

Agenda

Review Quiz 4 and Exam 1

Lecture: Exponentials

Lecture: Transformations of Functions

Project: Exponential Function

Review Questions

Finding the asymptotes of a rational function: Quadratic over linear

Graph all asymptotes of the following function.

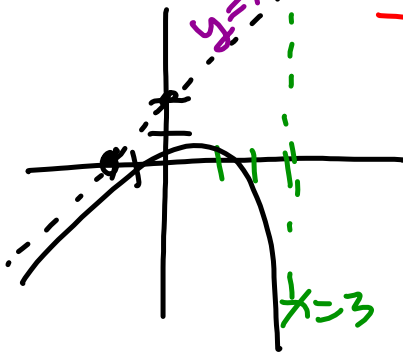
Quiz 4

$$f(x) = \frac{x^2 - x + 8}{x - 3} = 0$$

$$y = (x + 2) + \frac{14}{x - 3}$$

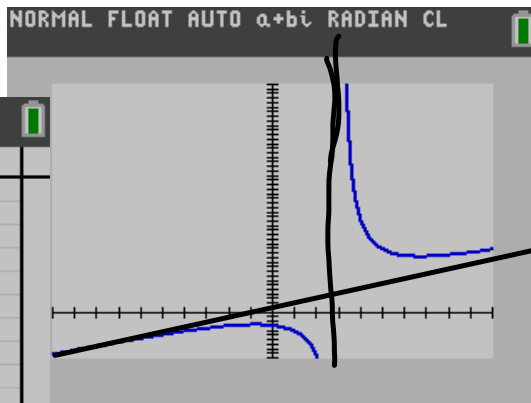
$$\begin{array}{r} x-3 \overline{) x^2 - x + 8} \\ \underline{-(x^2 - 3x)} \\ 2x \\ \underline{-(2x - 6)} \\ 14 \end{array}$$

$$\frac{8 - (-6) = 14}{1}$$



X	Y1	
0	-2.667	(1, -4)
1	-4	(2, -10)
2	-10	

X=



$$y = x + 2$$

Finding the asymptotes of a rational function: Quadratic over linear

Graph all asymptotes of the following function.

$$f(x) = \frac{x^2 - x + 8}{x - 3}$$

ZN: $1 \pm \sqrt{1-3}$ NOT REAL

ZD: 3

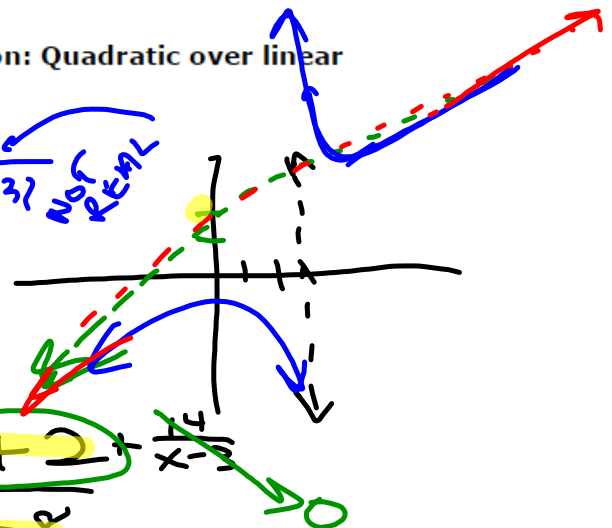
VA: $x = 3$

DN > DD Slant

$y = x + 2 + \frac{14}{x-3}$

$$\begin{array}{r} x-3 \overline{) x^2 - x + 8} \\ -(x^2 - 3x) \\ \hline 2x + 8 \\ -(2x - 6) \\ \hline 14 \end{array}$$

