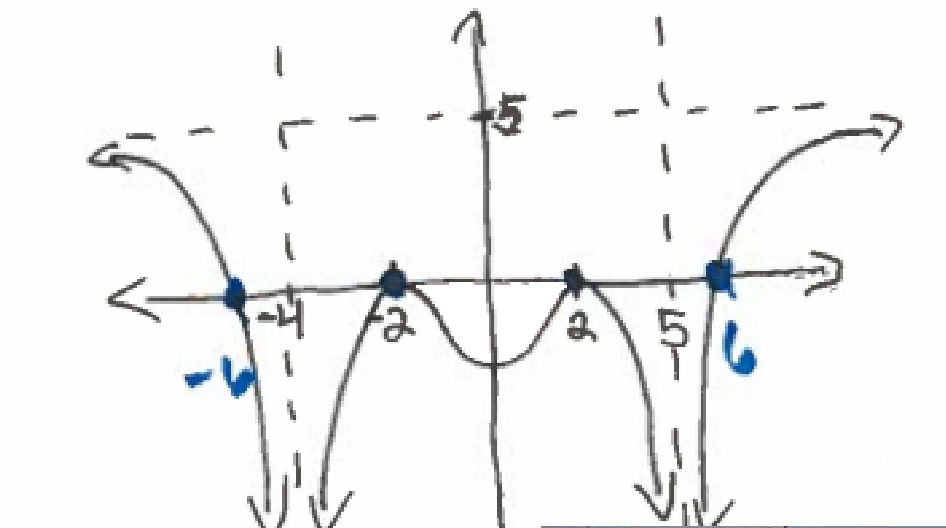
5

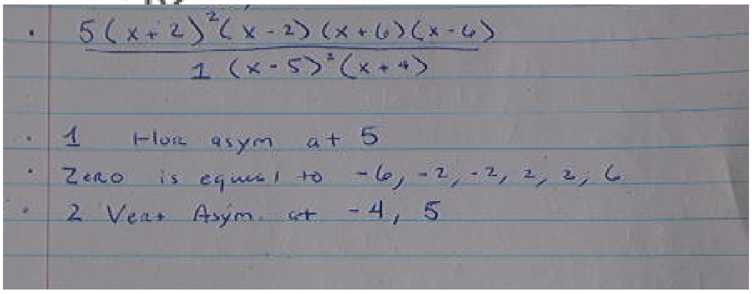




Victor
SaFa

$$\frac{-7(x+7)(x-2)^2}{(x+4)^2(x-6)^{\text{[illegible]}}}$$





- $$\frac{5(x+2)^2(x-2)(x+6)(x-6)}{1(x-5)^2(x+4)}$$

- 1 Hor asym at 5
- Zero is equal to -6, -2, -2, 2, 2, 6
- 2 Vert Asym. at -4, 5

SLANT Asymptote.

