

Agenda

Review of Quiz #9/10

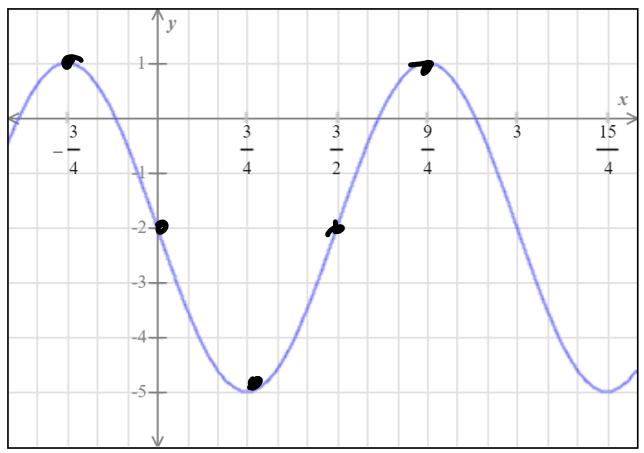
Inverse Trig Functions

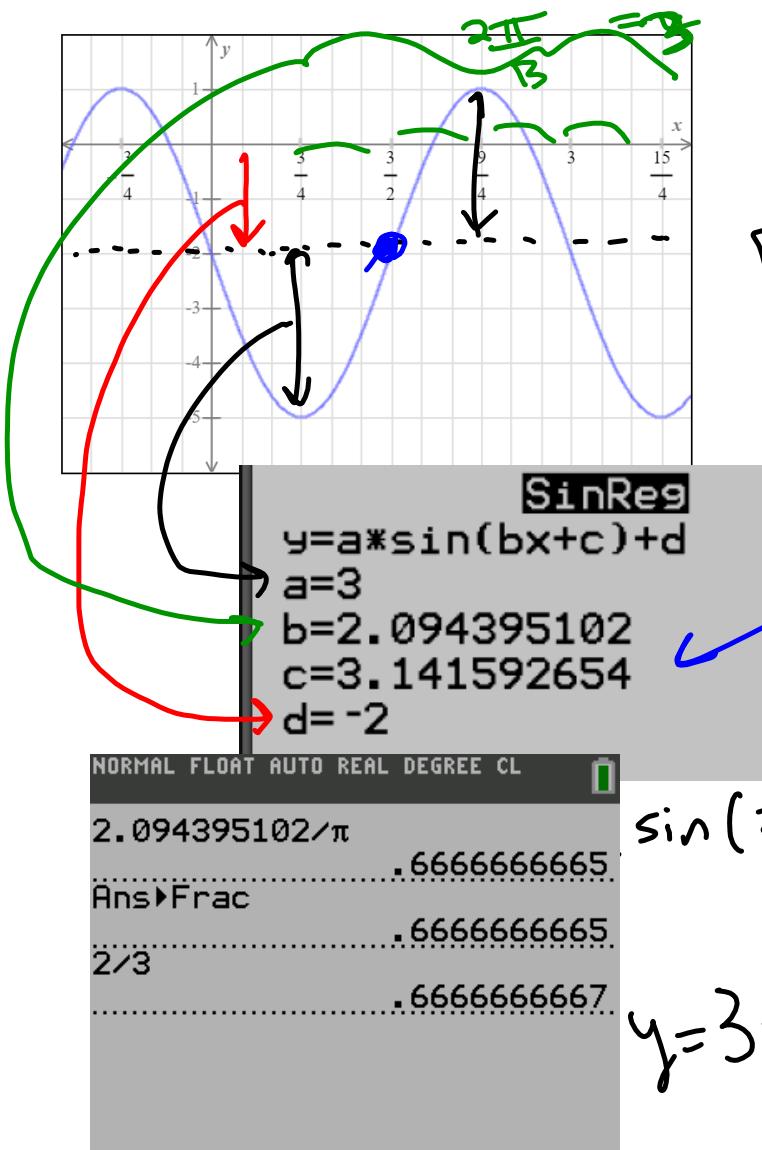
Right Triangle

Identities

Sum and Difference Formulas

Quiz 10 Review





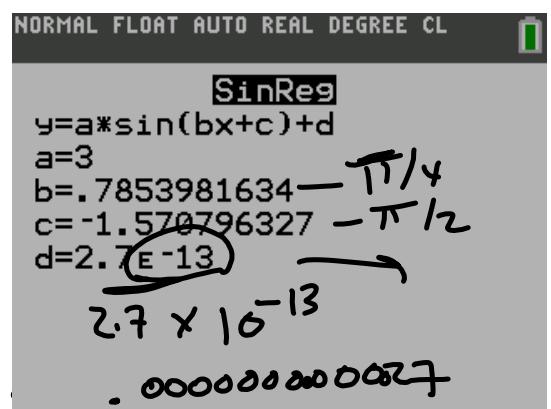
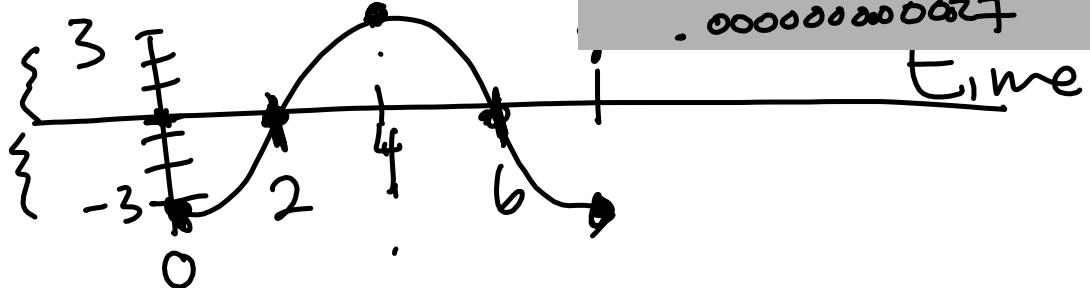
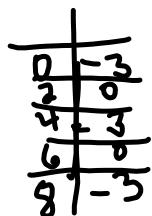
x	y
$\frac{3}{4}$	-5
$\frac{3}{2}$	-2
$\frac{9}{4}$	1
$\frac{3}{1}$	-2
$\frac{15}{4}$	-5

$C = \frac{3\pi}{4}$
 $\frac{2\pi}{3}$
 $\frac{\pi}{4}, \frac{2\pi}{3}, \frac{\pi}{2}$

$$\begin{aligned}
 & \text{Period} = \frac{2\pi}{B} \\
 & B = \frac{2\pi}{\text{Period}} = \frac{2\pi}{\frac{2\pi}{3}} = \frac{3}{2} \\
 & \sin\left(2.094395102x + \pi\right) - 2 \\
 & \quad \cancel{\frac{2\pi}{3}} \\
 & y = 3 \sin\left(\frac{2}{3}x + \pi\right) - 2
 \end{aligned}$$

A buoy floating in the ocean is bobbing in simple harmonic motion with amplitude 3 ft and period 8 seconds. Its displacement d from sea level at time $t=0$ seconds is -3 ft, and initially it moves upward. (Note that upward is the positive direction.)

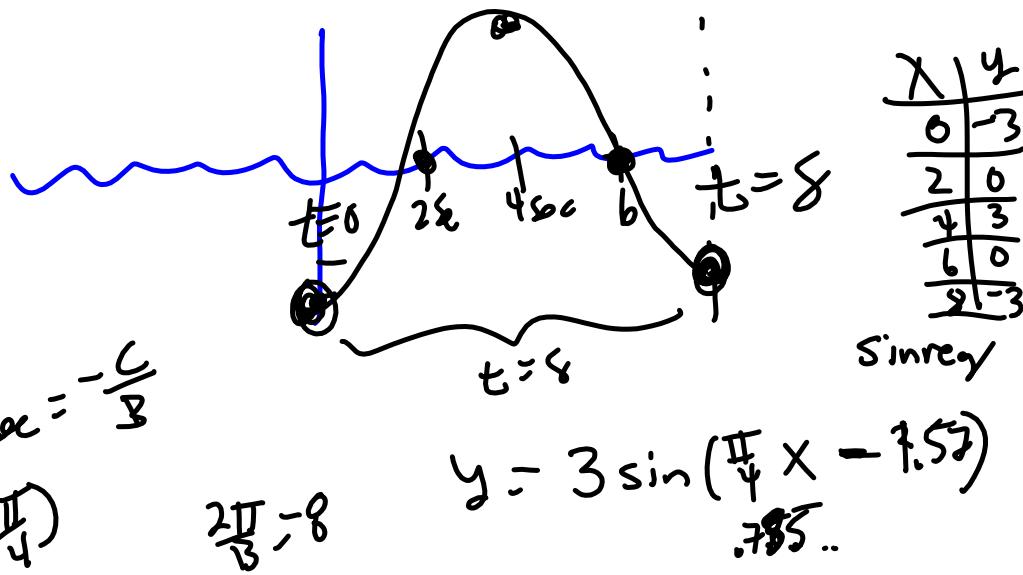
Give the equation modeling the displacement d as a function of time t .



$$y = 3 \sin\left(\frac{\pi}{4}x - \frac{\pi}{2}\right) + 0$$

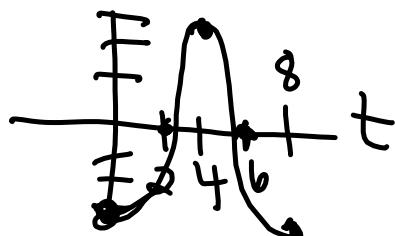
A buoy floating in the ocean is bobbing in simple harmonic motion with amplitude 3 ft and period 8 seconds. Its displacement d from sea level at time $t = 0$ seconds is -3 ft, and initially it moves upward. (Note that upward is the positive direction.)

