

Agenda

Homework/Quiz

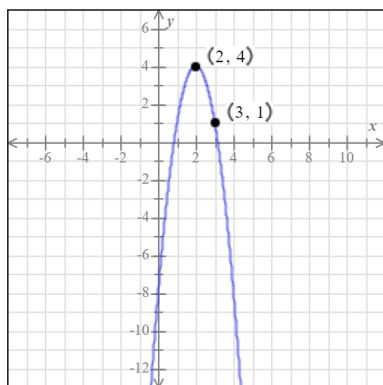
Lecture

Project

Homework

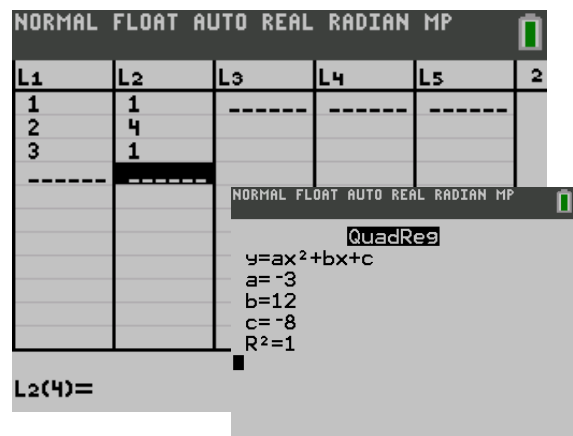
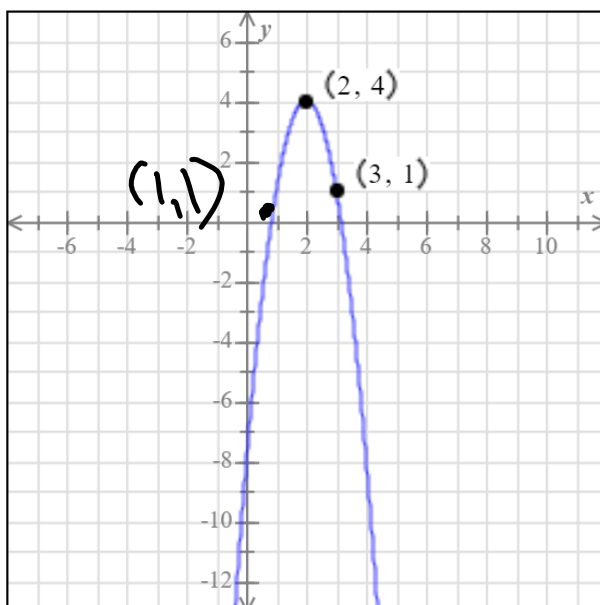
Writing the equation of a quadratic function given its graph

Find the equation of the quadratic function f whose graph is shown below.



Writing the equation of a quadratic function given its graph

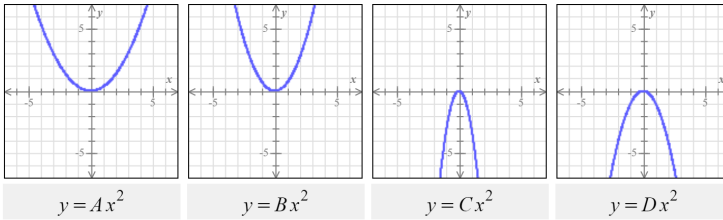
Find the equation of the quadratic function f whose graph is shown below.



$$y = -3x^2 + 12x - 8$$

How the leading coefficient affects the shape of a parabola

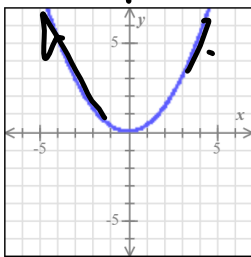
Look at the graphs and their equations below. Then fill in the information about the leading coefficients A , B , C , and



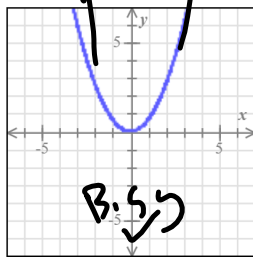
	A	B	C	D
(a) For each coefficient, choose whether it is positive or negative	Select One ▼	Select One ▼	Select One ▼	Select One ▼
(b) Choose the coefficient closest to 0	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
(c) Choose the coefficient with the greatest value	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

How the leading coefficient affects the shape of a parabola

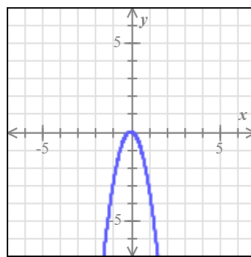
Look at the graphs and their equations below. Then fill in the information about the leading coefficients A , B , C , and D .



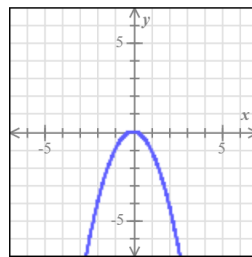
$y = Ax^2$



$y = Bx^2$



$y = Cx^2$



$y = Dx^2$

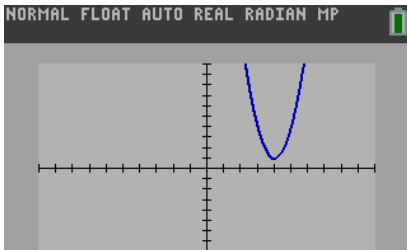
	A	B	C	D
(a) For each coefficient, choose whether it is positive or negative	Select One <input checked="" type="checkbox"/> <input type="checkbox"/>	Select One <input checked="" type="checkbox"/> <input type="checkbox"/>	Select One <input checked="" type="checkbox"/> <input type="checkbox"/>	Select One <input checked="" type="checkbox"/> <input type="checkbox"/>
(b) Choose the coefficient closest to 0	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
(c) Choose the coefficient with the greatest value	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>

Finding the maximum or minimum of a quadratic function

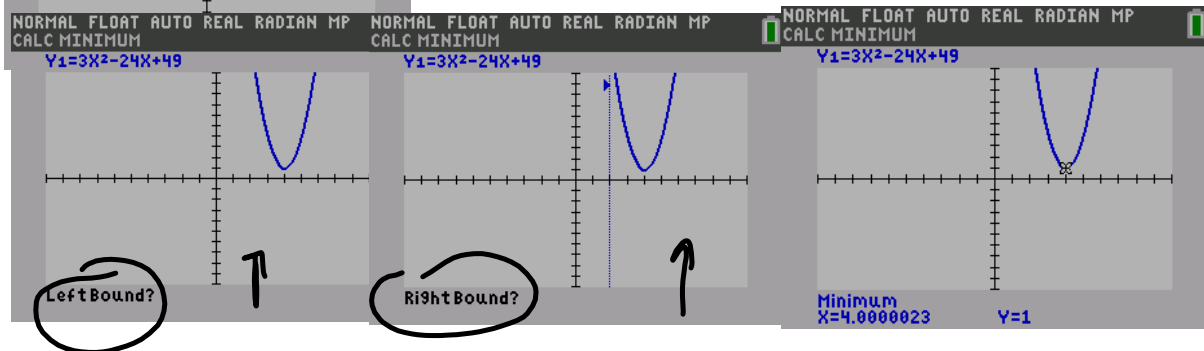
Answer the questions below about the quadratic function.

$$f(x) = 3x^2 - 24x + 49$$

Does the function have a minimum or maximum value? <input type="radio"/> Minimum <input type="radio"/> Maximum
Where does the minimum or maximum value occur? $x =$ <input type="text"/>
What is the function's minimum or maximum value? <input type="text"/>



$y = 3x^2 - 24x + 49$
 Calc: 3 min



ALEKS Calculator easier(?)