IF $DN > DD$

$$DR = DN - DD$$

$$LR = \frac{LN}{LD}$$

THEN SLANT ASYMTOTE

Has END Behavior

DR \ LR	POS	Neg
ODD	Disco RIGHT	Disco Left
EVEN	Happy	Sad

$$y = \frac{1x^2 - 1}{3x}$$

$DN = \underline{2}$

$DD: \underline{1}$

Slant

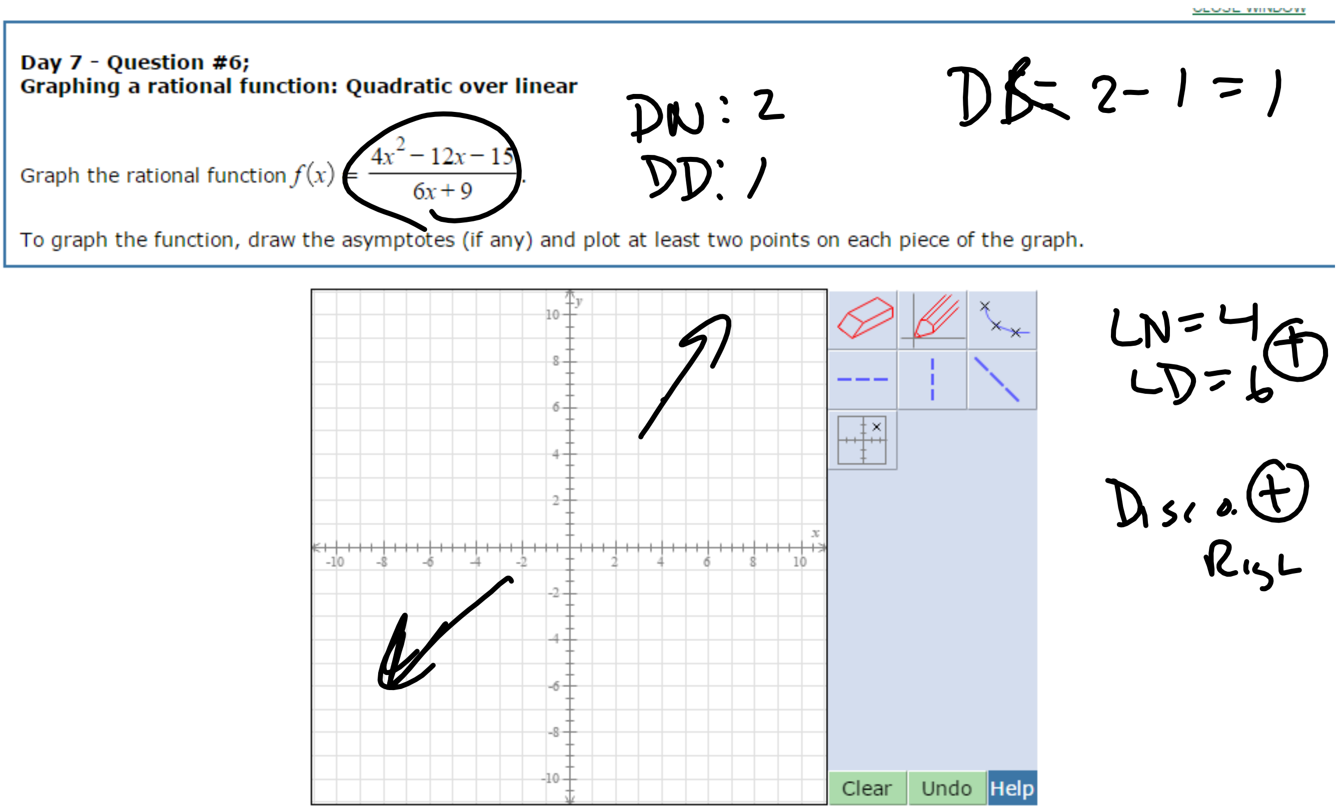
Degree of Slant $\nearrow$ 1

$DR = DN - DD = 2 - 1 = \underline{1}$

Disco.

$LR = \frac{LN}{LD} = \frac{1}{3}$ ⊕

Disco Right



Day 7 - Question #6;
Graphing a rational function: Quadratic over linear

Graph the rational function $f(x) = \frac{4x^2 - 12x - 15}{6x + 9}$.

To graph the function, draw the asymptotes (if any) and plot at least two points on each piece of the graph.

DN: 2
DD: 1

D~~B~~= 2-1 = 1

LN=4 ⊕
LD=6

Disc a. ⊕
Rigt

Day 7 - Question #6;
Graphing a rational function: Quadratic over linear

Graph the rational function $f(x) = \frac{4x^2 - 12x - 15}{6x + 9}$.

To graph the function, draw the asymptotes (if any) and plot at least two points on each piece of the graph.

$$6x+9 \overline{)\,4x^2 - 12x - 15}$$

$$\boxed{\frac{2}{3}x - 3} + \frac{12}{6x+9}$$

$$-(4x^2 + 6x)$$

$$-18x - 15$$

$$-(-18x - 27)$$

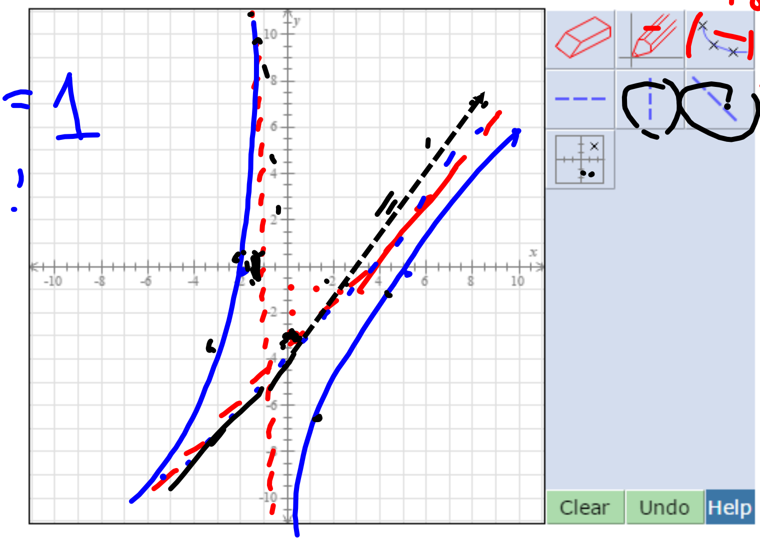
$$12$$

DR = 2-1 = 1

Disco!

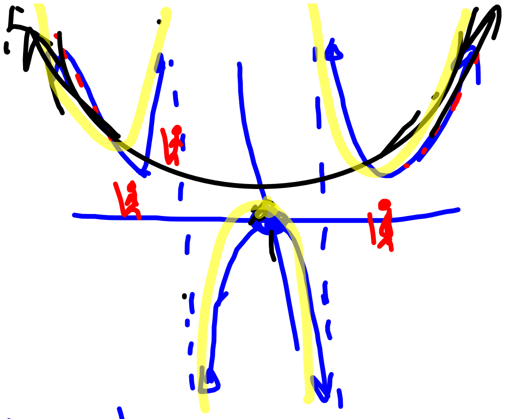
LR = $\frac{4}{6}$

Right



EX.

$$y = \frac{x^4}{x^2 - 1}$$



DN = 4   LN = 1   ZN = 0

DD = 2   LD = 1   ZD = 1, -1

DR = 2 = 4 - 2 $\boxed{\text{even}}$ LR = 1 ⊕   VA

ENDS HAPPY

$$x^2 + 1 + \frac{1}{x^2 - 1}$$

$$x^2 - 1 \,\overline{)\, x^4}$$

$$-(x^4 - x^2)$$

$$x^2$$

$$-(x^2 - 1)$$

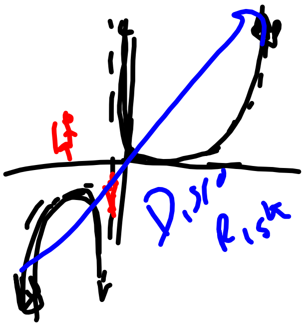
$$1$$

END???