$-x + (+3x)$
 $8 + 6$



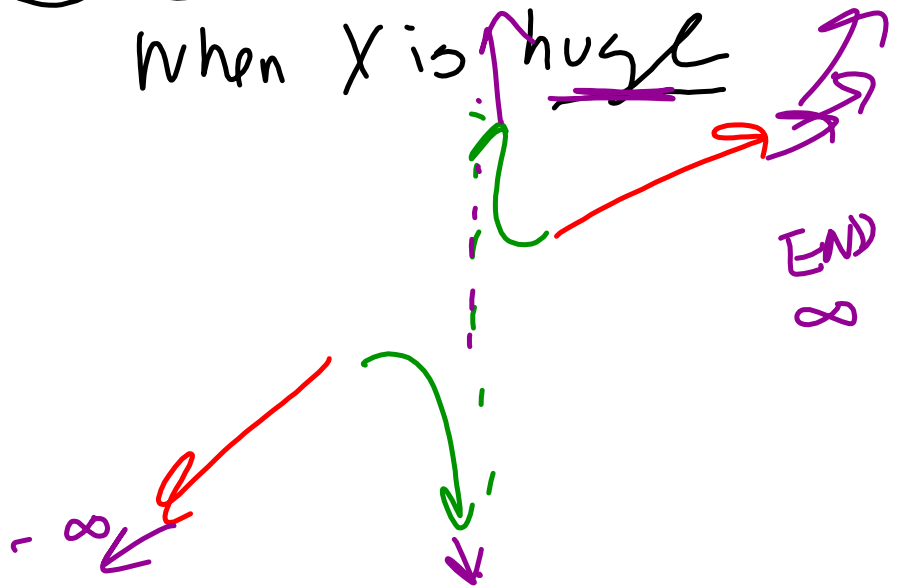
$\frac{x^2 - x + 8}{x - 3}$

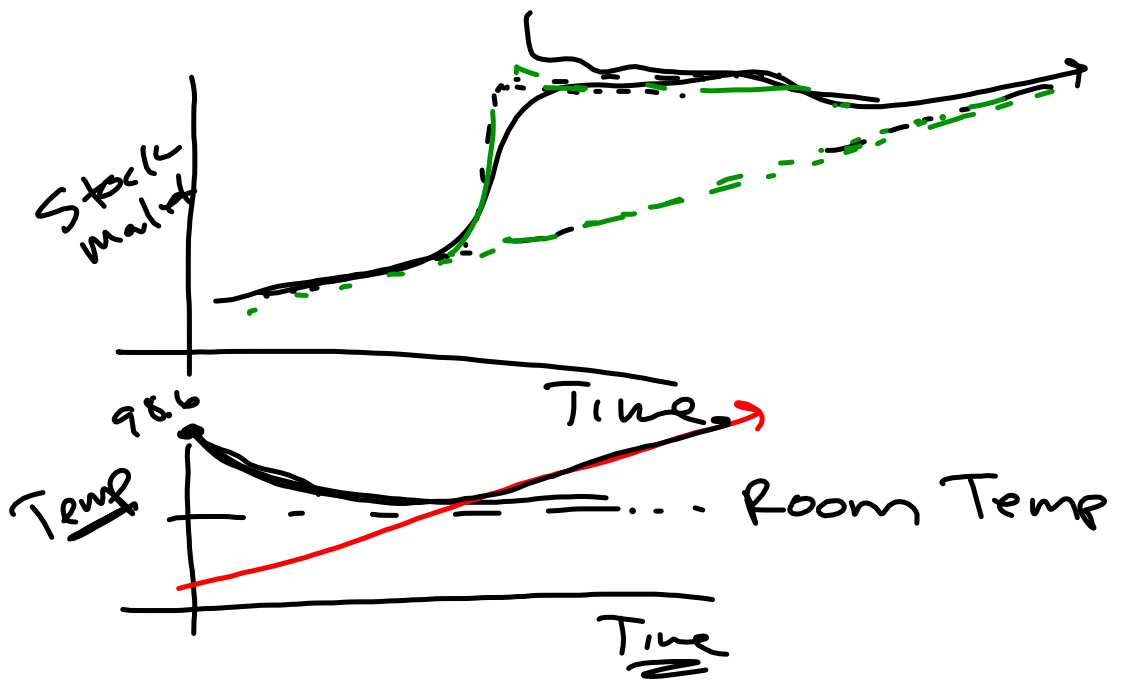
Division: $\frac{3}{2}$ Improper $1\frac{1}{2}$ Mix.

$\frac{x^2 - x + 8}{x - 3} = x + 2 + \frac{14}{x - 3}$

(Note: The original image has some scribbles and annotations like "Q" and "R" over the partial fractions, and "4" and "3" under the denominator terms.)

When x is huge





1

Solve the following inequality.

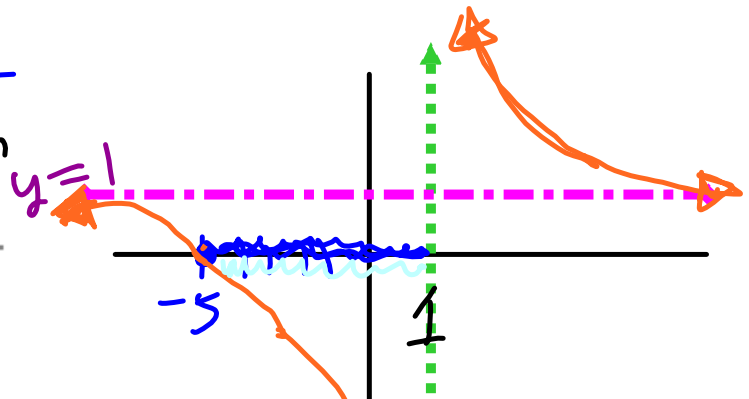
$$y = \frac{x+5}{x-1} \leq 0$$

$= 0$ Zeros: -5
water, beach
 $= 0 \checkmark A$ $x=1$

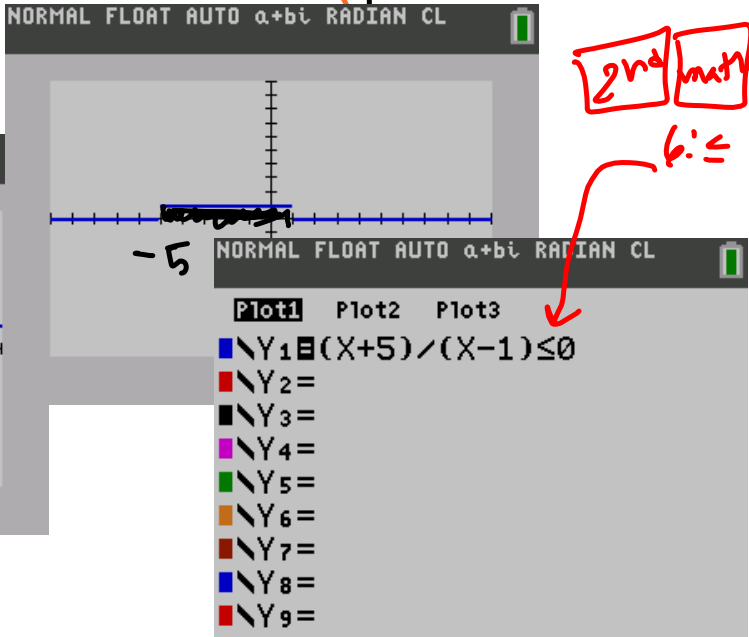
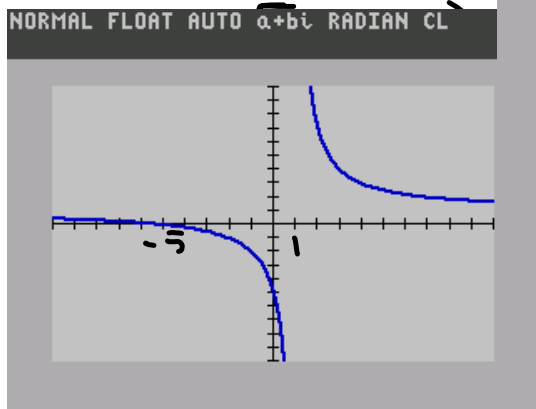
Write your answer using interval notation.

Your answer:

$$(-\infty, -5] \cup [1, \infty)$$



water.



Solve the following inequality.