Give the equation modeling the displacement d as a function of time t .



A buoy floating in the ocean is bobbing in simple harmonic motion with amplitude 3 ft and period 8 seconds. Its displacement d from sea level at time $t=0$ seconds is -3 ft, and initially it moves upward. (Note that upward is the positive direction.)

Give the equation modeling the displacement d as a function of time t .



$$A = 3 \rightarrow$$

$$\text{Period} = 8 = \frac{2\pi}{B} \rightarrow \frac{3\pi}{4}$$

$$t = 0$$

$$d = -3$$

L1	L2
0	-3
2	0
4	3
6	0
8	-3

$$y = a \sin(bx + c) + d$$

$$a = 3 \leftarrow$$

$$b = .7853981634 \leftarrow$$

$$c = -1.570796327 \leftarrow$$

$$d = 2.7e-13 \rightarrow$$

$$\begin{matrix} \pi/4 \\ -\pi/2 \\ 0 \end{matrix}$$

$$d = 3 \sin\left(\frac{\pi}{4}x - \frac{\pi}{2}\right)$$

$$d = -3 \cos\left(\frac{\pi}{4}x\right) \quad \frac{2\pi}{B} = 8$$

$$y = 2 \tan \frac{x}{2}$$

$$y = \sec\left(x + \frac{\pi}{4}\right)$$

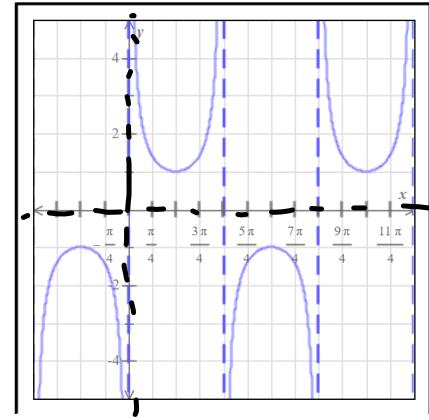
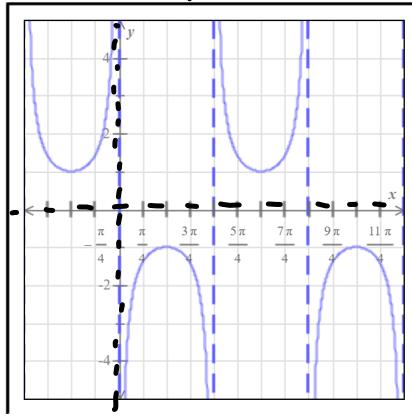
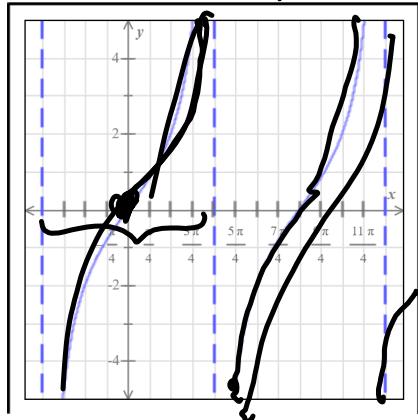
tan / cu

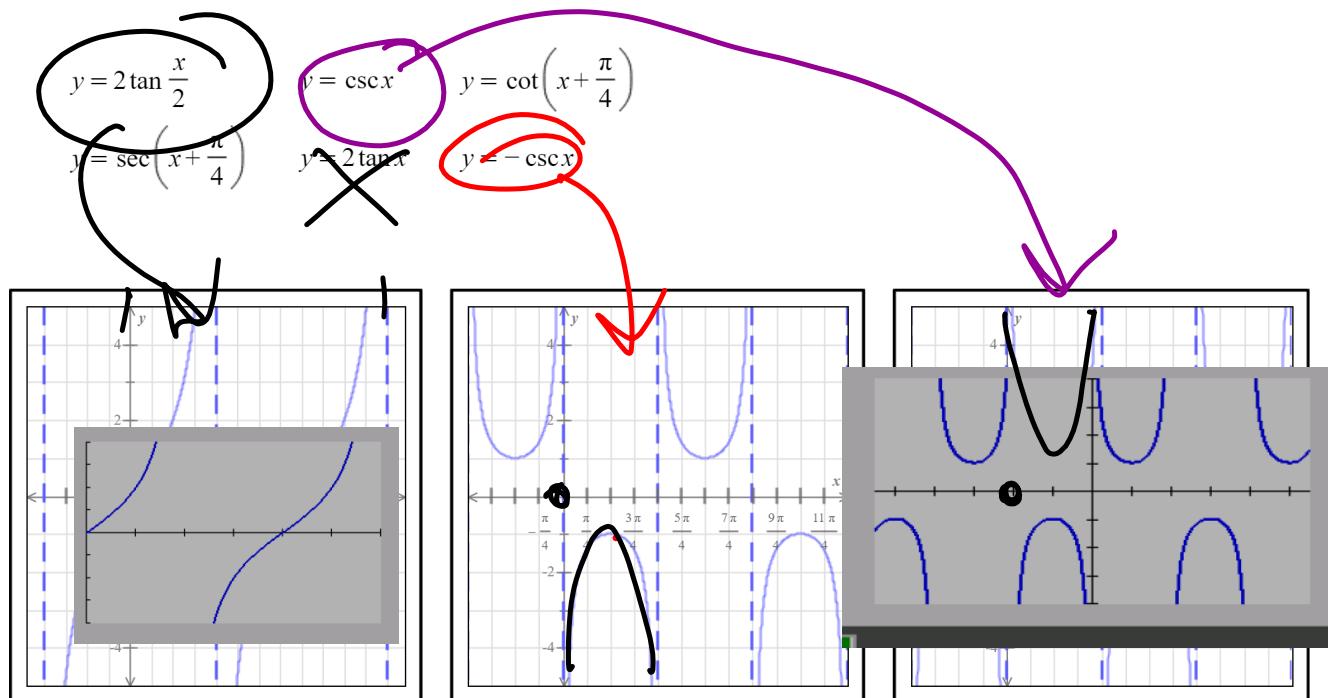
$$y = \csc x$$

~~$$y = \cot\left(x + \frac{\pi}{4}\right)$$~~

$$y = -\csc x$$

sec / csc



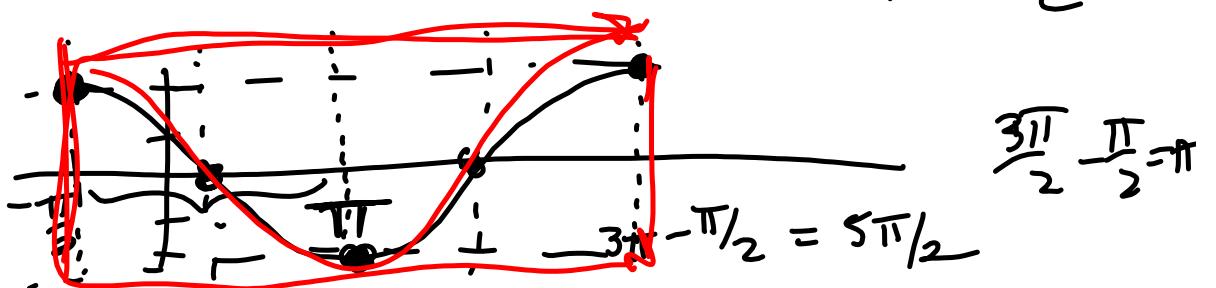


Graph the function $y = 2 \cos\left(\frac{2}{3}x + \frac{\pi}{3}\right)$.