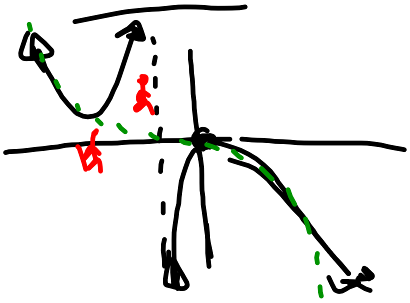
$$\frac{-x^6}{x+1}$$

DR: 6 - 1 = 5 odd

LR: -1 Neg

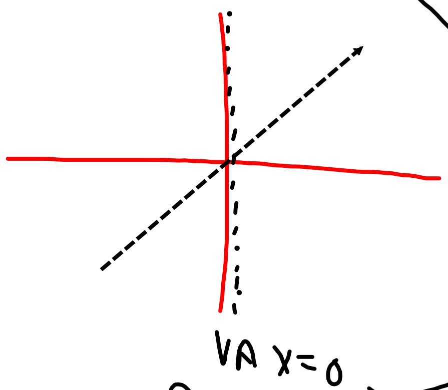


Disco Left



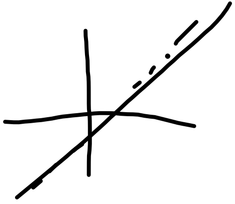
Ex $y = \frac{x^2+1}{x}$ ZN: NONE

ZD: 0



$$\begin{array}{r|l} & x \quad + \frac{1}{x} \\ x & x^2+0x+1 \\ & -x^2 \\ \hline & \qquad\quad 1 \end{array}$$

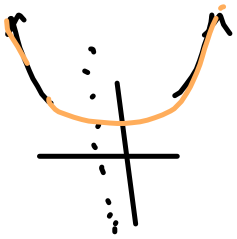
$$\frac{x^2+1}{x} = \boxed{x} + \frac{1}{x}$$



Ex $\frac{x^3+2x-7}{x+1}$.

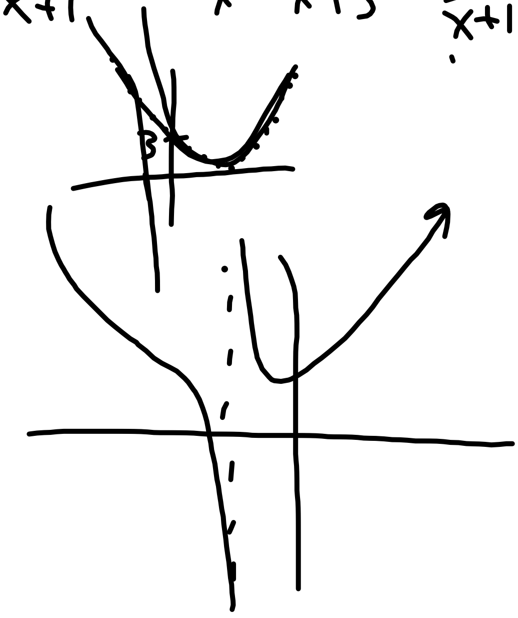
$$\begin{array}{r|l} & \qquad x^2 \\ x+1 & x^3+0x^2+2x-7 \\ & -(x^3+x^2) \\ \hline & \qquad -x^2 \end{array}$$