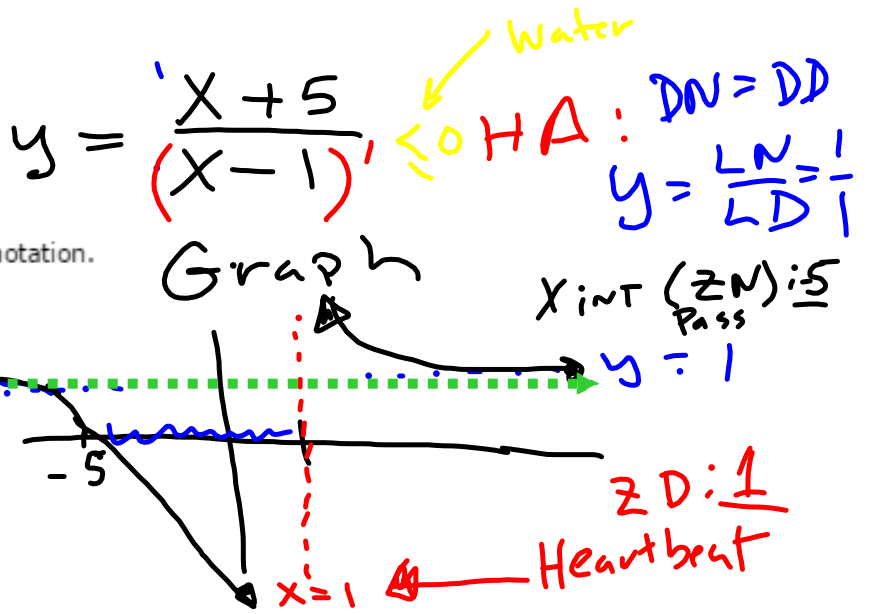
$$\frac{x+5}{x-1} \leq 0$$

Write your answer using interval notation.

Your answer:

~~$(-\infty, 5] \cup [1, \infty)$~~

$[-5, 1)$
court shore.



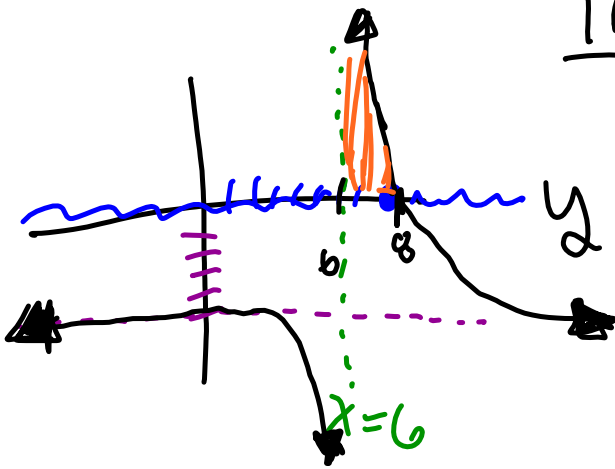
Solve the following inequality.

$$\frac{10}{x-6} \geq 5$$

$$\frac{10}{x-6} - 5 \geq 0$$

LCD: $x-6$

$$\frac{10 - 5(x-6)}{x-6} \geq 0$$



$$y = \frac{40 - 5x}{(x-6)}$$

$= 0 \quad x = 8$
 ≥ 0 ← greater than
= land
= Beach.
 $= 0 \quad x = 6$

HA: $y = \frac{-5}{1} = -5$

$(6, 8]$

Solve the following inequality.

$$\frac{10}{x-6} \geq 5$$

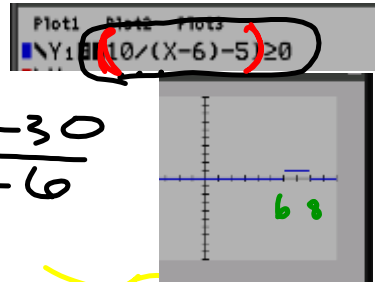
$$\frac{10}{x-6} - 5 \geq 0$$

Y:

$$y = \frac{10}{x-6} - 5$$

$$= \frac{10}{x-6} - \frac{5x-30}{x-6}$$

$$y = \frac{40-5x}{x-6}$$



≥ 0 LAND

ZN: 8 (xint)

ZD: 6

VA: $x=6$

H/A: DN=DD
[N] = -5

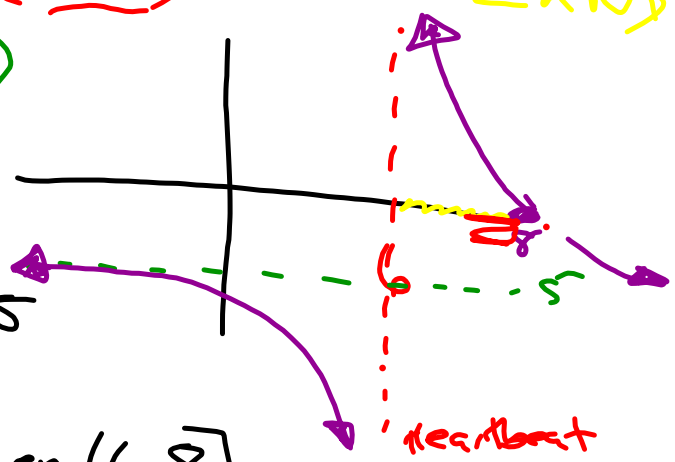
HA: $y = -5$

Answer $(6, 8]$

Not Answer (witten)

$$(-\infty, 6) \cup [8, \infty)$$

$$y \leq 0$$



Graphing a rational function with more than one vertical asymptote

Graph the following rational function.

$$f(x) = \frac{16}{x^2 - 2x - 8}$$

ZN: NONE

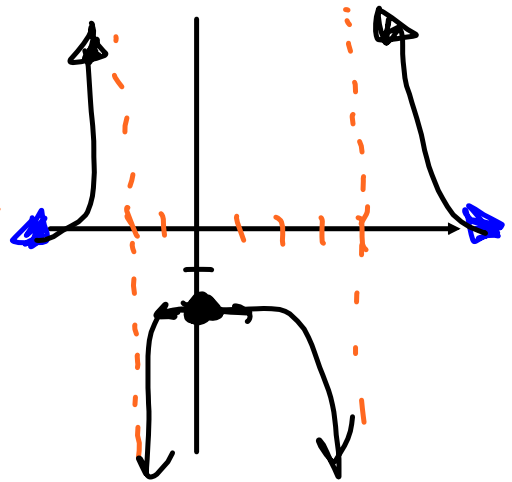
$$(x-4)(x+2) = 0 \quad \text{ZD: } x = 4, -2$$

To graph the function, draw the horizontal and vertical asymptotes (if any), plot the intercepts (if any), and plot at least one point on each side of each vertical asymptote.