Finding the maximum or minimum of a quadratic function

Answer the questions below about the quadratic function.

$$f(x) = 3x^2 - 24x + 49$$

Does the function have a minimum or maximum value?
<input checked="" type="radio"/> Minimum <input type="radio"/> Maximum
Where does the minimum or maximum value occur?
$x =$ <input type="text" value="4.0000..."/>
What is the function's minimum or maximum value?
<input type="text" value="1"/>

Where x
 When x

What y

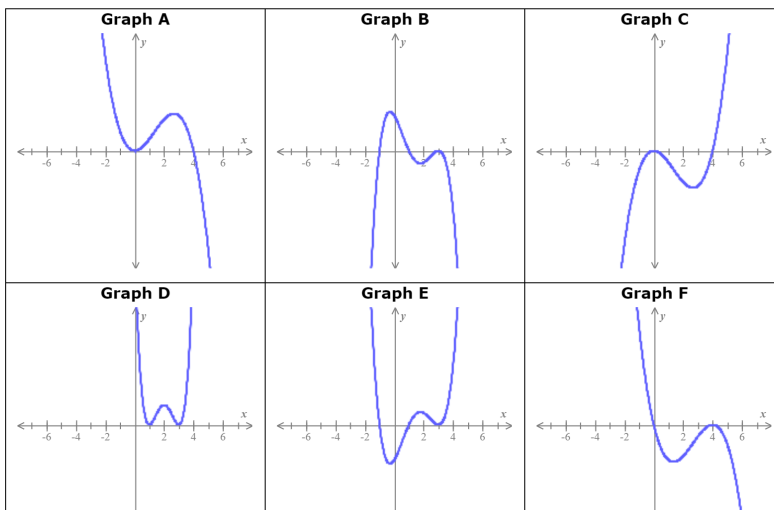
Matching graphs with polynomial functions

Consider the following polynomial functions.

$$g(x) = (x-3)^2(x^2-1)$$

$$h(x) = -x^3 + 4x^2$$

Choose the graph of each function from the choices below.



Matching graphs with polynomial functions

Consider the following polynomial functions.

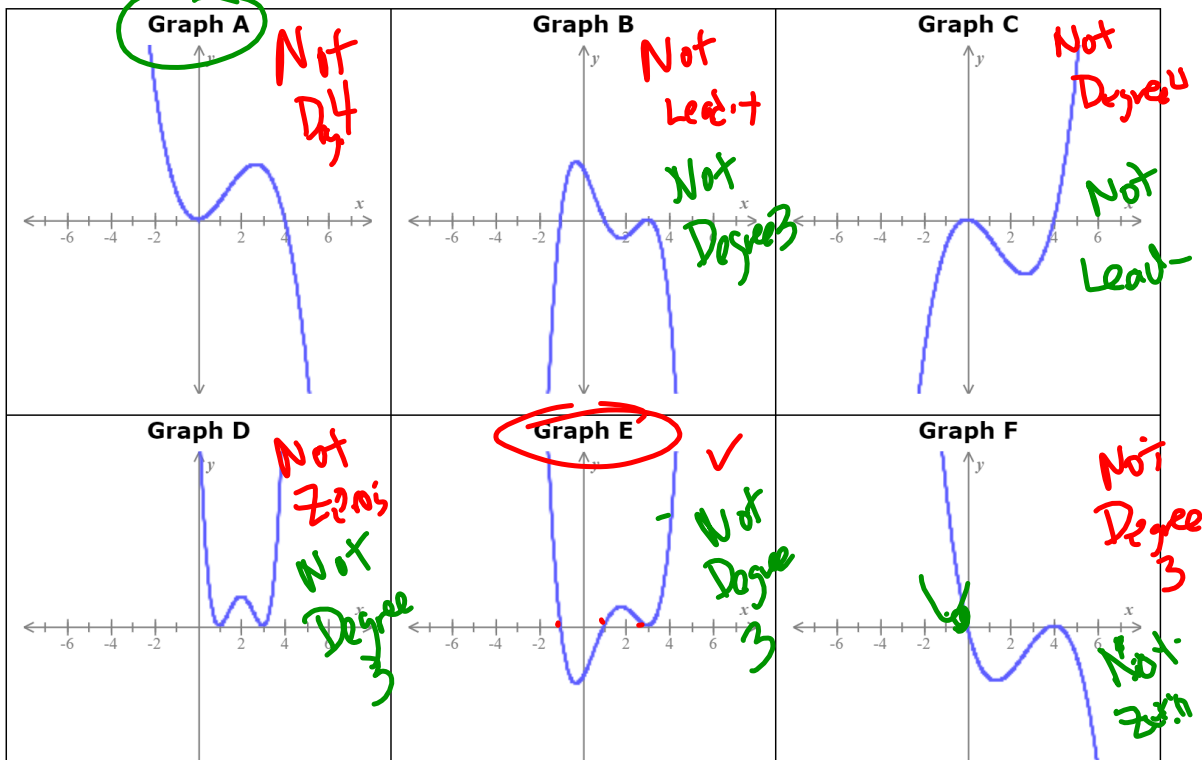
$$g(x) = (x-3)^2(x^2-1)$$

$$h(x) = -x^3 + 4x^2$$

Degree: **4**
Degree: **3**

$(x-3)(x-3)(x-1)(x+1)$
Lead: 1
 $-x^2(x-4)$
Lead: -1

Choose the graph of each function from the choices below.



Lecture: Zeros & Inequalities

Math?

Precalc?

Function = Job

Data, Graphs, Equations

✓ $f(x) = ax + b$ Linear

✓ $f(x) = ax^2 + bx + c$ Quadratic

✓ $f(x) = ax^n + bx^{n-1} + \dots$ Polynomial

→ $3x^3 + 5x + 7$

Degree: n Lead: a

Faces: n or less

Zeros: n or less

Max/Turns: $n-1$ or less

3 or more
faces

Zeros

Fundamental Theorem of Algebra ★

Poly degree "n"

$$a(x-x_1)(x-x_2)(x-x_3)\dots(x-x_n)$$

n factors

fully factored form

$$= (x-1)(x-2)(x+3)$$

zeros: 1, 2, -3

$$= x^3 + ax^2 + \dots + b$$

Ex $y = (x-1)(x-1) = x^2 - 2x + 1$

zeros: 1, 1

Two zeros: multiplicity one repeated time.