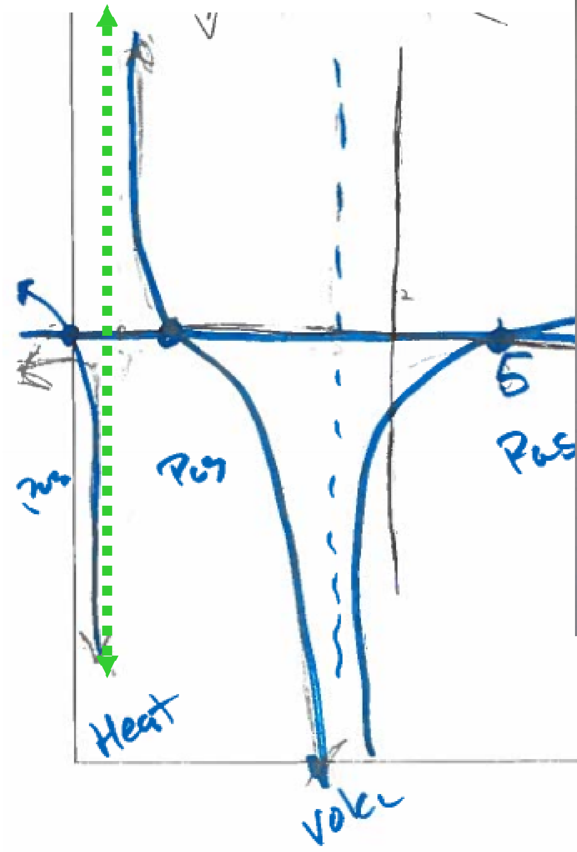
Ex $\boxed{\frac{x^3+2x-1}{x+1}}$

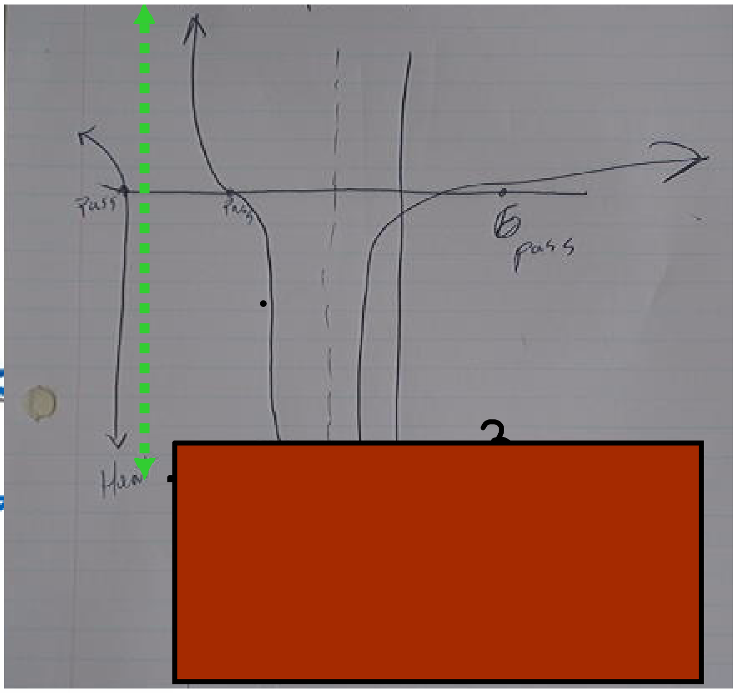


$$\begin{array}{r} x^2 - x + 3 \\ x+1 \overline{) x^3 + 0x^2 + 2x - 1} \\ -(x^3 + x^2) \\ \hline -x^2 + 2x \\ -(-x^2 - x) \\ \hline 3x - 1 \\ -(3x+3) \end{array} \quad , -\frac{4}{x+1}$$

$\frac{x^3+2x-1}{x+1} = x^2-x+3-\frac{4}{x+1}$







[redacted]

Lecture Properties of Logs

1. Definition
2. Sum/Product
3. Ladder
4. Change of Base
5. Log of Both Sides

Properties of Inverses

Defined

$$\log x \equiv \log_{10} x$$

$$\ln x \equiv \log_e x$$

Properties of Logs

$y = B^x \rightarrow x = \log_B y$

1. Definition of Log

Forward: Solve for an exponent

EX: Find the inverse for $3^x = y$ ⓅⒾ

EX: The How long will it take for an investment to double at 5%

Reverse: Get rid of a log

EX: Find x if 3 = lnx

① $x = \log_3 y$

② $y^{-1} = \log_3 x$

$P = Q e^{RT}$

$2 = 1 e^{.05T}$

$2 = e^{.05T}$

ⓅⒾ

$.05T = \log_e(2)$

$.05T = \ln 2$

$T = \ln 2 / .05 \approx 13.8$

Get Rid of Log

Ex $3 = \ln_e X$

P1

Base $= e^3 = x$

Ex

$\underline{5} = \log_{\underline{2}}(x-1)$

P1

Base $= \underline{2}^{\underline{5}} = X-1$

$32 = x-1$

$33 = x$

2. Sum/Product Property

Forward: Combine a log
- EX: Find a single log for log2+log5
- EX: Find x if log2+logx =3

Reverse: Find the components of a log
- EX: If log2=.3 find log 20

$$\underbrace{\log 2+\log 5}$$

$$\log(2\cdot 5)=\log 10$$

$$\log(\underline{2}\times 10^{\boxed{2}})=\boxed{\log \underline{2}}+\log 10^{2}$$

$$.3 \quad + \quad \underline{2}$$

$$\log A+\log B+\log C$$

$$=\log(A\cdot B\cdot C)$$

$$\log_{\square} 2 + \log_{\square} x = 3$$

P2 Sum Prod

$$\log(2x) = 3$$

P1 Definit

$$10^{\boxed{3}} = \boxed{2x}$$

Arith

$$1000 = 2x$$

$$x = 500$$

$\log A + \log B = \log A \cdot B$

2. Sum/Product Property

Forward: Combine a log

- EX: Find a single log for log2+log5 $= \log_{10} 10 = 1$
- EX: Find x if log2+logx =3