Then click on the graph icon.

$$\begin{aligned} \text{DN} &= 0 \\ \text{DD} &= 2 \end{aligned}$$

$$\begin{aligned} \text{HA} \\ y &= 0 \end{aligned}$$



y intercept (x = 0)

$$f(0) = \frac{16}{0 - 0 - 8} = -2$$

Graphing a rational function with more than one vertical asymptote

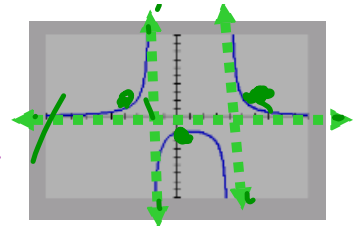
Graph the following rational function.

$$f(x) = \frac{16}{x^2 - 2x - 8} = \frac{16}{(x-4)(x+2)}$$

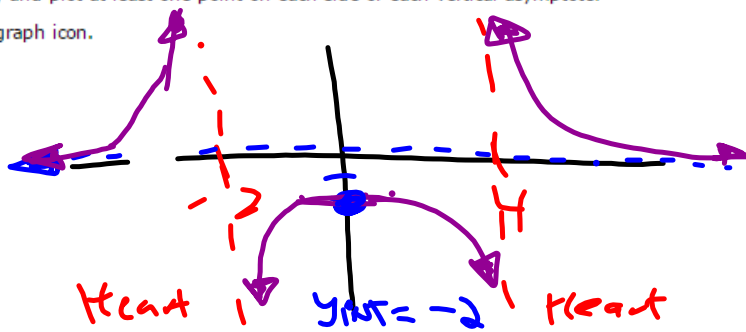
Z.N.: None

To graph the function, draw the horizontal and vertical asymptotes (if any), plot the intercepts (if any), and plot at least one point on each side of each vertical asymptote.

Then click on the graph icon.



DD > DN
HA: $y = 0$



$$y_{int} (x=0) = \frac{16}{20} = \frac{4}{5}$$

Graphing rational functions with holes

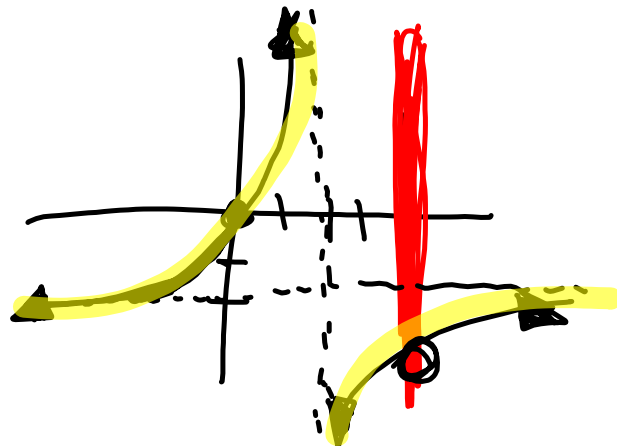
Graph the rational function $f(x) = \frac{-2x^2 + 8x}{x^2 - 6x + 8}$.

$$y = \frac{-2x}{x-2}$$

$$\text{HA: LN: } \frac{-2}{1} = -2$$
$$\text{LD: } 1$$

$$-2x = 0 \quad \text{ZM: } 0$$

$$\frac{-2x(x-4)}{(x-4)(x-2)} = 0$$



Graphing rational functions with holes

Graph the rational function $f(x) = \frac{-2x^2 + 8x}{x^2 - 6x + 8}$.

$$\frac{-2x \cancel{(x-4)}}{(x-2)\cancel{(x-4)}}$$

$$x \neq 4$$

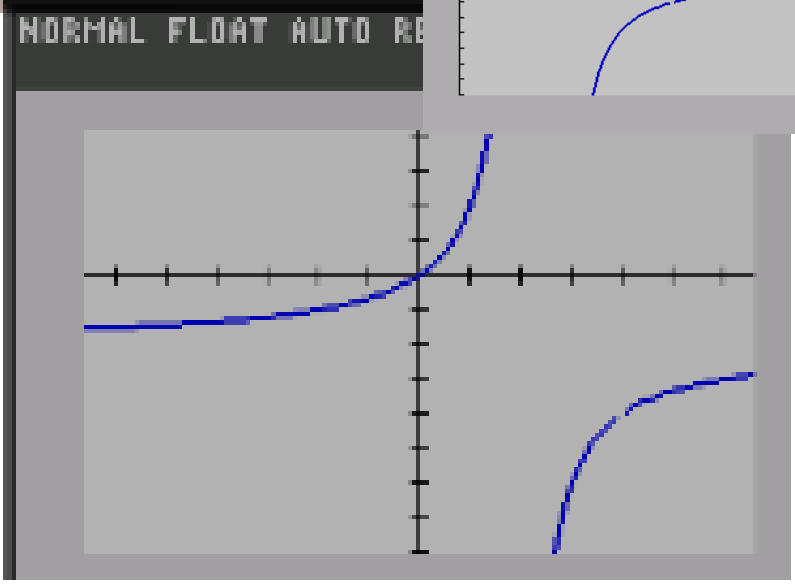
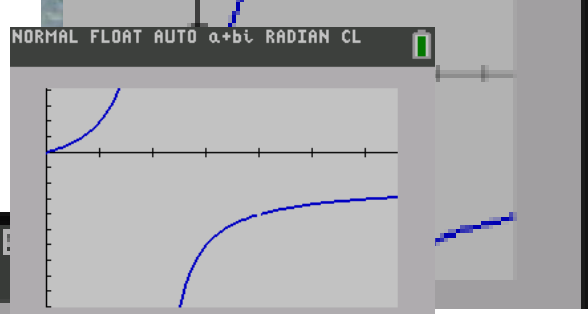
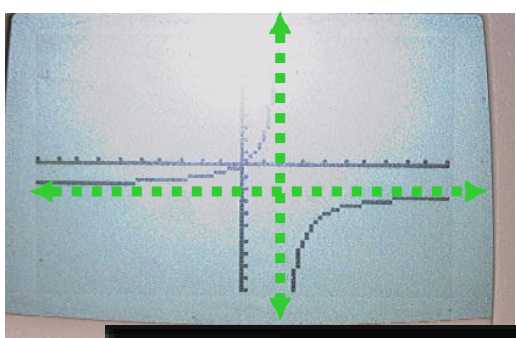
AT $x=4$

Hole $(\frac{-8}{2}) = -4$



$$y_1 = \frac{-2x^2 + 8x}{x^2 - 6x + 8}$$

C



Zero	Factor
2	$x-2$
$3+i$	$x-(3+i)$
$3-i$	$x-(3-i)$

$$(x-2)(x-3+i)(x-3-i)$$

These factors give the following polynomial in factored form.

$$f(x) = (x-2)(x-(3+i))(x-(3-i))$$

Note that this polynomial has degree 3.