③ Amp: 2

② Period $\frac{2\pi}{2/3} = 3\pi$

① P.S. $\frac{2}{3}x + \frac{\pi}{3} = 0 \rightarrow x = -\frac{\pi}{2}$

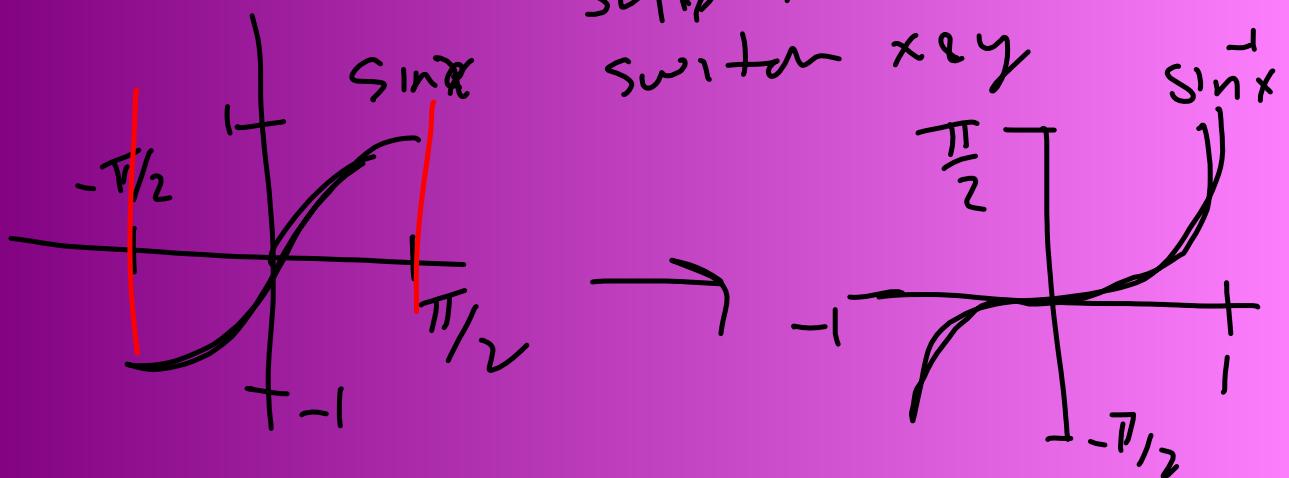


Inverse Trig Functions

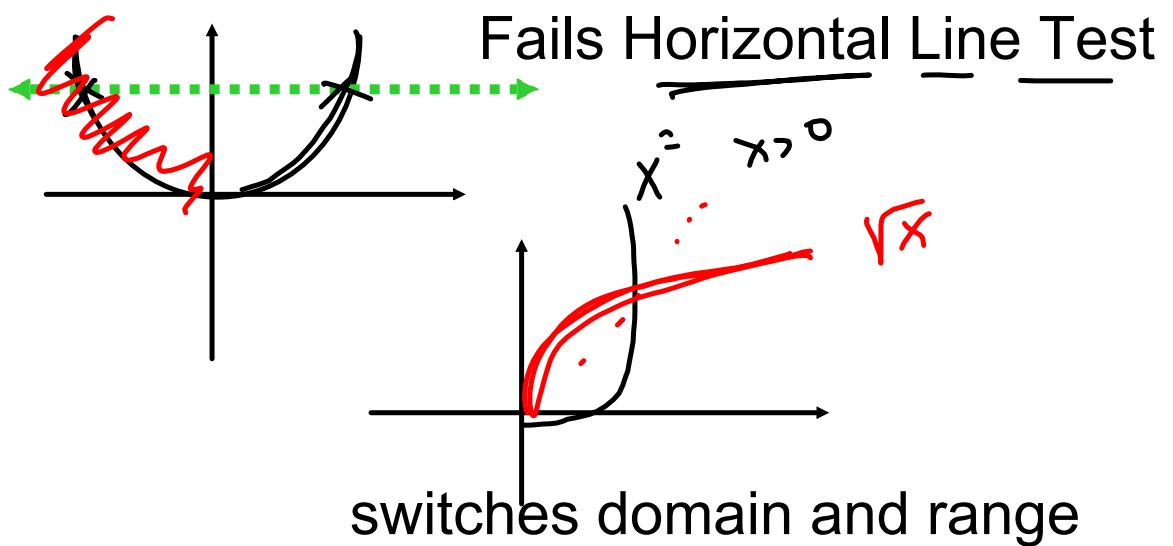
$$y = \sin x$$
$$\sin^{-1}(y) = x$$

Solve for x

switch x & y



Inverse Functions



domain: $[-90^\circ, 90^\circ]$

range: $[-1, 1]$

sine

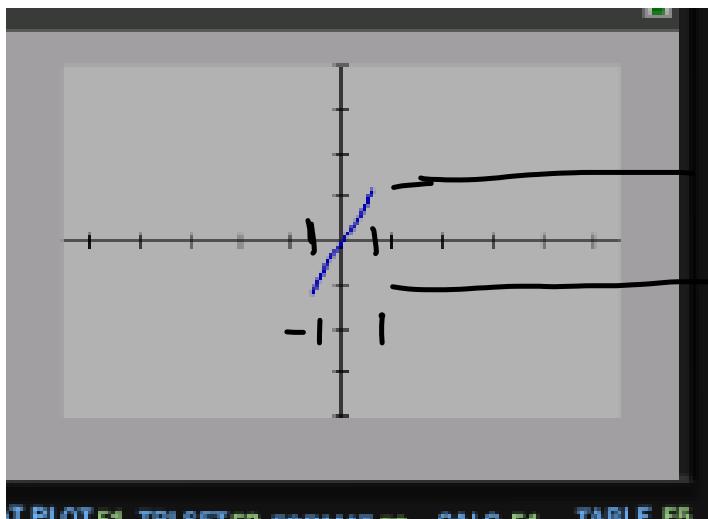
$-\pi/2$

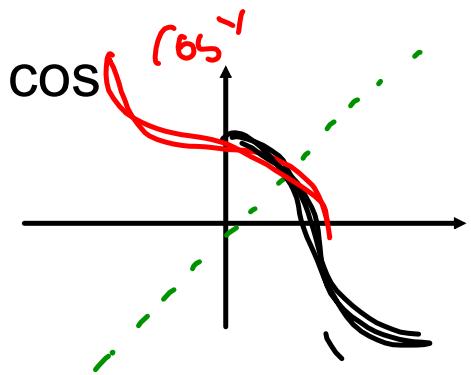
$\pi/2$

pass HLT ✓

$\sin^{-1} x = \arcsin x$ domain $[-1, 1]$

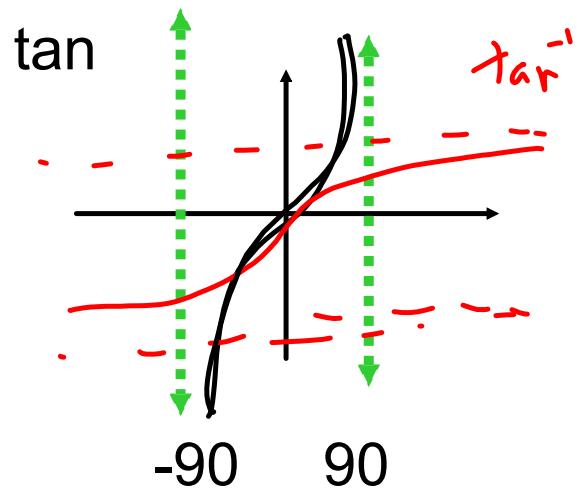
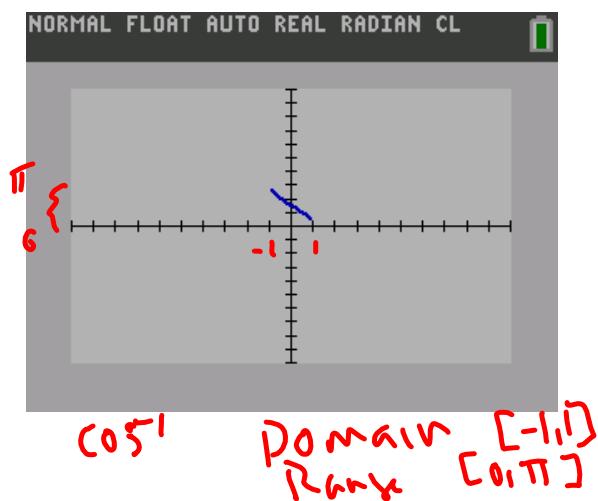
range: $[-90^\circ, 90^\circ]$





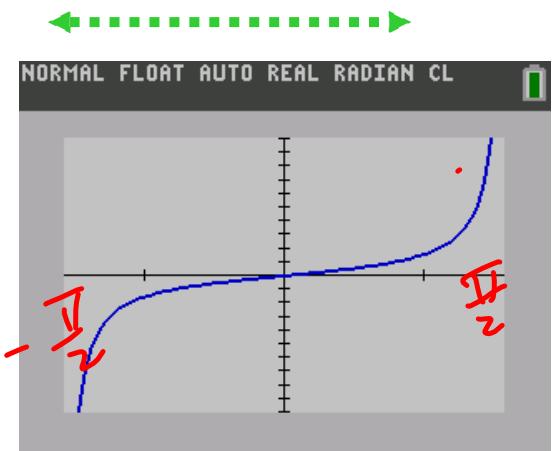
\cos^{-1}
domain $[0, 180^\circ]$

range $[-1, 1]$

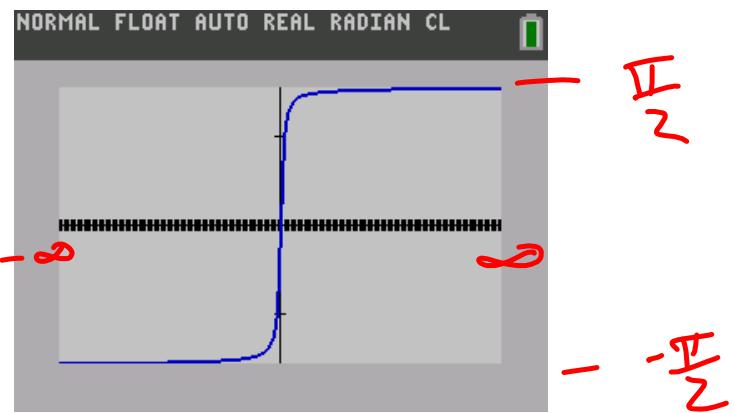


\tan^{-1}
domain $[-90^\circ, 90^\circ]$

range: $(-\infty, \infty)$



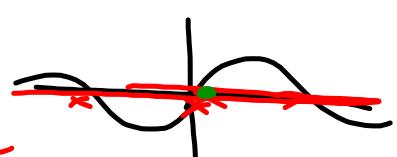
\tan^{-1}
Domain: $(-\infty, \infty)$
Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$
 $[-90^\circ, 90^\circ]$



Input \Rightarrow Value/ y

$$\sin x = y$$

Inverse or $x = \sin^{-1} y$



$\sin x = \frac{1}{2}$

$\pi/6$.5235987756	radian
$\sin^{-1}(.5)$.5235987756	values
$\sin^4(.5)$.	30° days

$x = \sin^{-1}(y)$

$\sin\left(\frac{\pi}{6}\right) = .5$

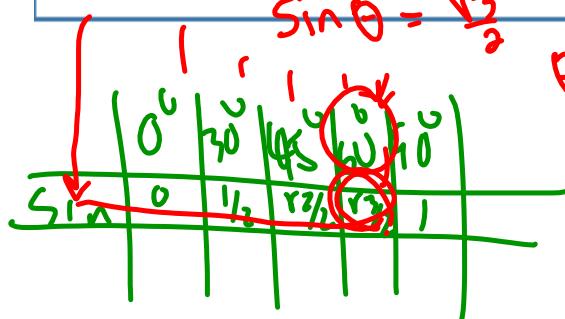
$\sin^{-1}(.5) = \frac{\pi}{6} = 30^\circ$

$\sin^{-1}(.5) = \theta$
 $\frac{1}{2} = \sin \theta$

Values of inverse trigonometric functions

Find the exact value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$. $\theta =$

Write your answer in radians in terms of π .