Touches

Imaginary (Unit i)

$$\sqrt{-1} = i$$

so $(i)^2 = -1$

Complex

$$\underbrace{a}_{\text{real}} + \underbrace{bi}_{\text{imaginary}}$$

Ex $3 + 2i$

$$\text{Ex } (3+2i)^2$$

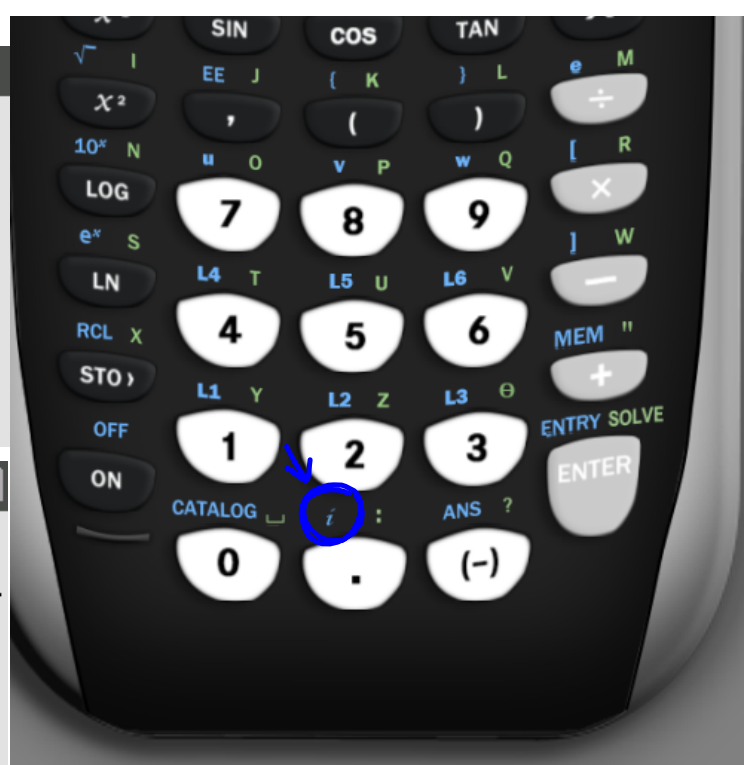
$$(3+2i)(3+2i) = \frac{9}{9} + \frac{6i}{+12i} + \frac{6i}{-4} + \frac{4i^2}{-4}$$

$$5 + 12i$$

MODE:

```
NORMAL FLOAT AUTO a+bi RADIAN MP
SCREEN VIEW
MATHPRINT CLASSIC
NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNCTION PARAMETRIC POLAR SEQ
THICK DOT-THICK THIN DOT-THIN
SEQUENTIAL SIMUL
REAL a+bi re^(θi)
FULL HORIZONTAL GRAPH-TABLE
FRACTIONTYPE: n/d Un/d
ANSWERS: AUTO DEC FRAC-APPROX
GO TO 2ND FORMAT GRAPH: NO YES
STAT DIAGNOSTICS: OFF ON
STAT WIZARDS: ON OFF
SET CLOCK 02/12/20 7:23PM
```

```
NORMAL FLOAT AUTO a+bi RADIAN MP
(3+2i)^2
.....5+12i
```



CONJUGATES (complex)
 $(a + bi)(a - bi)$

FOIL

$$a^2 - \cancel{abi} + \cancel{abi} - b^2 \cdot i^2$$

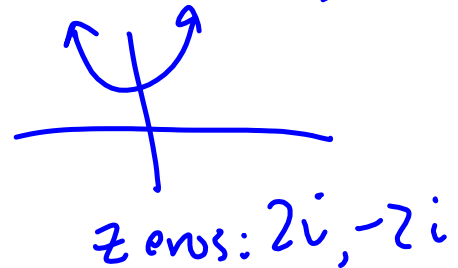
$a^2 + b^2$ real $i^2 = -1$

$$y = (x^2 - 4) = (x+2)(x-2)$$

Real Factors
Zero: 2, -2

$$y = (x^2 + 4) = (x - 2i)(x + 2i)$$

Complex Factors



$$= ax^2 + bx + c$$

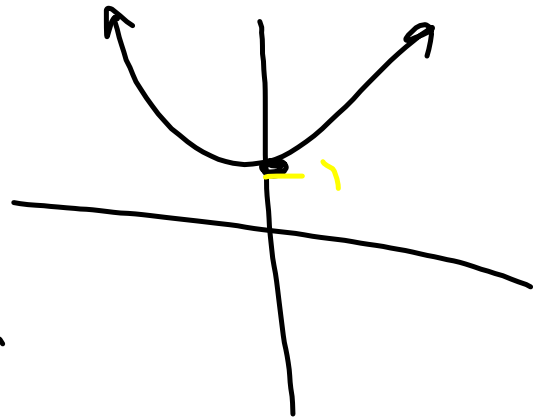
$$\stackrel{\text{ex}}{=} (x-i)(x+i)$$

Zeros $\pm i$

$$\rightarrow x^2 + \cancel{x(-i)} - \cancel{x(i)} - i^2$$

$$x^2 - i^2$$

$$x^2 - (-1) = x^2 + 1$$



$$y = x^2 + 1$$

$$0 = x^2 + 0x + 1$$

$$x = \frac{\pm \sqrt{-4}}{2} = \frac{\pm 2i}{2}$$

$$= \pm i$$

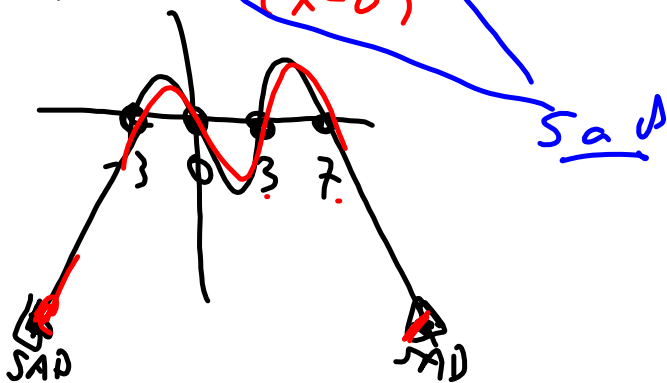
Find all real zeros of the function.

0, 3, -3, 7
PASS PASS PASS PASS
 Degree: 4

$$f(x) = -2x(x^2 - 9)(x - 7)$$

$$-2x(x-3)(x+3)(x-7) \quad \text{END: SAD}$$

Begin



$n \backslash a$	Lead +	Lead -
odd	Disco Right	Disco Left
even	Happy ☺	Sad ☹

Conjugates
 $(A-B)(A+B)$

$(A^2 - B^2)$ ← diff of squares

$$(x-a)(x-b)$$

~~$(x-1+i)(x-1-i)$~~

~~$((x-1)+i)((x-1)-i)$~~

Zeros
 $1+i$
 $1-i$

$$(x-1)^2 - i^2$$

$$(x-1)^2 + 1 = x^2 - 2x + 2$$

$A=1 \quad B=-2 \quad C=2$

$$x = \frac{+2 \pm \sqrt{4 - 4(2)}}{2}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$\therefore \frac{2 \pm 2i}{2} = \frac{2}{2} \pm \frac{2i}{2} = 1 \pm i$$

$$(X - \underline{\underline{a+bi}})(X - \underline{\underline{a-bi}})$$

$$(X - a - bi)(X - a + bi)$$

$$((X - a) - bi)((X - a) + bi)$$

$$(X - a)^2 - b^2$$

$$(X - a)^2 + b^2$$

$n=3$ cubic Poly
with zeros.