Reverse: Find the components of a log

- EX: If log2= 3 find log 20

$\underbrace{\log 2 + \log x}_{P2} = 3$

$\log_{10}(2x) = 3$

P1

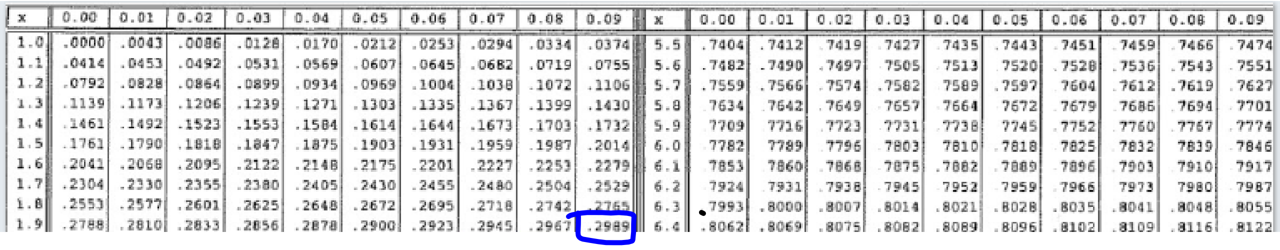
$10^3 = 2x$ $\boxed{x = 500}$

If $\log 2 = .3$

Find $\text{Log}\, 20$

$$\log_{10} 20 \overset{\text{P2}}{=} \log_{10} 10 + \log 2$$

$$1 + .3 = 1.3$$



x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0	.0000	.0043	.0086	.0128	.0170	.0212	.0253	.0294	.0334	.0374	5.5	.7404	.7412	.7419	.7427	.7435	.7443	.7451	.7459	.7466	.7474
1.1	.0414	.0453	.0492	.0531	.0569	.0607	.0645	.0682	.0719	.0755	5.6	.7482	.7490	.7497	.7505	.7513	.7520	.7528	.7536	.7543	.7551
1.2	.0792	.0828	.0864	.0899	.0934	.0969	.1004	.1038	.1072	.1106	5.7	.7559	.7566	.7574	.7582	.7589	.7597	.7604	.7612	.7619	.7627
1.3	.1139	.1173	.1206	.1239	.1271	.1303	.1335	.1367	.1399	.1430	5.8	.7634	.7642	.7649	.7657	.7664	.7672	.7679	.7686	.7694	.7701
1.4	.1461	.1492	.1523	.1553	.1584	.1614	.1644	.1673	.1703	.1732	5.9	.7709	.7716	.7723	.7731	.7738	.7745	.7752	.7760	.7767	.7774
1.5	.1761	.1790	.1818	.1847	.1875	.1903	.1931	.1959	.1987	.2014	6.0	.7782	.7789	.7796	.7803	.7810	.7818	.7825	.7832	.7839	.7846
1.6	.2041	.2068	.2095	.2122	.2148	.2175	.2201	.2227	.2253	.2279	6.1	.7853	.7860	.7868	.7875	.7882	.7889	.7896	.7903	.7910	.7917
1.7	.2304	.2330	.2355	.2380	.2405	.2430	.2455	.2480	.2504	.2529	6.2	.7924	.7931	.7938	.7945	.7952	.7959	.7966	.7973	.7980	.7987
1.8	.2553	.2577	.2601	.2625	.2648	.2672	.2695	.2718	.2742	.2765	6.3	.7993	.8000	.8007	.8014	.8021	.8028	.8035	.8041	.8048	.8055
1.9	.2788	.2810	.2833	.2856	.2878	.2900	.2923	.2945	.2967	$\boxed{.2989}$	6.4	.8062	.8069	.8075	.8082	.8089	.8096	.8102	.8109	.8116	.8122