Degrees
 $60^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{3}$

Radians

$$1.04 / \pi = .3333$$

Math IDtr $\rightarrow \frac{1}{3}$

$$1.04 = \frac{1}{3}\pi \approx \pi/3$$

Values of inverse trigonometric functions

Find the exact value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$. $\angle \theta$

Write your answer in radians in terms of π .

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$60^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{3}$$

in degree

$\sin^{-1}(\sqrt{3}/2)$

60

Composition of a trigonometric function with the inverse of another trigonometric function:
Problem type 1

Find the exact value of $\csc\left(\tan^{-1}\left(\frac{12}{5}\right)\right)$.

$i/\sin(\tan^{-1}(12/5))$	1.083333333
Ans►Frac	$13/12$

Composition of a trigonometric function with the inverse of another trigonometric function:
Problem type 1

Find the exact value of $\csc(\tan^{-1}(\frac{12}{5}))$.

$$\tan^{-1}(\frac{12}{5}) = \theta$$

[Explain](#)

$$1/\sin(\tan^{-1}(12/5)) = \text{hyp/opp} = 13/12$$

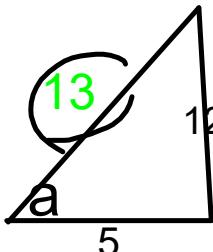
1/sin(tan⁻¹(12/5))
Ans>Frac
.....
13/12

$$\tan \theta = \frac{12}{5}$$

angle = $\tan^{-1}(12/5)$

$$\tan(\text{angle}) = 12/5$$

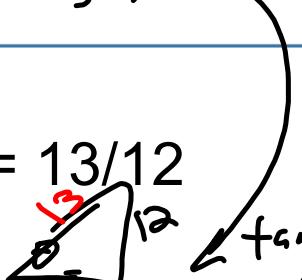
SOH
CAH
TOA



$\tan = \text{opp}/\text{adj}$

$$\frac{1}{\sin \theta} = \frac{\text{hyp}}{\text{opp}} = \frac{13}{12}$$

$$\tan^{-1}(\frac{12}{5}) = \theta$$

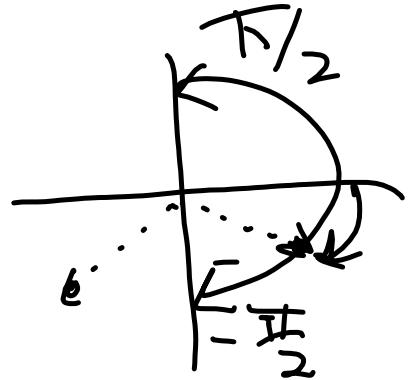


$$\tan \theta = \frac{12}{5}$$

Find the exact value of $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$. $= -\frac{\pi}{4}$

Write your answer in radians in terms of π .

$\sin^{-1}(-\sqrt{2}/2)$	$-\frac{\pi}{4}$
$-.7853981634$	
Ans/π	$-.25$
<input type="checkbox"/>	



Rewrite $\cos(\sin^{-1} 3w)$ as an algebraic expression in w .

$$\begin{array}{c} \Rightarrow \\ \theta \end{array}$$

$$\theta = \sin^{-1}(3w)$$

$$\sin \theta = 3w$$

SOH

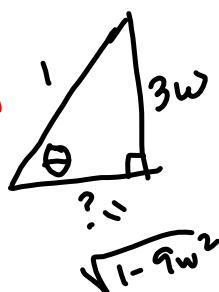
$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{3w}{1}$$

$$\cos \theta$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

CAH

$$\frac{\sqrt{1-9w^2}}{1}$$



$$\begin{aligned} ?^2 + (3w)^2 &= 1 \\ ?^2 &= 1 - 9w^2 \\ ? &= \sqrt{1 - 9w^2} \end{aligned}$$



QUESTION



Rewrite $\cos(\sin^{-1} 3w)$ as an algebraic expression in w .

$$\theta = \sin^{-1}(3w)$$
$$\sin(\theta) = \frac{3w}{1} = \frac{\text{opp}}{\text{hyp}}$$



$$?^2 + (3w)^2 = 1$$

$$\sqrt{1-9w^2}$$

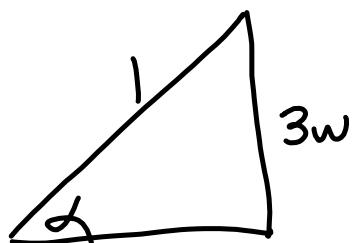
$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} \rightarrow \frac{\sqrt{1-9w^2}}{1}$$



QUESTION

Rewrite $\cos(\underbrace{\sin^{-1} 3w}_\alpha)$ as an algebraic expression in w .

$$\sin^{-1} 3w = \alpha$$



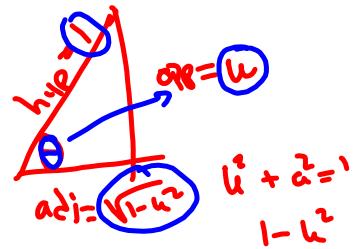
$$\frac{\text{opp}}{\text{hyp}} = \frac{3w}{1} = \sin \alpha$$

$$\begin{aligned}\cos \alpha &= \frac{\text{adj}}{\text{hyp}} \\ &= \sqrt{1 - 9w^2}\end{aligned}$$

$$\begin{aligned} ?^2 + (3w)^2 &= 1^2 \\ ?^2 &= 1 - 9w^2 \\ ? &= \sqrt{1 - 9w^2}\end{aligned}$$

Rewrite $\cot(\sin^{-1} u)$ as an algebraic expression in u .

$$\begin{aligned}\sin^{-1} u &= \theta \\ \sin \theta &= \frac{u}{1} \\ \text{SOH}\end{aligned}$$



$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{\text{opp}}{\text{adj}}} = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{1-u^2}}{u}$$

$\theta = \sin^{-1} u$

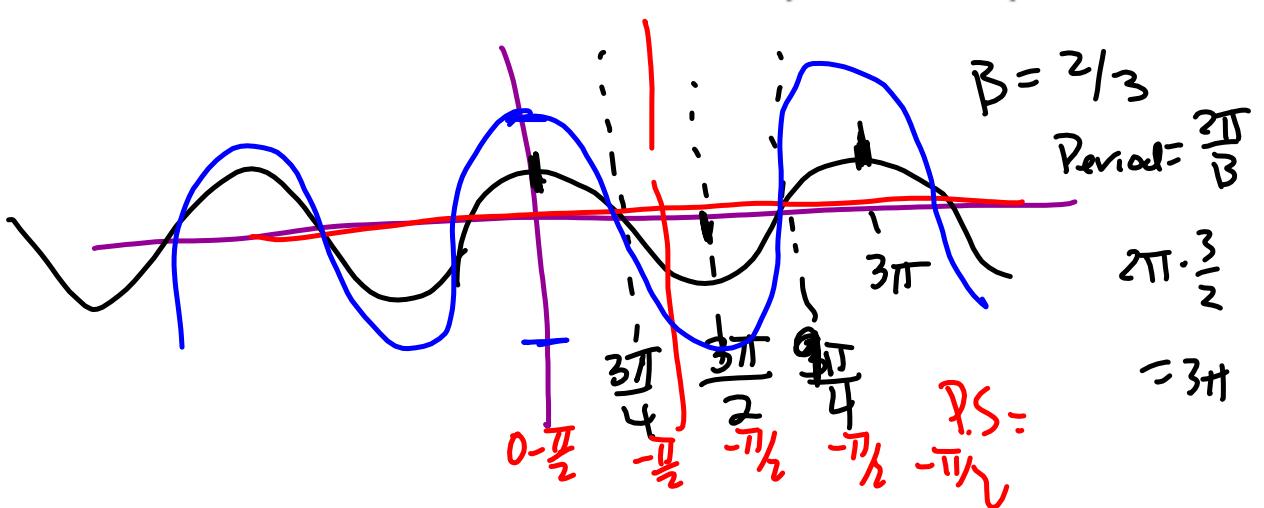
Rewrite $\cot(\sin^{-1} u)$ as an algebraic expression in u .

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{adj}}{\text{opp}}$$

$$\cot(\sin^{-1} u) = \frac{\sqrt{1-u^2}}{u}$$

$$\begin{aligned} ?^2 + u^2 &= 1 \\ ? &= \sqrt{1-u^2} \end{aligned}$$

Graph the function $y = 2 \cos\left(\frac{2}{3}x + \frac{\pi}{3}\right)$.