1 , $1+i$, $1-i$ ^{Imaginal}

Factors $(x-1)(x-(1+i))(x-(1-i))$

$$(x-1)(x-1-i)(x-1+i)$$

$$(x-1) \underbrace{(x-1-i)(x-1+i)}$$

$$(x-1) \left((x-1)^2 - i^2 \right)$$

$$x^2 - 2x + 1 + 1$$

$$(x-1)(x^2 - 2x + 2)$$

$$x-1$$

$$\begin{array}{r} x^3 - x^2 + 2x - 2 \\ x^3 - 2x^2 \quad 2x \end{array}$$

$$x^3 - 3x^2 + 4x - 2$$

Finding x- and y-intercepts given a polynomial function

Find all x-intercepts and y-intercepts of the graph of the function.

$$f(x) = 3x^3 - 12x^2 - 15x$$

If there is more than one answer, separate them with commas.

Click on "None" if applicable

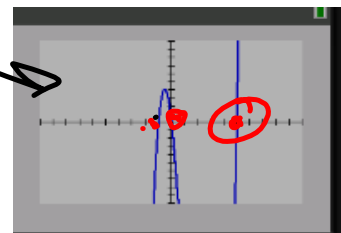
Finding x- and y-intercepts given a polynomial function

Find all x-intercepts and y-intercepts of the graph of the function.

$$f(x) = 3x^3 - 12x^2 - 15x$$

If there is more than one answer, separate them with commas.

Click on "None" if applicable



y-int (0,0)

x-int 0, 5, -1

Calc: 720

$$3x(x^2 - 4x - 5)$$
$$3x(x - 5)(x + 1)$$

Input interpretation:

factor $3x^3 - 12x^2 - 15x$

Result:

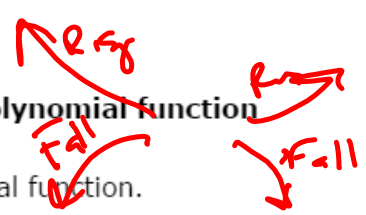
$$3x(x + 1)(x - 5)$$

Irreducible factorization:

$$3(x - 5)x(x + 1)$$

Plots:

Determining the end behavior of the graph of a polynomial function



Choose the end behavior of the graph of each polynomial function.

<p>(a) $f(x) = -5x^6 - x^4 + 8x^3 - 3x$ degree:6(even) Lead: -5 (negative)</p> <p style="text-align: right;"><u>sad</u></p>	<ul style="list-style-type: none"> <input type="radio"/> Falls to the left and rises to the right <input type="radio"/> Rises to the left and falls to the right <input type="radio"/> Rises to the left and rises to the right <input checked="" type="radio"/> <u>Falls to the left and falls to the right</u>
<p>(b) $f(x) = x^3 - 3x^2 - 2x - 9$</p> <p style="text-align: right;"><u>disco right</u></p>	<ul style="list-style-type: none"> <input checked="" type="radio"/> <u>Falls to the left and rises to the right</u> <input type="radio"/> Rises to the left and falls to the right <input type="radio"/> Rises to the left and rises to the right <input type="radio"/> Falls to the left and falls to the right
<p>(c) $f(x) = -5x(2x - 5)^2$ degree:3</p> <p style="text-align: right;"><u>disco left</u></p>	<ul style="list-style-type: none"> <input type="radio"/> Falls to the left and rises to the right <input checked="" type="radio"/> <u>Rises to the left and falls to the right</u> <input type="radio"/> Rises to the left and rises to the right <input type="radio"/> Falls to the left and falls to the right

Lead: -20(negative)

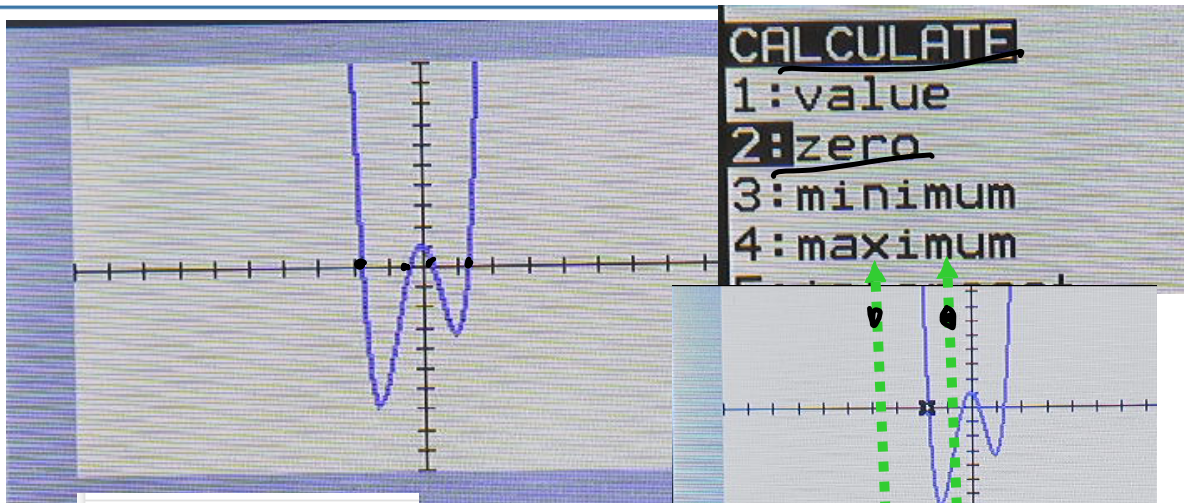
<p>(a) $f(x) = -5x^3 + 3x^2 + 5x + 2$</p> <p>disco left</p>	<ul style="list-style-type: none"> <input type="radio"/> Falls to the left and rises to the right <input type="radio"/> Rises to the left and falls to the right <input type="radio"/> Rises to the left and rises to the right <input type="radio"/> Falls to the left and falls to the right
<p>(b) $f(x) = 4x^5 - 3x^3 + 8x + 9$</p> <p>disco right</p>	<ul style="list-style-type: none"> <input type="radio"/> Falls to the left and rises to the right <input type="radio"/> Rises to the left and falls to the right <input type="radio"/> Rises to the left and rises to the right <input type="radio"/> Falls to the left and falls to the right
<p>(c) $f(x) = 2(x - 2)^2(x + 4)$</p> <p>disco right</p>	<ul style="list-style-type: none"> <input type="radio"/> Falls to the left and rises to the right <input type="radio"/> Rises to the left and falls to the right <input type="radio"/> Rises to the left and rises to the right <input type="radio"/> Falls to the left and falls to the right

Use the ALEKS graphing calculator to find all the zeros of the poly

$$f(x) = 4x^4 + 3x^3 - 9x^2 - 2x + 1$$

Round to the nearest hundredth.