log1=0 log 1.01= .0043 log 1.09=.0374

log11 = 1og(1.1x10)=log(1.1)+log(10)=.0414+1

$\log(6.31 \times 10^5) =$

$\log 6.31 + \log 10^{\textcircled{5}}$

$.7993 + 5 = 5.7993$

log 1.99 =

.2989

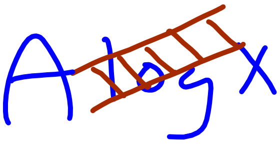
3. “Ladder” Property

Forward: Get rid of a coefficient

EX: Find 3log4+2log5

Reverse: Get rid of an exponent inside a log

EX: Find



$A \log x$

$3\log 4 + 2\log 5$

P3

$= \log 4^3 + \log 5^2$

P2

$= \log(4^3 \cdot 5^2)$

3. “Ladder” Property

$$\cancel{\log} A^{B} = B\log A$$

Forward: Get rid of a coefficient

EX: Find 3log4+2log5

Reverse: Get rid of an exponent inside a log

EX: Find

$$\downarrow$$

$$\ln x^{3}$$

$$= 3\,\underline{\ln x}$$

$$\underline{3}\log 4 + \underline{2}\log 5$$

$$\textcircled{P3}$$

$$\log 4^{3} + \log 5^{2}$$

$$\textcircled{P2}$$

$$\log(4^{3} \cdot 5^{2}) =$$

4. Change of Base

$$\boxed{\log_B A} = \frac{\log_C A}{\log_C B}$$

C "New" base

Forward: Evaluate a old bases in terms of a new one

EX: Find

EX: Graph $y = \log_3(x)$

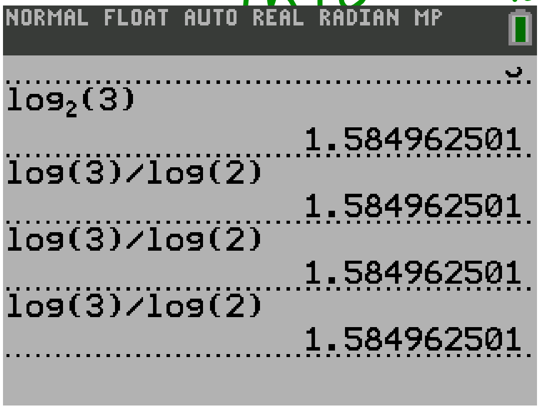
$y = \log_{10}(x) / \log(3)$

$= \ln(x)/\ln(3)$

Reverse: Divide logs of a same base to get a single log

EX: Simplify lnx/ln10

$$\frac{\ln x}{\ln 10} = \log_{10} x$$



```
NORMAL FLOAT AUTO REAL RADIAN MP
log2(3)
                    1.584962501
log(3)/log(2)
                    1.584962501
log(3)/log(2)
                    1.584962501
log(3)/log(2)
                    1.584962501
```

logBase

Change of Base

P4 Rev.

$$\frac{\log_c A}{\log_c B} = \log_B A$$

←

$$\log\left(\frac{A}{B}\right) = \log\left(A \times B^{-1}\right)$$

P2

$$= \log A + \log B^{-1}$$

P3

$$\log\left(\frac{A}{B}\right) = \log A - \log B$$

$$\boxed{\log\left(\frac{A}{B}\right) = \log A - \log B}$$

Subtract / Division

$$A \cancel{\times} B = A \times B^{\textcircled{-1}}$$

↓

$$\frac{A}{B} = A \cdot \frac{1}{B}$$

$\boxed{x = y}$ then $\boxed{\log_B x = \log_B y}$

5. Log of Both Sides

Forward: Take the log of both sides

EX: Derive the change of base formula

Reverse: Drop the logs from both sides

EX: If logx=log4 find x.

$\log x = \log 4$

PS

$x = 4$

$$\log_B A = \frac{\log_c A}{\log_c B}$$

$\log_B A = x$

P1

$B^x = A$

PS

$\log_c B^x = \log_c A$

P3

$x \log_c B = \log_c A$

Algebra

$$x = \frac{\log_c A}{\log_c B} = \log_B A$$

$$\log x = \log 3$$

Prop ⑤

$$x = 3$$

Eg

$$\left(\sqrt{x}\right)^2 = \left(\sqrt{2}\right)^2$$

$$x = 2$$

Properties of Inverses

$$\boxed{\begin{array}{l}(f \circ f^{-1})(x) = x \\ (f^{-1} \circ f)(x) = x\end{array}}$$

$$e^{\ln x} = x$$

$$\ln e^{x} = x$$

$$\log_{10} 10^{x} = x$$

$$10^{\log_{10} x} = x$$

Ex

$$4^{3\log_4 7} \quad \boxed{P3}$$

$$= 4^{\log_4 (7^3)}$$

Prop of Inverse

$$= 7^3$$

$f^{-1}(x)$ - $y^{-1} = \log_B x$	$f(x)$ $y = B^x$

$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(B^x)$$

$$= \boxed{\log_B(B^x) = x}$$

Ex:

$$\log_{10} 10^{300} = 300$$

$f^{-1}(x)$ $y^{-1} = \log_B x$ | $f(x)$ $y = B^x$

$$f(f^{-1}(x)) = f(\log_B x) = B^{\log_B x}$$

$$\boxed{B^{\log_B x} = x}$$

$\log_{10} 10$ -

Groupwork

Percent Growth (ln b)