Values of inverse trigonometric functions

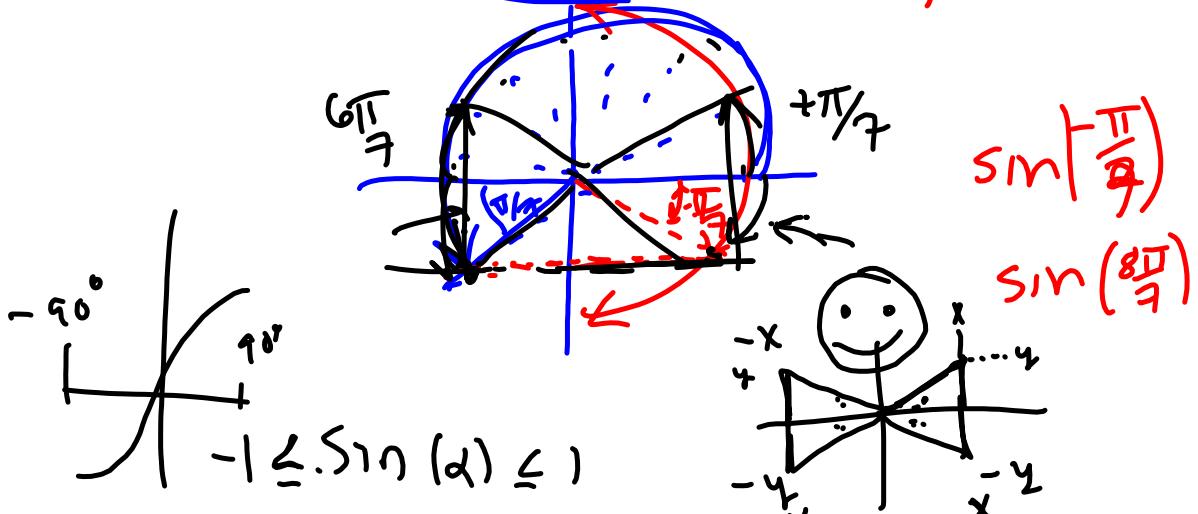
Find the exact value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$. $\approx 60^\circ$ In degree
Write your answer in radians in terms of π .