If there is more than one answer, separate them with commas.



Real solution:
 $x = \frac{1}{4}$

Solution:
 $x = \frac{1}{4}$

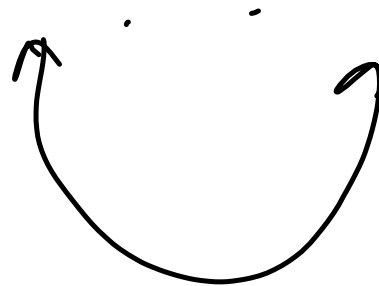
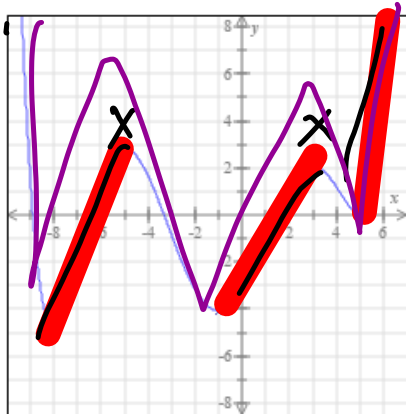
Complex solutions:
 $x \approx -1.8019$
 $x \approx -0.44504$
 $x \approx 1.2470$

warning.. .25000002

calculator crud

.24999999

$$\begin{aligned} \overline{3} &= \frac{1}{3} \\ \overline{6} &= \frac{2}{3} \\ \overline{9} &= \frac{3}{3} = 1 \end{aligned}$$



(a) The function f is increasing over which intervals? Choose all that apply. x values

- $(-\infty, -8)$
 $(-8, -5)$
 $(-5, -1)$
 $(-1, 3)$
 $(-1, 5)$
 $(5, \infty)$

(b) The function f has local maxima at which x-values? If there is more than one value, separate them with commas.

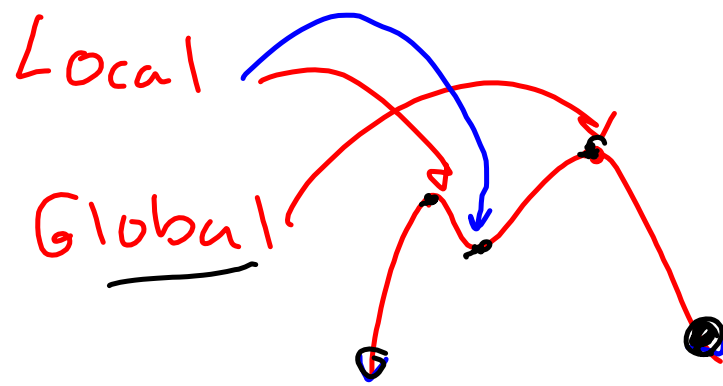
$x = -5, 3$

(c) What is the sign of the leading coefficient of f ?

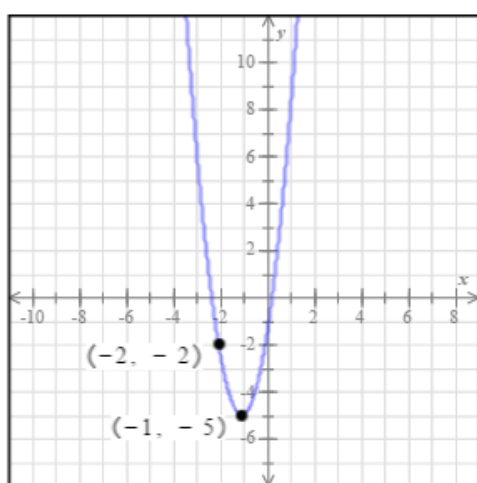
Select One: positive... happy Panda

(d) Which of the following is a possibility for the degree of f ? Choose all that apply.

- ~~4~~
 5
 6
 7
 8
 9
- ~~4~~
 6
 8
- faces



Find the equation of the quadratic function f whose graph is shown below.



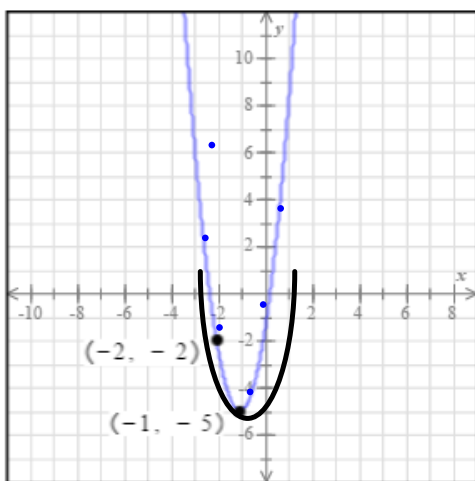
The graph of a quadratic function is a parabola.

Any quadratic function f whose graph has vertex (h, k) can be written in

$$f(x) = a(x - h)^2 + k, \quad \text{where } a \neq 0.$$

In the current problem, the vertex is $(-1, -5)$.

Find the equation of the quadratic function f whose graph is shown below



vertex $(-1, -5)$

move x^2 to left $(x+1)^2$

down by 5, $(x+1)^2 - 5$

$$y = a(x+1)^2 - 5$$

$$-2 = a(-2+1)^2 - 5$$

$$-2 = a - 5 \quad \text{so } a = 3$$

The graph of a quadratic function is a parabola.

Any quadratic function f whose graph has vertex (h, k) can be written in

$$f(x) = a(x-h)^2 + k, \quad \text{where } a \neq 0.$$

In the current problem, the vertex is $(-1, -5)$

Find a polynomial $f(x)$ of degree 3 with real coefficients and the following zeros.

3, $2i$

$$\begin{aligned} & (x-3)(x-2i)(x+2i) \\ &= (x-3)\underbrace{(x-2i)(x+2i)}_{(x^2+4)} \\ &= x^3 + 4x - 3x^2 - 12 \end{aligned}$$

Find a polynomial $f(x)$ of degree 3 with real coefficients and the following zeros.

3, $2i$

Zeros = 3 by FTA

$\left. \begin{array}{l} \uparrow \\ \uparrow \end{array} \right\} 3, 0+2i, 0-2i$