Domain, Range, Asymptotes

End Behaviour

$$P = \boxed{Q}\, e^{R\,t}$$

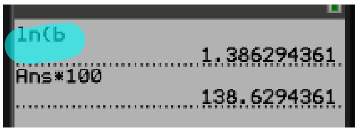
$$Y = \boxed{A} * B^{x}$$

R = Rate

Exp Regossi

$$e^{R} = B \qquad P1$$

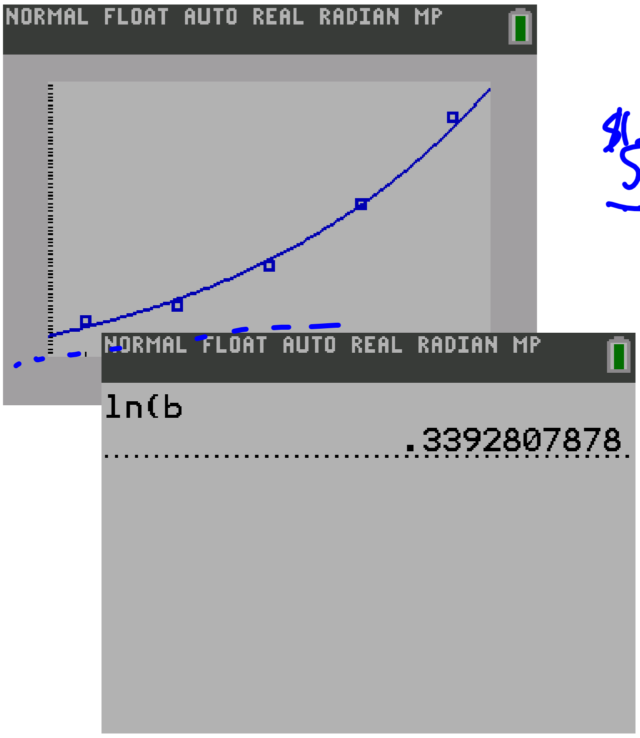
$$R = \log_{e}(B) = \boxed{\ln B}$$



```
ln(b
                  1.386294361
Ans*100
                  138.6294361
```

According to my exponential regression,

LG sales is growing by 138.63% a year

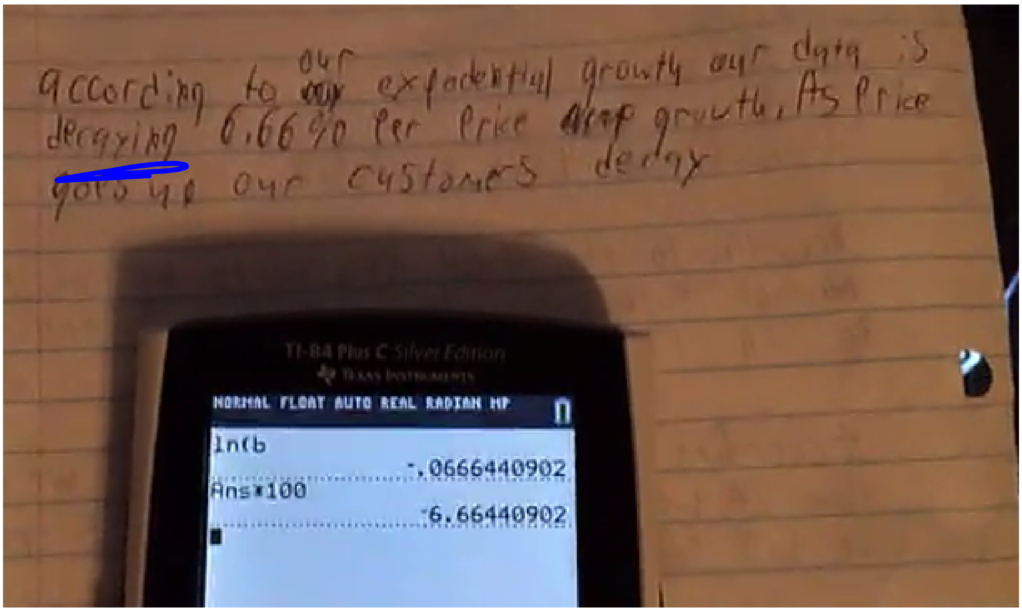


```
ln(b
                .3392807878
```

$r = 5\%$

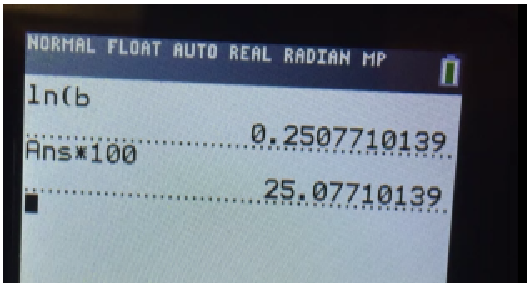
$\$500 \xrightarrow[13]{} 250 \xrightarrow[13]{} 125 \rightarrow \$62$

$= r$ 33.92



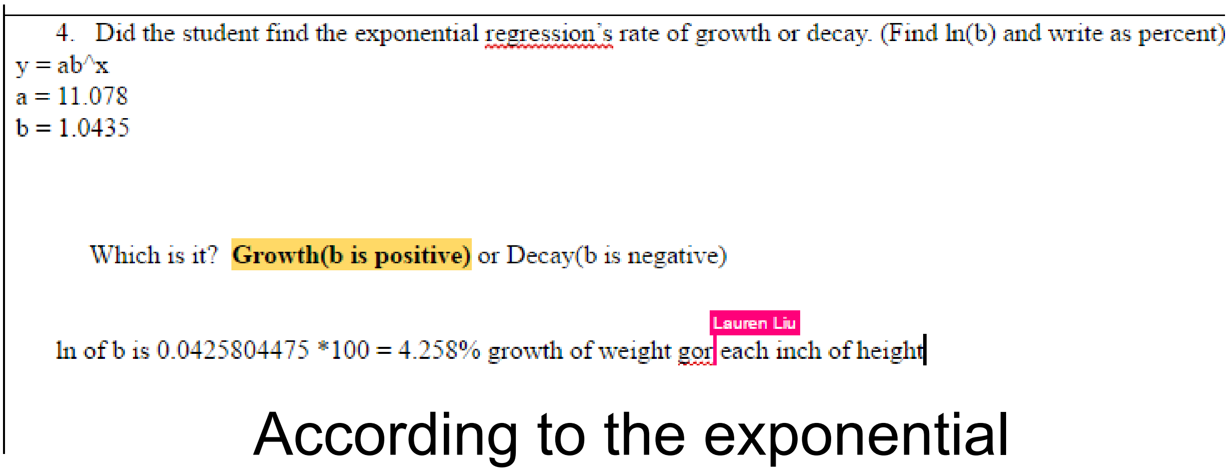
according to ~~our~~ our expodential growth our data is decaying 6.66% per price ~~drop~~ growth, As price goes up our customers decay

```
ln(b
                        ⁻.0666440902
Ans*100
                        ⁻6.66440902
```



```
NORMAL FLOAT AUTO REAL RADIAN MP
ln(b
                    0.2507710139
Ans*100
                     25.07710139
■
```