$\sin(60^\circ) = \frac{\sqrt{3}}{2}$
Radian

$$\frac{60}{180} \frac{\pi}{\pi} = \frac{\pi}{3}$$

$\sin^{-1}(\sqrt{3}/2)$ 1.047197551
 $\pi/3$ 1.047197551

$$\sin^{-1} \left(\sin \left(\frac{8\pi}{7} \right) \right) = -\frac{\pi}{7}$$



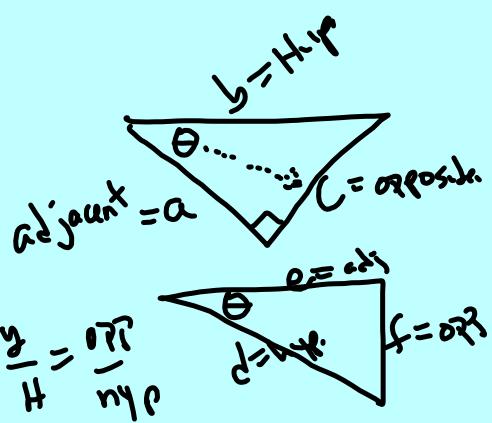
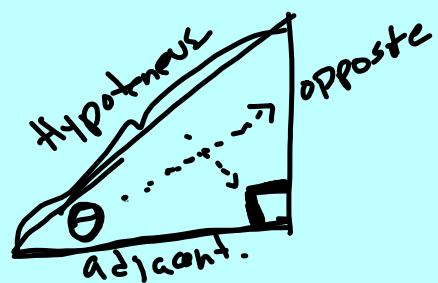
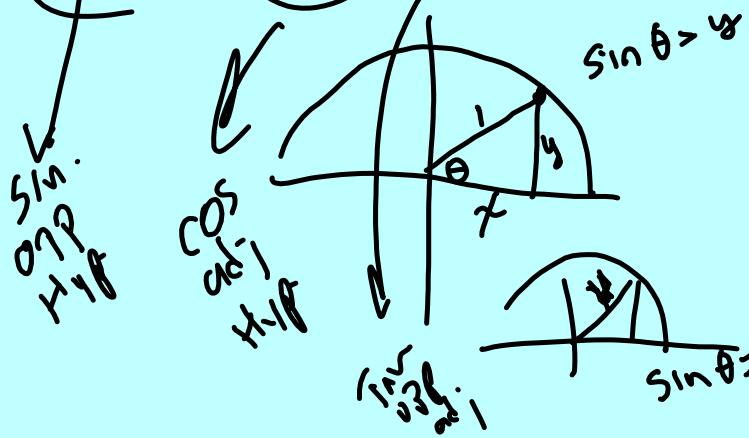
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sin-1(sin(8π/7))
.....-.4487989505.
Ans/π
.....-.1428571429
Ans▶Frac
.....-1/7.

```

Right Triangle Trig

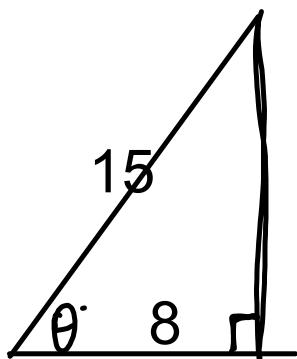
SOH CAH TOA



$$\sin \theta = \frac{y}{H} = \frac{opposite}{hypotenuse}$$

$$\cos \theta = \frac{x}{H} = \frac{adjacent}{hypotenuse}$$

Right Triangle Trig



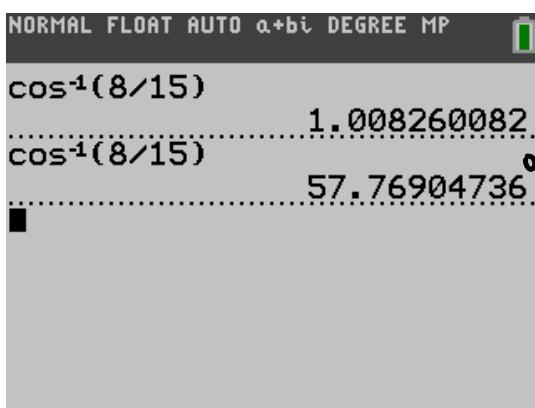
17 = hypotenuse

8 = adjacent

~~SOF~~ CATH ~~TAN~~

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{8}{17}$$

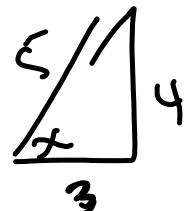
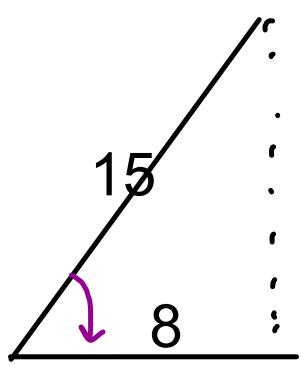
$$\theta = \cos^{-1}\left(\frac{8}{17}\right)$$



Right Triangle Trig

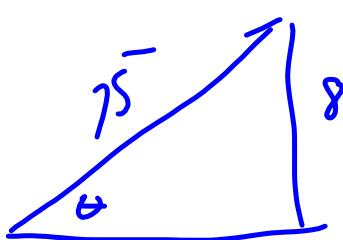
SOH. CAH. TOA

$$\frac{\text{adj}}{\text{hyp}} = 8/15 = \cos(a)$$

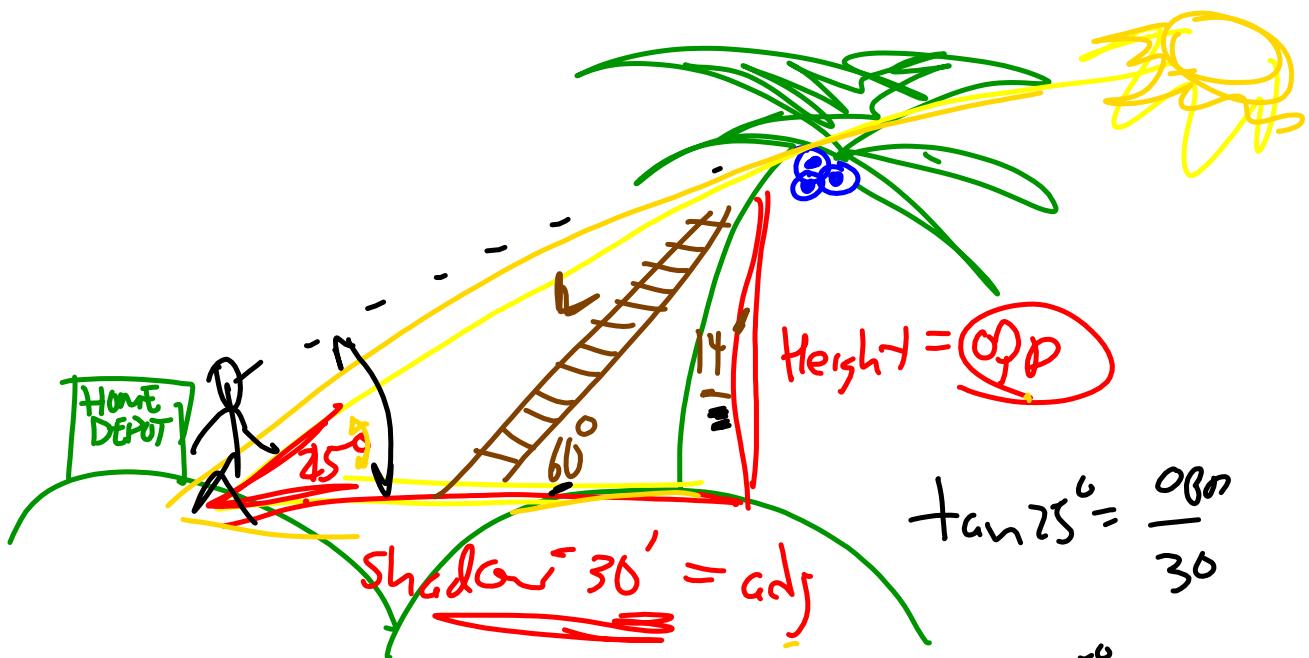


$$a = \cos^{-1}(8/15) =$$

$\cos^{-1}(8/15)$ 57.76904736



$$\tan^{-1}\left(\frac{4}{3}\right) = 53^\circ$$



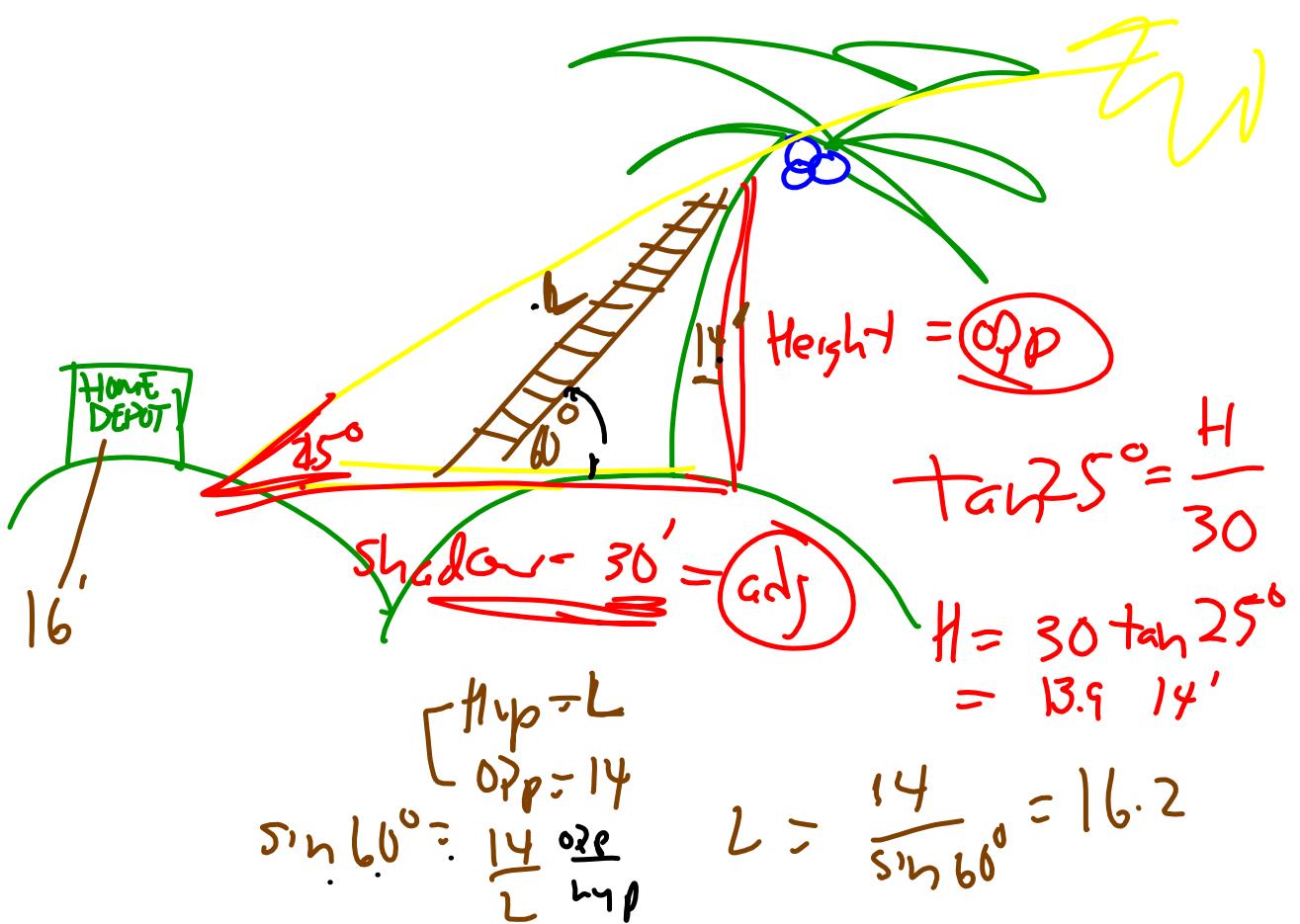
$$\tan 25^\circ = \frac{OPD}{30}$$

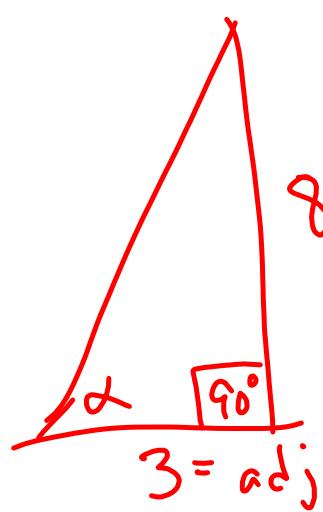
$$14 = .30 \tan 25^\circ = OPD$$

$$\begin{bmatrix} \text{hyp} = L \\ OPD = 14 \end{bmatrix}$$

$$\sin 60^\circ = \frac{14}{L}$$

$$L = \frac{14}{\sin 60^\circ} = 16.2$$





$$\tan \alpha = \frac{\text{opp}}{\text{adj}}$$

$$8 = \text{opp} \quad \tan \alpha = \frac{8}{3}$$

$$\alpha = \tan^{-1}\left(\frac{8}{3}\right)$$

IDENTITIES

$$\sec \theta \equiv \frac{1}{\cos \theta}$$
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

✓ Condition

$$\begin{array}{r} x+1=2 \\ -1 \quad -1 \\ \hline x=1 \end{array}$$

✓ Contradiction

$$\begin{array}{r} x+1=x+2 \\ -x \quad -x \\ \hline 1 \neq 2 \end{array}$$

✓ Identity

$$\begin{array}{r} x+1=2x+1-x \\ \underline{-1} \quad \underline{-1} \\ 0=0 \end{array}$$

Study of Functions

1. Linear, Quad, Cubic...Polynomial
2. Rational Functions
3. Exponential
4. Inverse/Transformations/Composite
5. Logs
6. Trig Functions

Function: Job

Domain: x: input

Range: y : Output

Max/Min/Increasing/Decreasing

Asymptotes/End Behaviour

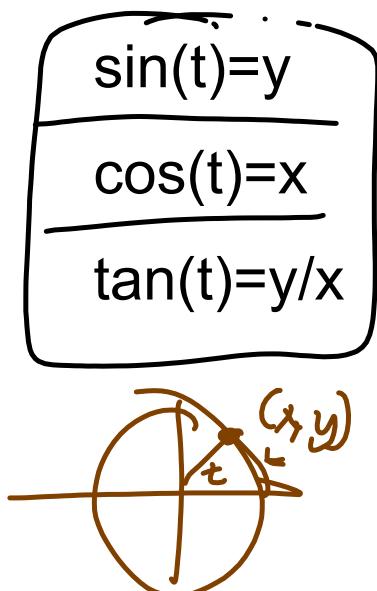
Geography of the Graph

Properties:

Log Properties

Trig Properties....

Reciprocal Identities



$$1/y = 1/\sin(t) = \csc(t)$$

$$1/x = 1/\cos(t) = \sec(t)$$

$$x/y = 1/\tan(t) = \cot(t)$$

$$1/\csc(t) = \sin(t)$$

$$1/\sec(t) = \cos(t)$$

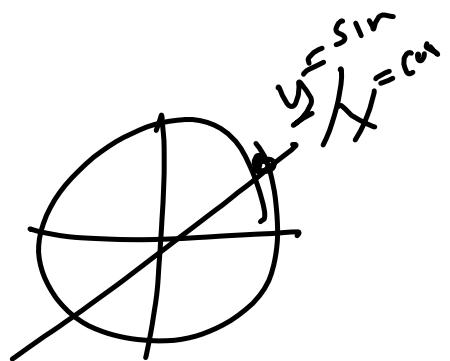
$$1/\cot(t) = \tan(t)$$

Quotient Identities

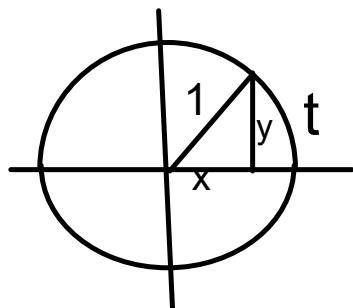


$$\tan(\theta) = \sin(\theta) / \cos(\theta)$$

$$\cot(\theta) = \cos(\theta) / \sin(\theta)$$



Pythagorean Identities



$$x^2 + y^2 = 1$$

★ $\boxed{\cos^2(t) + \sin^2(t) = 1}$

$$\frac{\cos^2(t)}{\cos^2(t)} + \frac{\sin^2(t)}{\cos^2(t)} = \frac{1}{\cos^2(t)}$$

★ $\boxed{1 + \tan^2(t) = \sec^2(t)}$

$$\cot^2(t) + 1 = \csc^2(t)$$

.....57.76904736.
..... $(\sin(4))^2 + (\cos(4))^2$1.
..... $(\sin(9))^2 + (\cos(9))^2$1.
.....

~~Notation~~ ^{Important} $(\sin(x))^2 \equiv \sin^2 x$ ~~$\sin x^3 = \sin x \cdot \sin x \cdot \sin x$~~ ^{NOT}

$$\begin{aligned} \sin^3(x) &= (\sin x)^3 \\ &= \underline{\sin x} \underline{\sin x} \underline{\sin x} \end{aligned}$$

$\sin^{-1}(x) \equiv \text{inverse } \underline{\sin}$

$$\begin{aligned} (\sin(x))^{-1} &= \frac{1}{\sin^2(x)} = \csc^2 x \\ &= \frac{1}{\sin x} \end{aligned}$$

$$\sin^{-2}(x) = \frac{1}{\sin^2(x)} = \csc^2(x)$$

Simplifying trigonometric expressions

Simplify.

$$\cancel{(\sec x - \sin^2 x \sec x)}$$

Use algebra and the fundamental trigonometric identities.

Your answer should be a number or use a single trigonometric function.

C4X

$$\begin{aligned} & \sec x (1 - \sin^2 x) \\ & (\sin^2 x + \cos^2 x - \sin^2 x) \\ & \sec x \cdot \cos^2 x \end{aligned}$$

$$\frac{1}{\cos x} \cdot \cos x = \frac{\cancel{\sec x} \cdot \cos x}{\cancel{\sec x}} = \cos x$$

Simplifying trigonometric expressions

Simplify.

$$\sec x - \sin^2 x \sec x = \cos x$$

Algebra = factors

Use algebra and the fundamental trigonometric identities.

Your answer should be a number or use a single trigonometric function.

$$\sec(x) (1 - \sin^2 x)$$

Pythagorean Identity

$$\sec(x) [\cancel{\sin^2 x + \cos^2 x} - \cancel{\sin^2 x}]$$

Algebra

$$\sec x \cdot \cos^2 x$$

Reciprocal

$$\frac{1}{\cos x} \cdot \cos^2 x$$

Algebra

$$\cos x$$

$$\frac{1}{x} \cdot \frac{x}{1} \therefore x$$