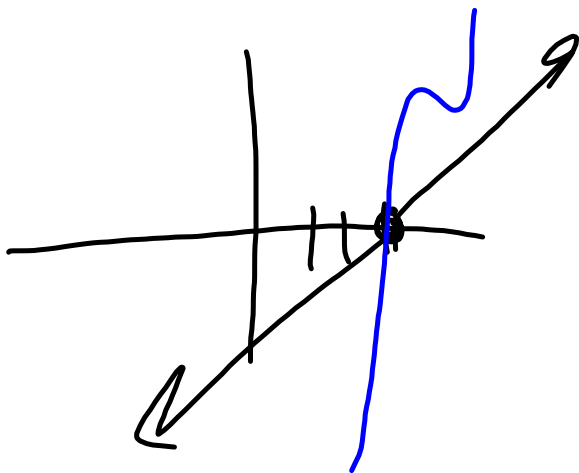
Zeros: 3, $2i$, $-2i$

Factors: $(x-3)(x-2i)(x+2i)$

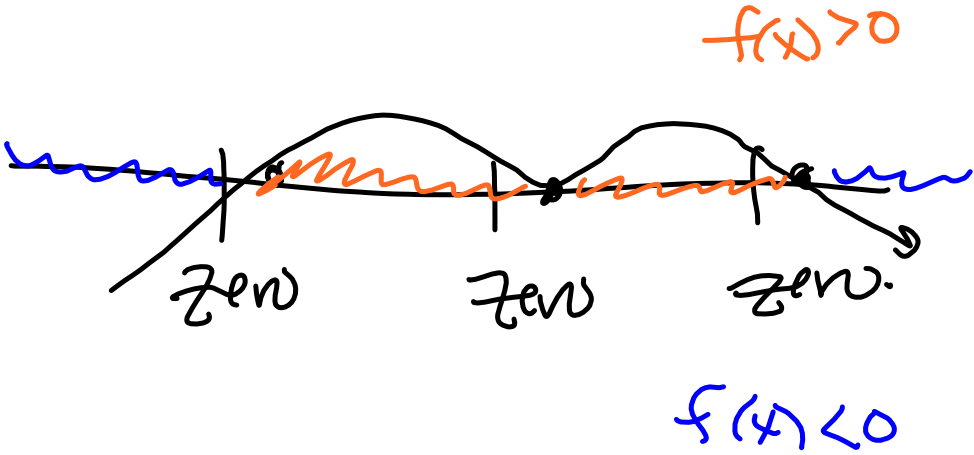
$$x^2 - \cancel{2xi} + \cancel{2xi} - 4i^2$$

$$(x-3)(x^2+4)$$

$$= x^3 - 3x^2 + 4x - 12$$



Polynomial Inequalities

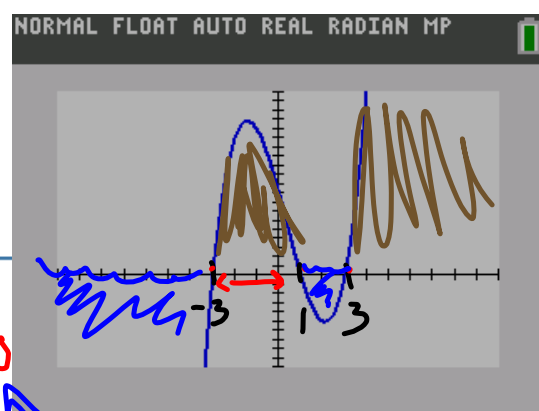


Solving a polynomial inequality

Solve the inequality.

$$x^3 - x^2 \geq 9x - 9$$

Write your answer as an interval or union of intervals.
If there is no real solution, click on "No solution".



$$y = x^3 - x^2 - 9x + 9 \geq 0$$

$$x^2(x-1) - 9(x-1)$$

$$(x^2 - 9)(x-1)$$

$$(x-3)(x+3)(x-1)$$

LAND

$$[-3, 1] \cup [3, \infty)$$

Solving a polynomial inequality

Solve the inequality.

$$x^3 - x^2 \geq 9x - 9$$

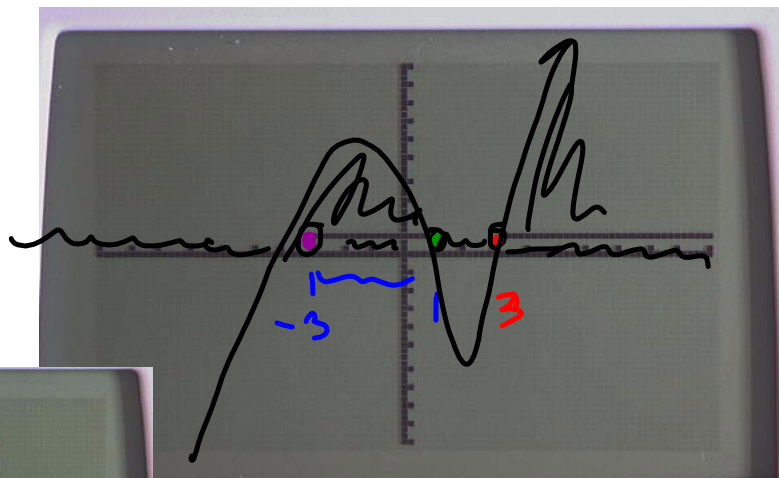
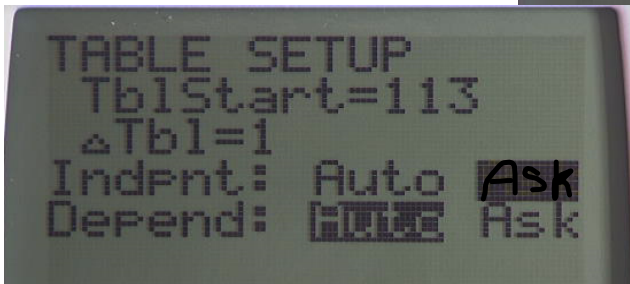
$$y = x^3 - x^2 - 9x + 9 \geq 0$$

Write your answer as an interval or union of intervals.
If there is no real solution, click on "No solution".

2nd math = test

$$y1 = (x^3 - x^2) \geq (9x - 9)$$

2nd window

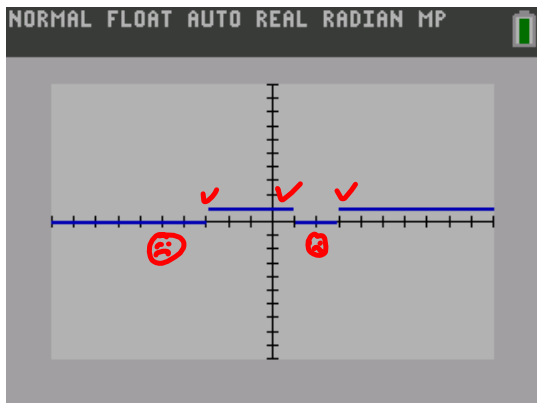


2nd graph

X	Y1
-3	0
-2	1
-1	1
1	0
2	0
3	1
2.99	0
3.01	1

Handwritten notes: "TRUE ✓" in blue next to the first two rows, and "TRUE ✓" in green next to the last two rows.

$$[-3, 1] \cup [3, \infty)$$



X	Y1
4	1
-3	1
-2	1
1	1
2	0
3	1
2.99	0
3.01	1

X=

Solving a polynomial inequality

Solve the inequality.

$$\underline{x^3 - x^2 \geq 9x - 9}$$

Write your answer as an interval or union of intervals.
If there is no real solution, click on "No solution".