According to the exponential regression, Chicken chops are growing by 25% a year



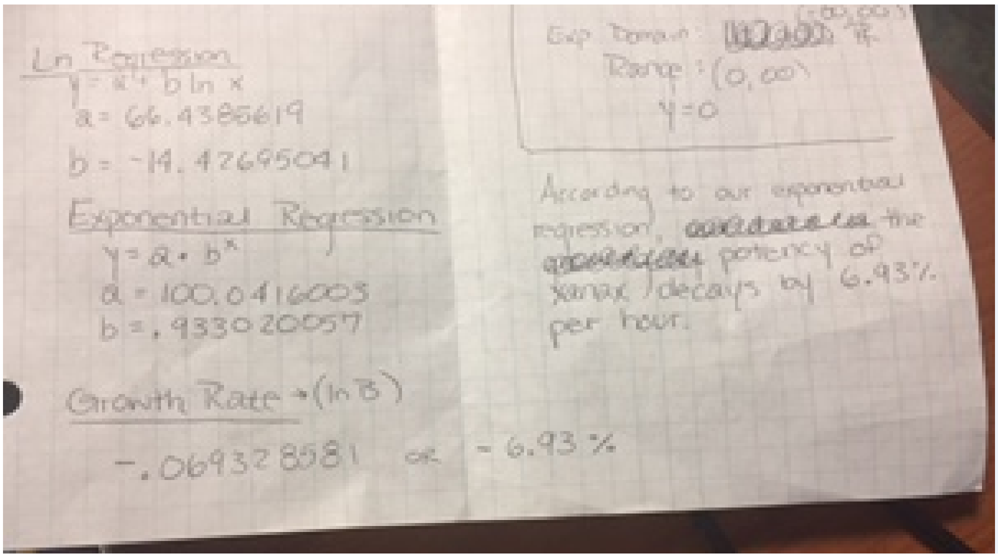
4. Did the student find the exponential regression's rate of growth or decay. (Find ln(b) and write as percent)

y = ab^x
a = 11.078
b = 1.0435

Which is it? **Growth(b is positive)** or Decay(b is negative)

ln of b is 0.0425804475 *100 = 4.258% growth of weight gor each inch of height

According to the exponential regression, a persons' weight should grow by 4.258% per inch of height



Ln Regression

$y = a + b\ln x$

$a = 66.4385619$

$b = -14.47695041$

Exponential Regression

$y = a \cdot b^x$

$a = 100.0416003$

$b = .933020057$

Growth Rate → (ln B)

$-.06932858 1$ or -6.93%

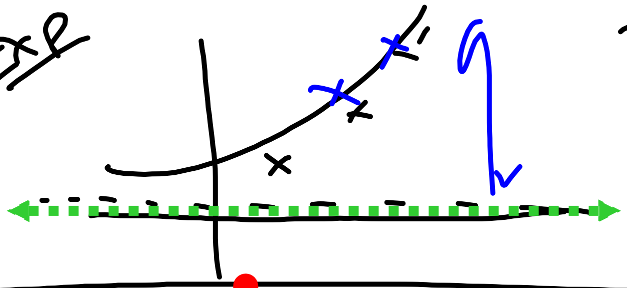
Exp Domain: $(-\infty, \infty)$ $\mathbb{R}$

Range: $(0, \infty)$

$y = 0$

According to our exponential regression, the potency of xanax decays by 6.93% per hour.

Exp

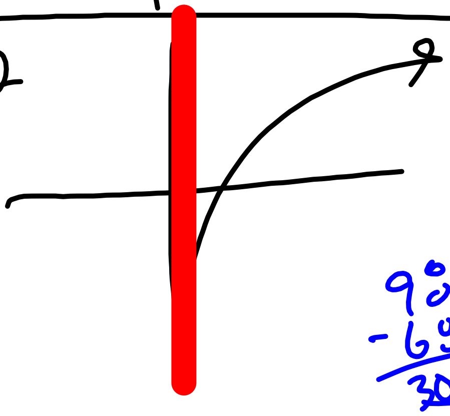


Domain : ℝ (-∞,∞)

Range : (0,∞)

Asymptote : y=0

Log



Domain: (0,∞)

Range (-∞,∞)

Asymptote x = 0

90
-60
30