Simplifying trigonometric expressions

Simplify.

$$y = \sec x - \sin^2 x \sec x$$

$$= \cos x = y$$

Use algebra and the fundamental trigonometric identities.
Your answer should be a number or use a single trigonometric function.

$$\sec x (1 - \sin^2 x) \quad \text{factoring}$$

$$\sec x (\cancel{\sin^2 x} + \cos^2 x - \cancel{\sin^2 x}) \quad \text{pythagorean id}$$

$$\sec x \cos^2 x \quad \text{algebra}$$

$$\frac{1}{\cancel{\cos x}} \cos^2 x \quad \text{reciprocal}$$

$$\cos x \quad \text{algebra}$$

Reciprocal identities:

$$\begin{aligned}\sin u &= \frac{1}{\csc u} & \cos u &= \frac{1}{\sec u} & \tan u &= \frac{1}{\cot u} \\ \csc u &= \frac{1}{\sin u} & \sec u &= \frac{1}{\cos u} & \cot u &= \frac{1}{\tan u}\end{aligned}$$

Quotient identities:

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean identities:

$$\sin^2 u + \cos^2 u = 1$$

$$\tan^2 u + 1 = \sec^2 u$$

$$\cot^2 u + 1 = \csc^2 u$$

Odd/Even function identities:

$$\sin(-u) = -\sin(u) \quad \cos(-u) = \cos(u) \quad \tan(-u) = -\tan(u)$$

$$\csc(-u) = -\csc(u) \quad \sec(-u) = \sec(u) \quad \cot(-u) = -\cot(u)$$

Odd

$$f(-x) = -f(x)$$

Ex $y = 4x^3$
 $y(-x) = 4(-x)^3 = -4x^3$

Even
 $f(-x) = f(x)$

Ex $y = 7x^2$
 $y(-x) = 7(-x)^2 = 7x^2$

$\sec(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos(x)} = \sec \text{ even}$

$\csc(-x) = \frac{1}{\sin(-x)} = \frac{1}{-\sin(x)} = -\csc \text{ odd}$

$$\begin{aligned}f(x) &= 5x^3 + 2x^2 \leftarrow \\f(-x) &= 5(-x)^3 + 2(-x)^2 \\&= -5x^3 + 2x^2 \leftarrow \\&\neq f(x) \quad \neq -f(x)\end{aligned}$$

sin

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

cos

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

$$\sin(-x) = -\sin x \quad \text{ODD}$$

$$*\cos(-x) = \cos x \quad \text{EVEN}$$

$$\tan(-x) = -\tan x \quad \text{ODD}$$

$$**\sec(-x) = \sec(x) \quad \text{EVEN}$$

Verifying a trigonometric identity

Complete the proof of the identity by choosing the Rule that justifies each step.

$$(1 + \cot^2 x) \tan x = \csc x \sec x$$

$\csc^2 x \cdot \tan x$ ←
Pythagorean ID

$\csc x \cdot \csc x \cdot \tan x$ → Reciprocal
 $(\csc x \cdot \sec x)$

$\csc x \cdot \frac{1}{\sin x} \cdot \tan x$ ← Quotient
 $\csc x \cdot \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x}$ Alg.
 $\csc x \cdot \frac{1}{\cos x}$

Verifying a trigonometric identity

Complete the proof of the identity by choosing the Rule that justifies each step.

$$(1 + \cot^2 x) \tan x = \csc x \sec x$$

Pythagorean ID
 $\csc^2 x \cdot \tan x$

Quotient

$$\csc^2 x \cdot \frac{\sin x}{\cos x}$$

Reciprocal

$$\csc x \cdot \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x}$$

Algebra

$$\csc x \cdot \frac{1}{\cos x}$$

Reciprocal

$$(\sec x - \csc x)$$

Verifying a trigonometric identity

Complete the proof of the identity by choosing the Rule that justifies each step.

$$(1 + \cot^2 x) \tan x = \csc x \sec x$$

$$\csc^2 x \tan x = \text{Pythagorean}$$

$$\csc^2 x \frac{\sin x}{\cos x} = \text{Quotient}$$

$$\cancel{\csc^2 x} \frac{\sin x}{\cancel{\cos x}} \sec x = \text{Reciprocal}$$

$$\csc x \cancel{\csc x} \frac{\sin x}{\sec x} = \text{algebra}$$

$$\csc x \frac{1}{\sin x} \sec x = \text{Reciprocal}$$

$$\frac{1}{\sin x}$$

$$\csc x \frac{\sec x}{\sin x} = \text{algebra}$$

verify

Statement	Rule
$(1 - \sin^2 x) \csc x$	
$= \cos^2 x \csc x$	Pythagorean
$= \cos^2 x \left(\frac{1}{\sin x} \right)$	Reciprocal
$= \cos x \left(\frac{\cos x}{\sin x} \right)$	Algebra
$= \cos x \cot x$	Quotient

control

Ctrl C

Paste ✓

Prove the identity.

$$(1 - \cos^2 x) \cot^2 x = \cos^2 x$$

Note that each Statement must be based on a Rule chosen from the Rule menu. To see a detail select the corresponding question mark.

Statement	Rule
$(1 - \cos^2 x) \cot^2 x$	
$= (\sin^2 x) \cot^2 x$	Pythagorean
$= (\sin^2 x) \frac{\cos^2 x}{\sin^2 x}$	Quotient
$= \cos^2 x$	Algebra

—

Thank you, your proof is complete.

$= \frac{1}{\sin x} - \sin x$	Reciprocal
$= \frac{1 - \sin^2 x}{\sin x}$	Algebra
$= \frac{\cos^2 x}{\sin x}$	Pythagorean
$= \cos x \left(\frac{\cos x}{\sin x} \right)$	Algebra
$= \cot x \cos x$	Quotient

Statement	Rule
$= \frac{1}{\cos x} - \frac{\cos x}{\cos x}$	Algebra
$= \frac{1 - \cos^2 x}{\cos x}$	Algebra
$= \frac{\sin^2 x}{\cos x}$	Pythagorean
$= \sin x \frac{\sin x}{\cos x}$	Algebra
$= \sin x \tan x$	Quotient
Thank you, your proof is complete.	

Statement	Rule
$\sec^2 x (1 - \sin^2 x)$	
$= \sec^2 x (\cos^2 x)$	Pythagorean
$= \frac{1}{\cos^2 x} (\cos^2 x)$	Reciprocal
Click here to validate this line.	

<input type="checkbox"/>	<input type="checkbox"/> <input checked="" type="checkbox"/>	π
<input type="checkbox"/> \cos	<input type="checkbox"/> \sin	<input type="checkbox"/> \tan