We multiply the factors to get the answer.

$$(x-2)(x-(3+i))(x-(3-i))$$

$$(x-2)(x^2-6x+10)$$

$$= (x-2)(x^2-6x+10)$$

[More](#)

$$\begin{array}{r}
 x^2 - 6x + 10 \\
 \underline{x - 2} \\
 x^3 - 2x^2 + 12x - 20 \\
 + 3x^2 - 6x^2 + 10x \\
 \hline
 x^3 - 8x^2 + 22x - 20
 \end{array}$$

Lecture: Exponential Functions

$$f(x) = B^x$$

Diagram illustrating the components of an exponential function $f(x) = B^x$. The base B is labeled "base" and the exponent x is labeled "exponent".

40 mins
60 min
80 min

300 $\downarrow \times 2$
600 $\downarrow \times 2$
1200 $\downarrow \times 2$

2^x
 2^4

Interest Rate Formulas

I = interest

P = principal

R = rate

T = time

per cent
 $5 / 100 = .05$

Simple
I = P R T

$\$100$
 $\underline{\$70} \rightarrow .05$ per yr.
 2 yr

$$I = 100 \cdot .05 \cdot 2 \\ = \$100$$

Compound Interest

P = Ending Amount

Q = Starting Amount

R = Rate

N = Means..... Annual

T = Time

Quarterly

Monthly

Daily

Interest

N = 1

N = 4

N = 12

N = 365

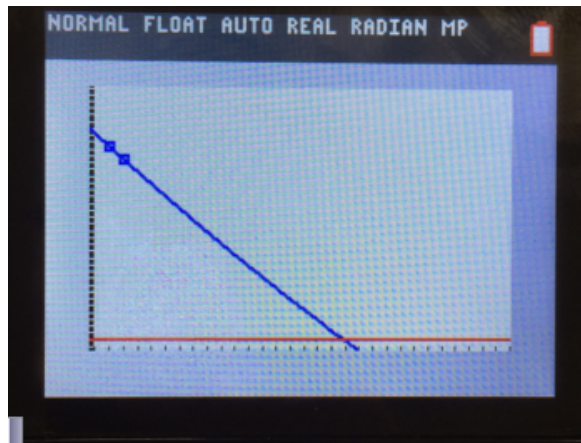
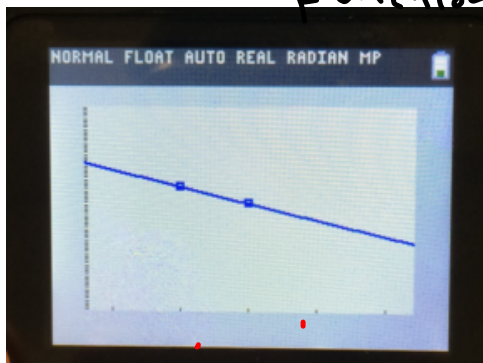
$$P = Q \left(1 + \frac{R}{N} \right)^{NT}$$

$$= Q \left(1 + \frac{R}{N} \right)^{N \cdot T}$$

$$= \text{\$} 1000 \left(1 + \frac{.02}{365} \right)^{\underline{365 \cdot 5}}_{5415}$$

	\$ 100	at .5%	= 100.50
1 yr	100	(1.005)	= 100.50
2 yrs	<u>100</u>	$(\underline{1.005})(\underline{1.005})$	= <u>101.005</u>
10 yrs	100	$(1.005)^{10}$	= 105.114
100 yrs	100	$(1.005)^{100}$	= 164.67

EXponential Functions



exponential decay

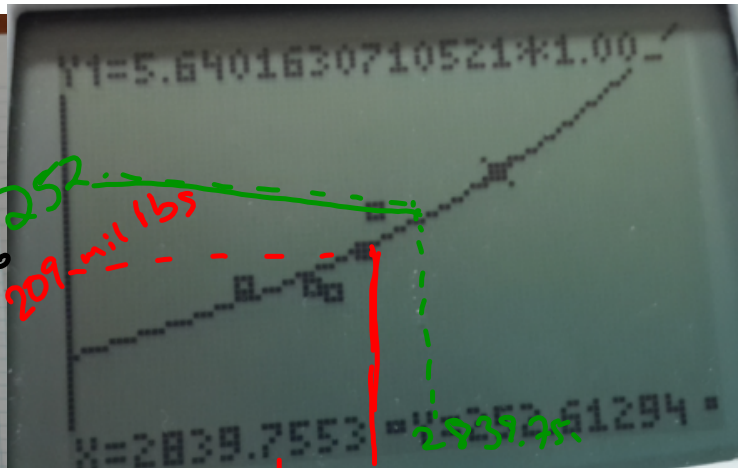
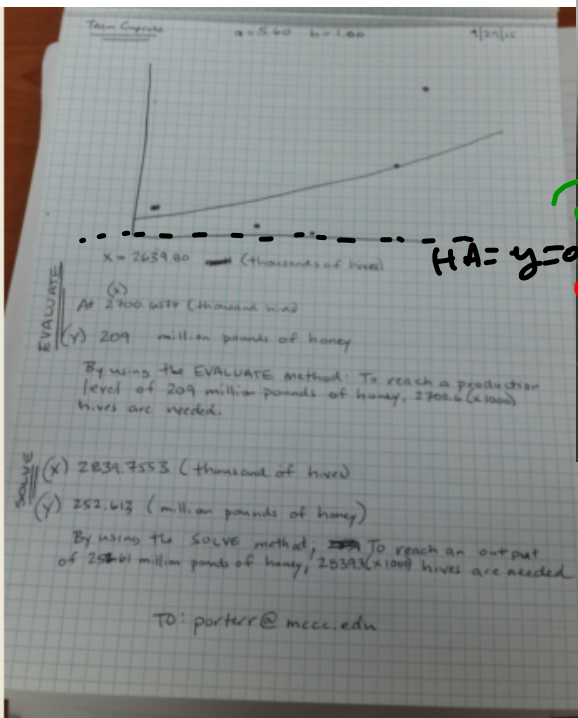
$$y = A \cdot B^x$$

School Board

In the year 2006 there were 492 recorded convictions and according to our regression there is a steady decrease leading to 400 convictions in the year 2023.