Degree

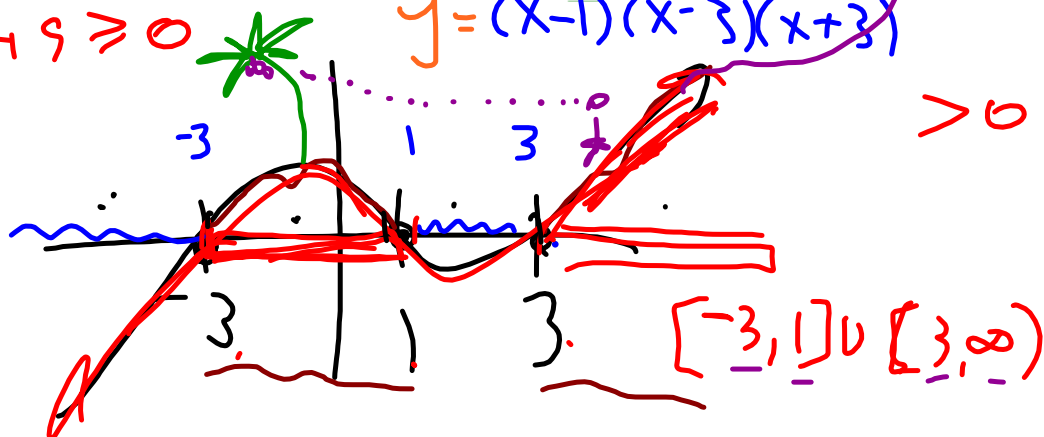
$$y = x^3 - x^2 - 9x + 9 \geq 0$$

$$x^2(x-1) - 9(x-1)$$

$$(x-1)(x^2-9)$$

$$y = (x-1)(x-3)(x+3)$$

$$x^3 - x^2 - 9x + 9 \geq 0$$



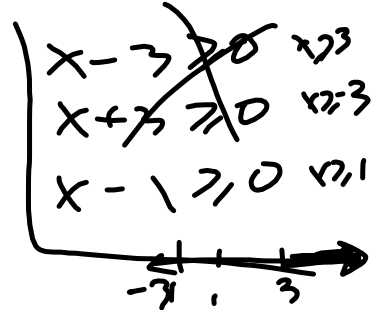
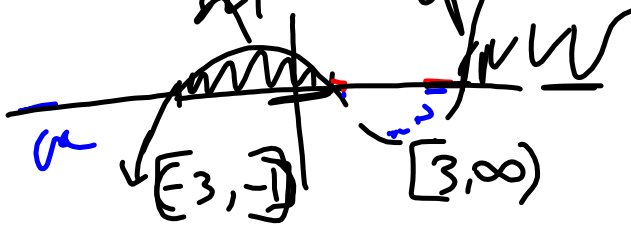
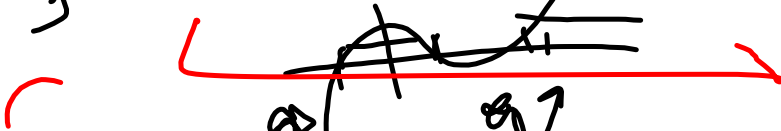
$$x^3 - x^2 \geq 9x - 9$$

$$x^3 - x^2 - 9x + 9 \geq 0$$

$$x^2(x-1) - 9(x-1) \geq 0$$

$$(x^2 - 9)(x-1) \geq 0$$

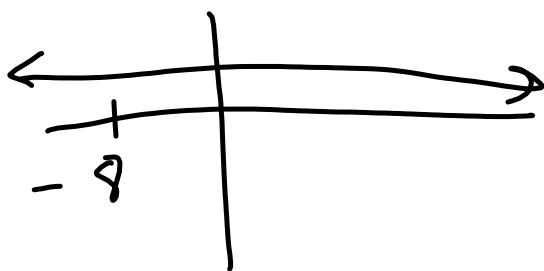
$$\Rightarrow (x-3)(x+3)(x-1) \geq 0$$



9. Solve the inequality.

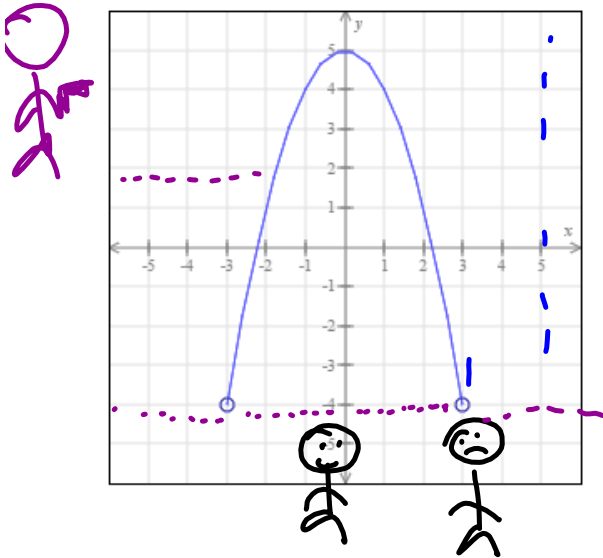
$$y = \frac{-(x+8)}{(-x-8)(x^2+5)(x+8)} \leq 0$$

Write your answer as an interval or union of intervals.



$$(-\infty, \infty)$$

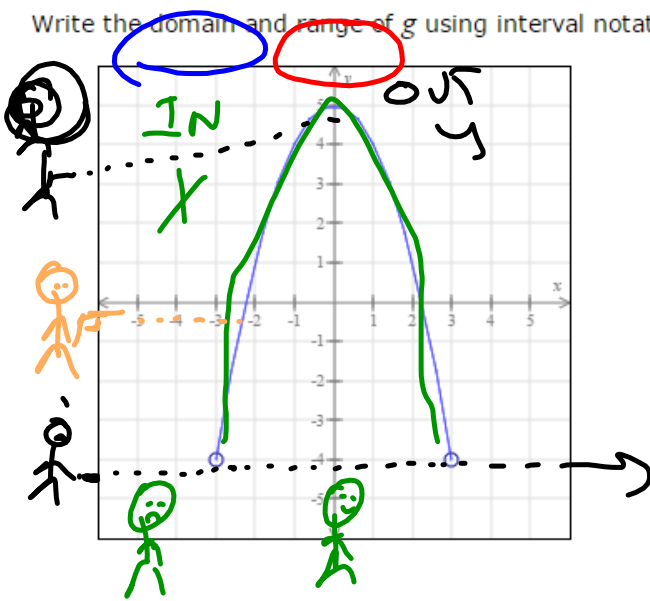
The entire graph of the function g is shown in the figure below.
Write the domain and range of g using interval notation.



Domain
 $(-3, 3)$

Range
 $[-4, 5]$

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 $(-3, 3)$

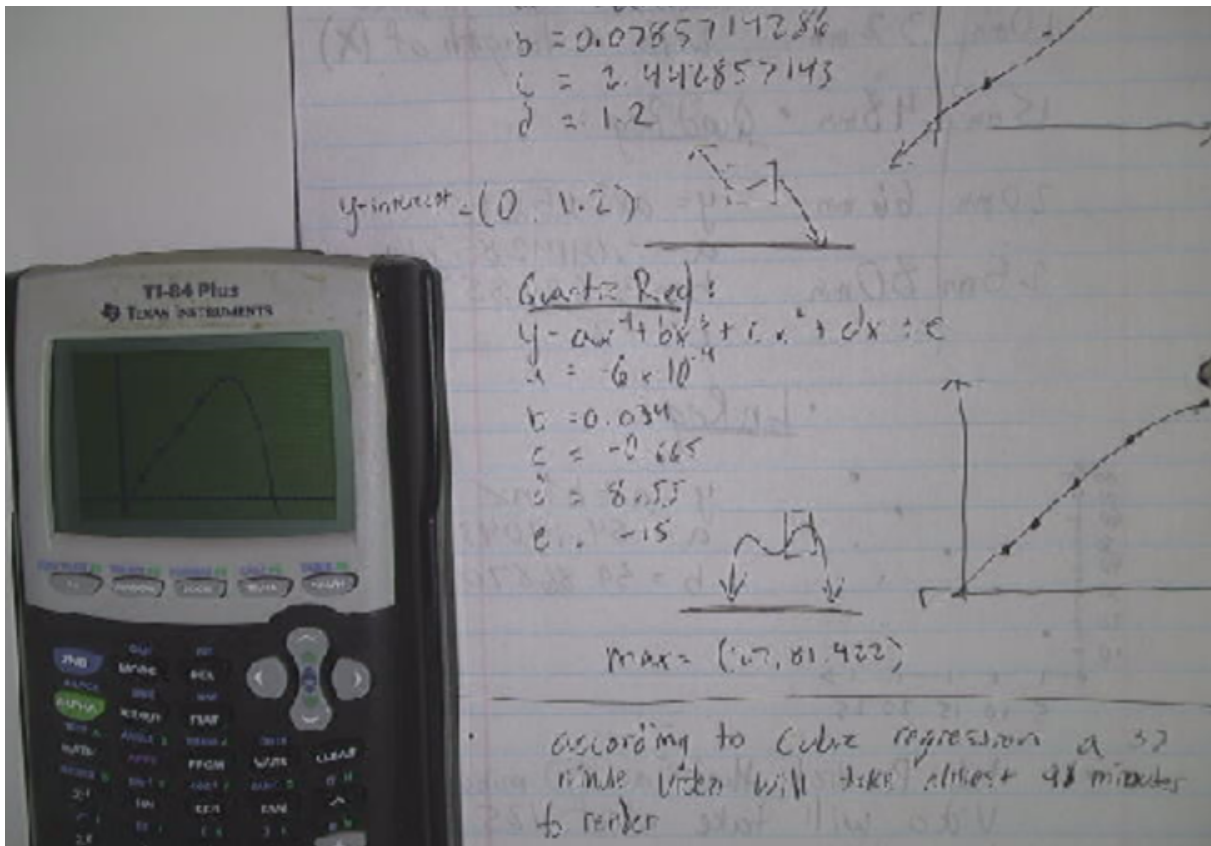
Range
 $(-4, 5]$

Project Work



Realistic Domain: $[0, 100]$

Realistic Range: $[0, 50]$

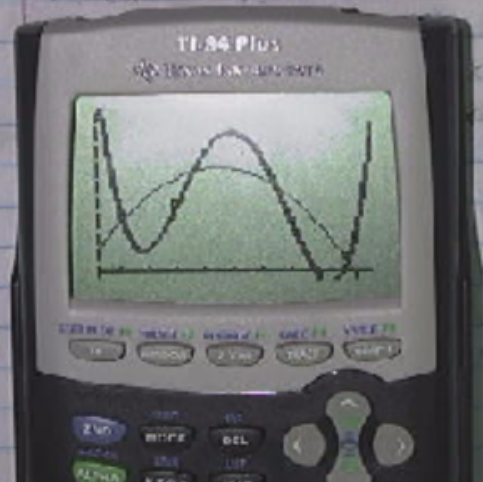


Cubic $8x^3 - 2x^2 - 84x$ Max 31.802 @ 36.487
 Quartic $2x^4 - 2x^2 - 84x$ zero $= 39.82$ & 40.821
 Max $= 182.00$ at 43.069 at 36.905

$$2x^3 - 2x^2 - 84x = x^3 + 8x^2 + 15x$$

$$2x(x^2 - x - 42) = x(x^2 + 8x + 15)$$

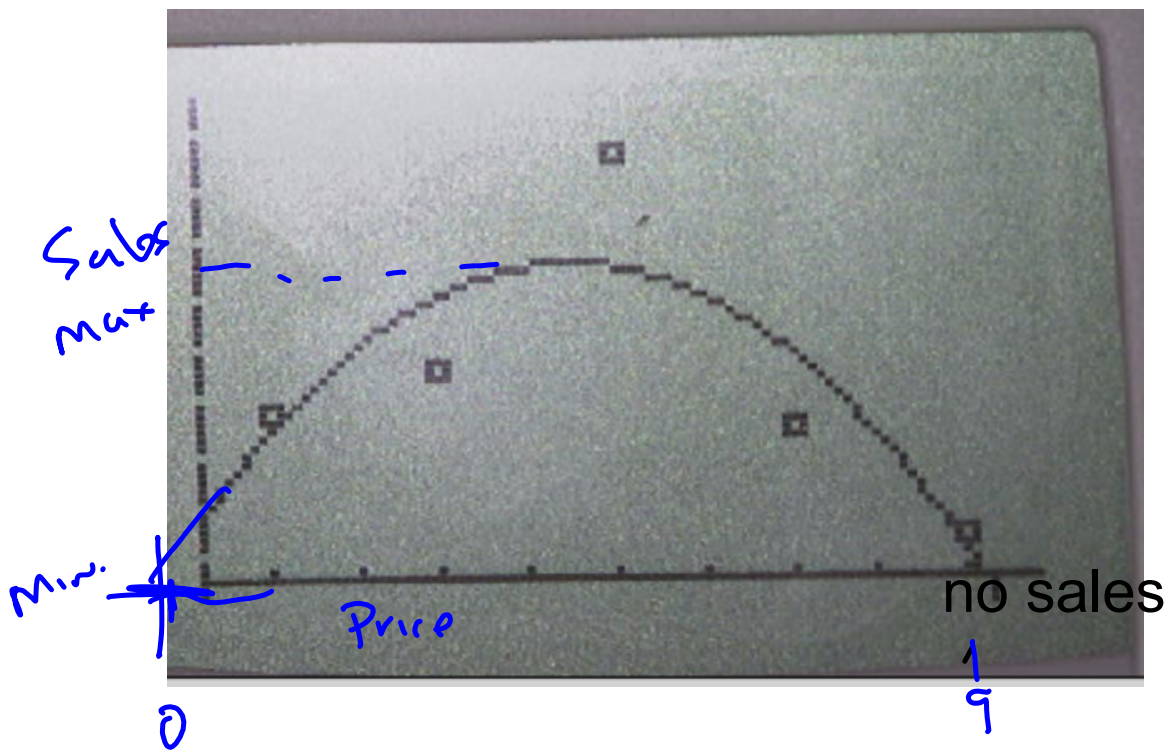
$$2x(x-7)(x+6) = x(x+5)(x+3)$$



$$= (x^2 - 9x - 5)$$

$$= (x - 5)(x + 1)$$

$$3x - 9$$



Find Max/min on Calculator