X	Y1
7	486
8	480.07
9	474.22
10	468.44
11	462.72
12	457.08
13	451.51
14	446
15	440.56
16	435.19
17	429.88

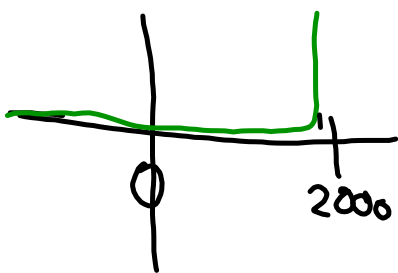
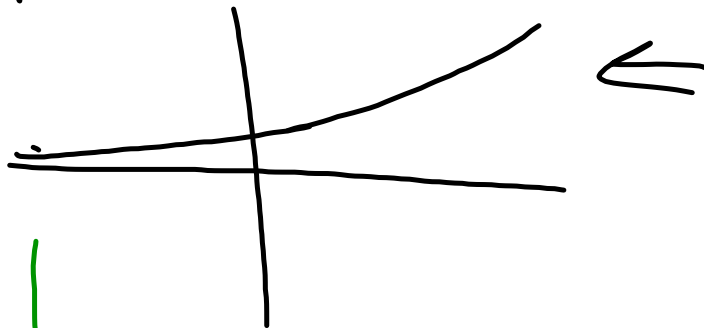
X=17



Exponential Growth | 2700 Hives

Get 209 lbs
 Evaluating
 Pick X → get y
 Want y = 252 lbs
 Solve
 Pick y → get X

Exponential Growth



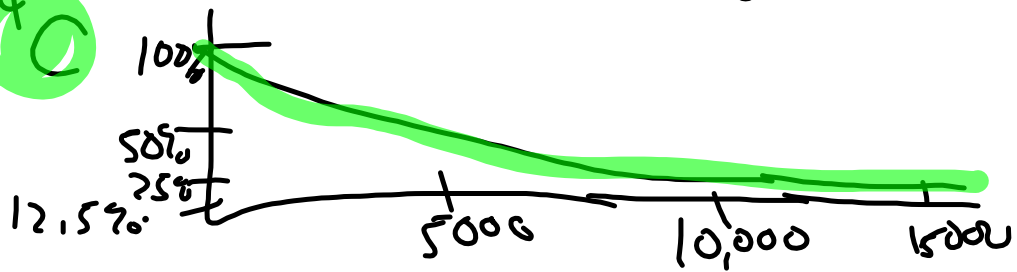
1.008

5% interest

$$\$100(1.05) = \$105$$
$$100(1.05)^{13} = 200.$$

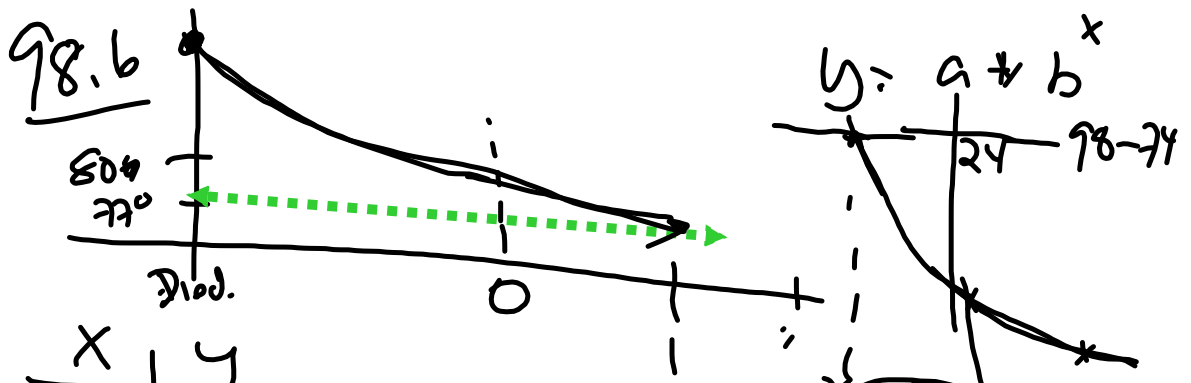
Exponential Decay

^{14}C



carbon dating

20,000 6.25%
25,000 3.125%



X	y	
0	80°	-74° = 6
1	77°	-74° = 3
∞	74°	-74° = 0

$$\begin{array}{r} 98.6 \\ -74 \\ \hline 24.6 \end{array}$$

0	6
1	3

$$x = -1.9$$

Lecture : Transformation of Functions

Transformation of Functions

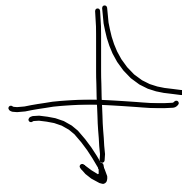
Person -----> Werewolf

Parent Functions:

$$f(x) = x^2$$



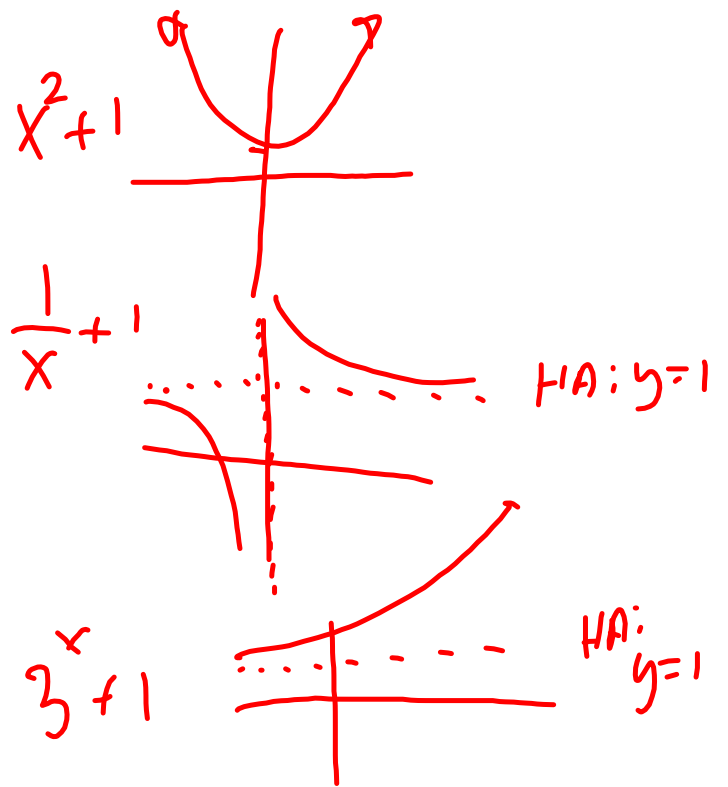
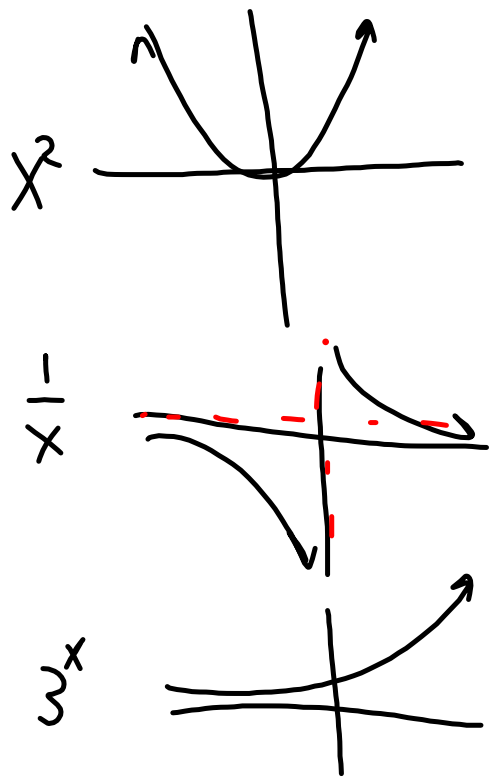
$$g(x) = 1 / x$$



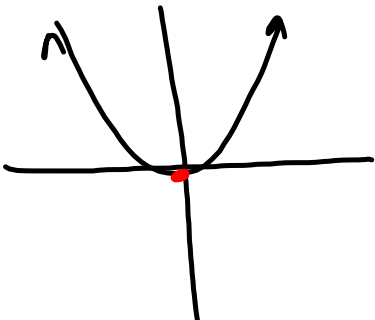
$$h(x) = 1.2^x$$



	Inside	Outside
+	Left	Raise
-	Right	Lower
$2, 3, 4 \dots \times 1^+$	"Smush"	Stretch
$\frac{1}{2}, \frac{1}{3}, \dots \times 1^-$	"Pull"	Shrink
\rightarrow	Reflects Y-axis	Reflects X-axis \curvearrowright

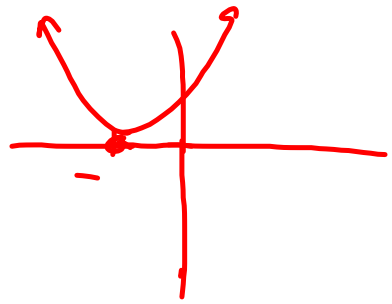


x^2

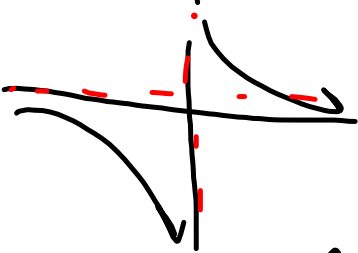


Left by 1

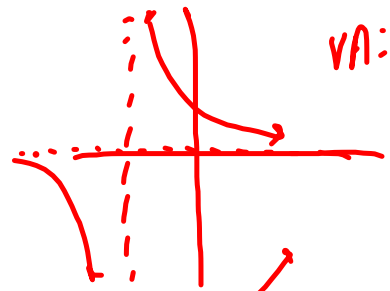
$$(x+1)^2$$



$x-1$

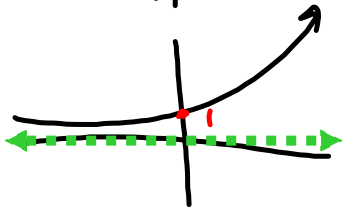


$$\frac{1}{x+1}$$



VA: $x = -1$

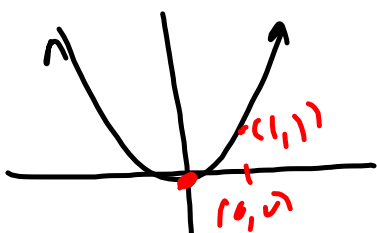
$3x$



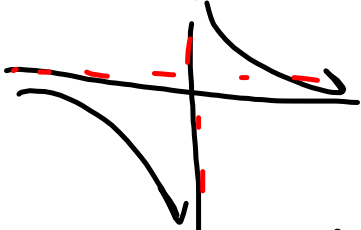
$$3^{x+1}$$



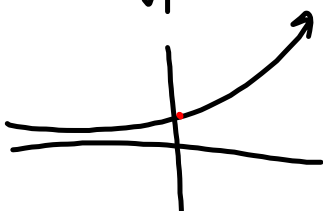
x^2



$\frac{1}{x}$

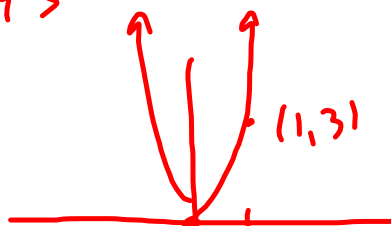


3^x



СИМЕТРИЯ ОТ 3

$$3x^2$$

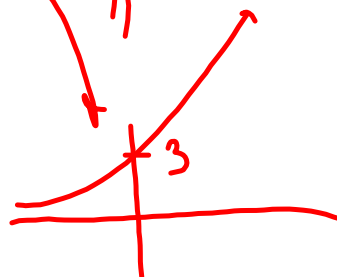


$$\frac{3}{x}$$



$$3 \cdot 3^x$$



$$3^{x+1}$$



Ex $3(x-1)^2 + 7$ Vertex $(1, 7)$

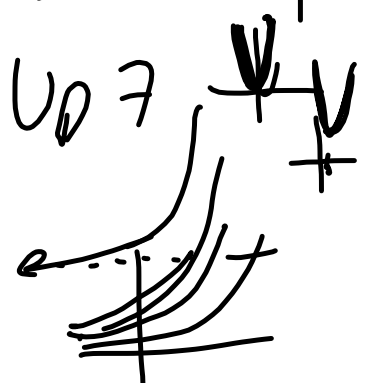
Group: Parent: x^2 

Order:

(PEMDAS) - 1 Inside Right by 1
 3 Outside Stretch 
 7 Outside Up 7 

Parent 3^x
 $3 \cdot 3^{x-1} + 7$

HA: $y = 7$



natural exponent

$$"e" = 2.71\dots \text{ (like } \pi \text{)}$$

Formula

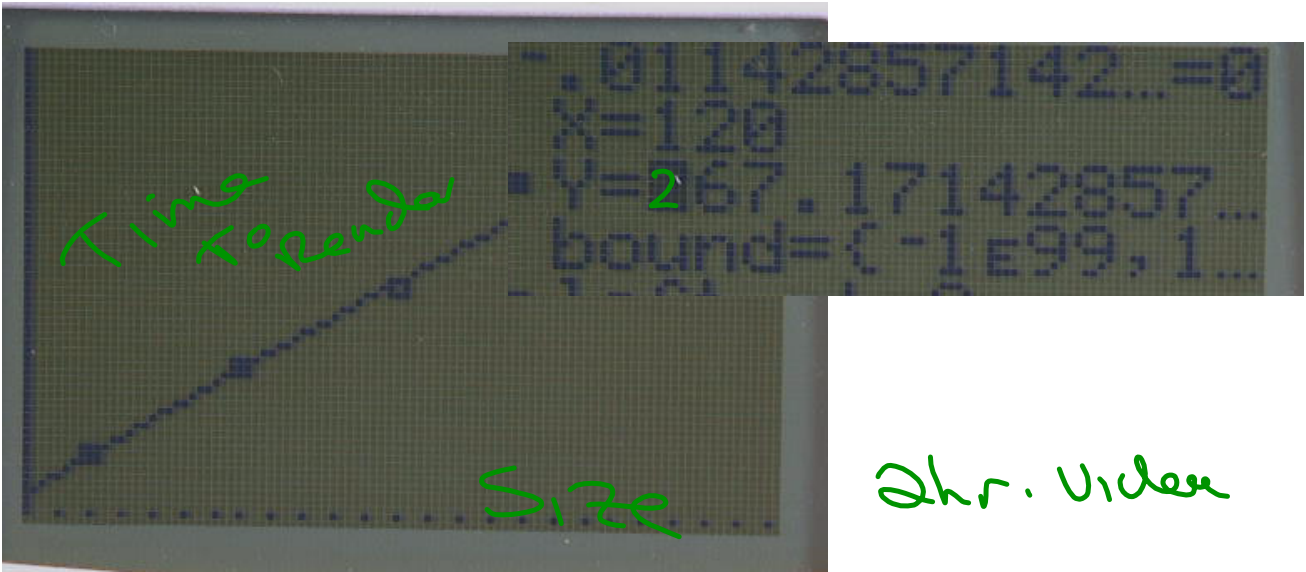
$$P = Q e^{RT}$$

(Compound
Continuously)
 $N = \infty$

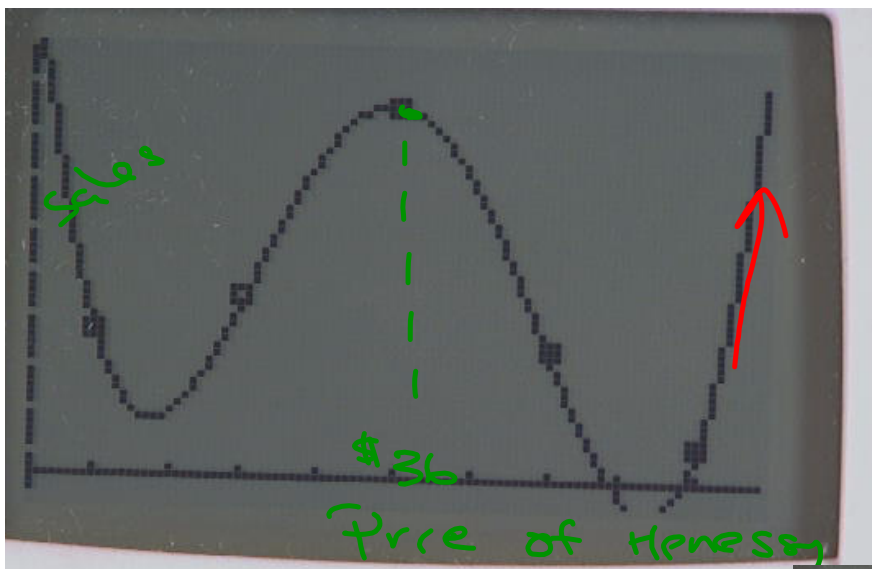


Project

Predict using Exponential regression



2hr. Video



```

. 3515625000000..=0
. X=31.817331389...
  Y=100
  bound=(-1E99, 1...
. left-rt=-4E-8

```

```

. 3515625000000..=0
. X=42.330944367...
  Y=100
  bound=(-1E99, 1...
. left-rt=6E-8

```

```
4.19698014303...=0
X=0
▪ Y=-.9889145750...
bound=(-1E99, 1...
▪ left-rt=0
```

```
4.19698014303...=0
▪ X=6.754354992...
Y=.001
bound=(-1E99, 1...
▪ left-rt=1E-14
```

```
4.19698014303...=0
▪ X=2661.2342458...
Y=100
bound=(-1E99, 1...
▪ left-rt=0
```

```
4.19698014303...=0
▪ X=40.407107758...
Y=.1
bound=(-1E99, 1...
▪ left-rt=0
```

so that if the price drops to \$175 we are
to sell 17 Calculators

According to the cubic Regression we can expect
that if we sell our calculators for \$175 then
we will sell zero

$$-.002x^3 + .445x^2 - 33.25x + 834.419...$$

exponential decay slows as price goes up
Sales goes down