<input type="checkbox"/> \cot	<input type="checkbox"/> \sec	<input type="checkbox"/> \csc
(<input type="checkbox"/>)		
x	↶	?

Statement	Rule
$\sec^2 x (1 - \sin^2 x)$	
$= \sec^2 x (\cos^2 x)$	Pythagorean
$= \frac{1}{\cos^2 x} (\cos^2 x)$	Reciprocal
$= 1$	Algebra

$$\sec x - \sin x \tan x = \cos x$$

Note that each Statement must be based on a Rule chosen from the Rule menu. To see a detailed explanation select the corresponding question mark.

Statement	Rule
$\sec x - \sin x \tan x$	
$= \frac{1}{\cos x} - \sin x \tan x$	Reciprocal
$= \frac{1}{\cos x} - \sin x \frac{\sin x}{\cos x}$	Quotient
$= \frac{1 - \sin^2 x}{\cos x}$	Algebra
$= \frac{\cos^2 x}{\cos x}$	Pythagorean
$= \cos x$	Rule ?

Statement	Rule
$\frac{\sin x}{1 - \cos x}$	
$= \frac{\sin x}{1 - \cos x} \frac{1 + \cos x}{1 + \cos x}$	Algebra
$= \sin x \frac{1 + \cos x}{1 - \cos^2 x}$	Algebra
$= \sin x \frac{1 + \cos x}{\sin^2 x}$	Pythagorean
$= \frac{1 + \cos x}{\sin x}$	Algebra

Statement	Rule
$= \sin x \frac{1 + \cos x}{\sin^2 x}$	Pythagorean
$= \frac{1 + \cos x}{\sin x}$	Algebra
$= \frac{1}{\sin x} + \frac{\cos x}{\sin x}$	Algebra
$= \csc x + \frac{\cos x}{\sin x}$	Reciprocal
$= \csc x + \cot x$	Quotient
Thank you, your proof is complete.	

odd

$$f(-x) = -f(x) \text{ ex: } y = x^3$$

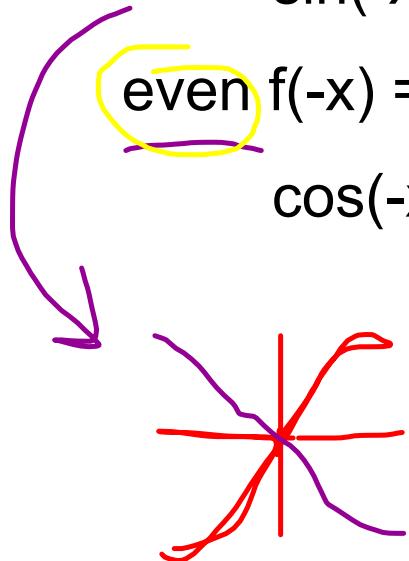
$$\sin(-x) = -\sin x$$

$$(-x)^3 = -x^3$$

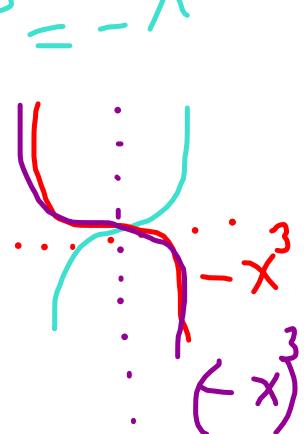
even

$$f(-x) = f(x) \text{ ex: } y = x^2$$

$$\cos(-x) = \cos x$$



$$\sin(-x) - \sin(x)$$



sum and difference identities

$$\sin(\alpha + \beta) = \underline{\sin \alpha \cos \beta} + \underline{\cos \alpha \sin \beta}$$

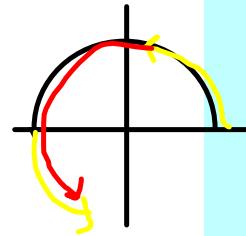
$$\sin(\alpha - \beta) = \underline{\sin \alpha \cos \beta} - \underline{\cos \alpha \sin \beta}$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$



$$\sin(2x) = 2\sin x \cos x$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\begin{aligned} \cos(2x) &= \\ &\cos^2 x - \sin^2 x \end{aligned}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Statement	Rule
$\cos\left(x - \frac{\pi}{3}\right) + \sin\left(\frac{\pi}{6} - x\right)$	
$= \cos(x)\cos\left(\frac{\pi}{3}\right) + \sin(x)\sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{6} - x\right)$	Sum and Difference
$= \cos(x)\cos\left(\frac{\pi}{3}\right) + \sin(x)\sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{6}\right)\cos(x) - \cos\left(\frac{\pi}{6}\right)\sin(x)$	Sum and Difference
$= \cos(x).5 + \sin(x)\frac{\sqrt{3}}{2} + .5\cos(x) - \frac{\sqrt{3}}{2}\sin(x)$	Evaluation
$= \cos x$	Algebra

Prove the identity.

$$\frac{\cos(x-y)}{\sin x \cos y} = \cot x + \tan y$$

Super
Ugly ✓

Note that each Statement must be based on a Rule chosen from the Rule menu. To see a detailed corresponding question mark.

Statement	Rule
$\frac{\cos(x-y)}{\sin x \cos y}$	
$= \frac{\cos x \cos y + \sin x \sin y}{\sin x \cos y}$	Sum and Difference
$= \frac{\cos x \cos y}{\sin x \cos y} + \frac{\sin x \sin y}{\sin x \cos y}$	Algebra
$= \frac{\cos x}{\sin x} + \frac{\sin y}{\cos y}$	Algebra
$= \cot x + \tan y$	Quotient

$$\sin\left(\frac{\pi}{3} + x\right) - \cos\left(\frac{\pi}{6} + x\right) = \sin x$$

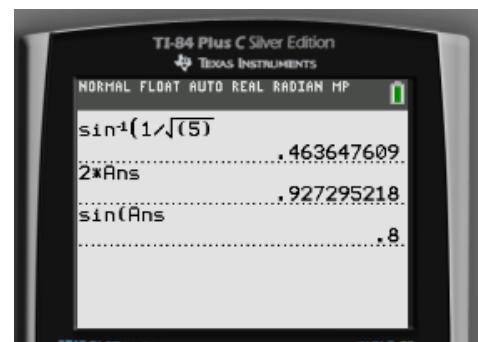
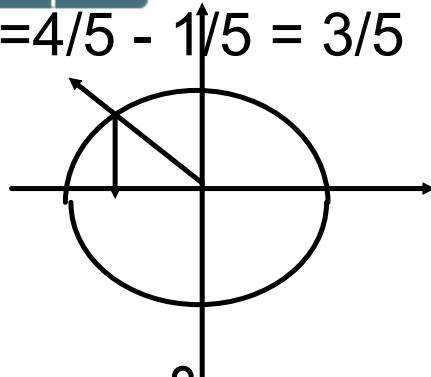
Statement	Rule
$\sin\left(\frac{\pi}{3} + x\right) - \cos\left(\frac{\pi}{6} + x\right)$	
$= \sin\left(\frac{\pi}{3}\right)\cos(x) + \sin(x)\cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{6} + x\right)$	Sum and Difference
$= \sin\left(\frac{\pi}{3}\right)\cos(x) + \sin(x)\cos\left(\frac{\pi}{3}\right) - \left(\cos\left(\frac{\pi}{6}\right)\cos(x) - \sin\left(\frac{\pi}{6}\right)\sin(x)\right)$	Sum and Difference
$= \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$	Evaluation
$= \sin x$	Algebra

Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ if $\sin x = \frac{1}{\sqrt{5}}$ and x terminates in quadrant II.

$$\begin{aligned}\sin 2x &= 2\sin x \cos x = 2 \left(\frac{1}{\sqrt{5}}\right) \left(\frac{2}{\sqrt{5}}\right) \\ \cos 2x &= \cos^2(x) - \sin^2(x) = \frac{4}{5} - \frac{1}{5} = \frac{3}{5} \\ \tan 2x &= \frac{\sin(2x)}{\cos(2x)}\end{aligned}$$

$$2 \left(\frac{1}{\sqrt{5}}\right) \left(\frac{2}{\sqrt{5}}\right)$$

$$\times \quad \swarrow \quad ?$$



$$\begin{array}{c} \sin x \cancel{\cos x} \\ \sin x \frac{1}{\cos x} \quad \text{recip} \\ \frac{\sin x}{\cos x} \quad \text{Alg} \\ + \tan x \quad \text{Quotient} \end{array}$$

$$(1) \sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$(2) \sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$(3) \cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$(4) \cos(u-v) = \cos u \cos v + \sin u \sin v$$

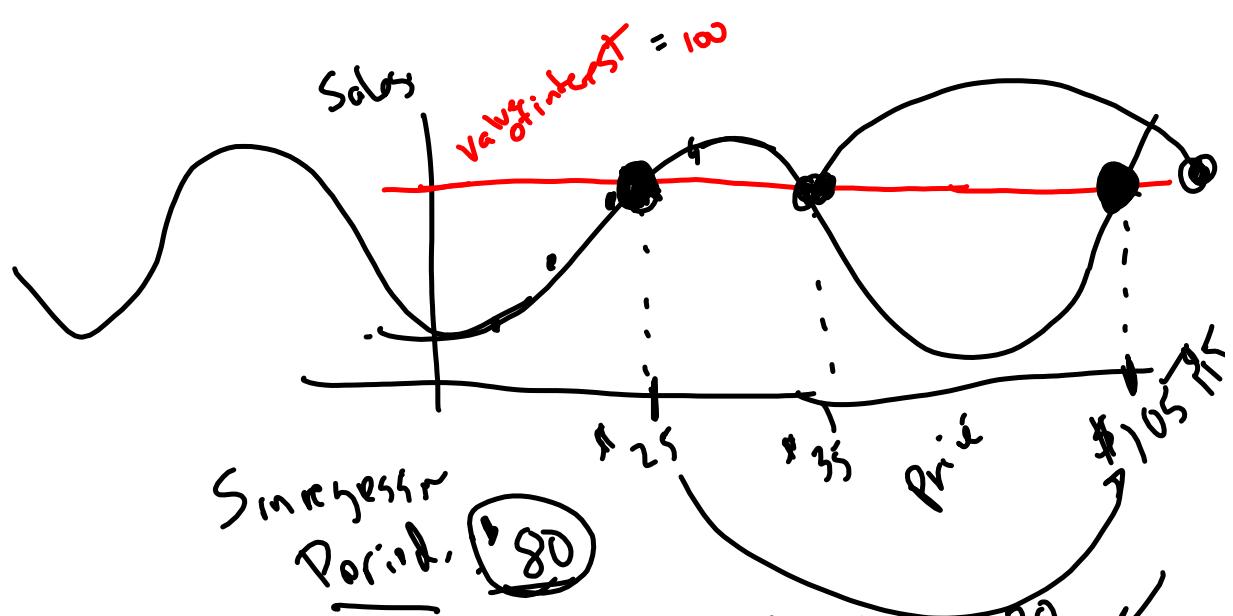
$$\cancel{\sin(2x)} = \sin(x+x) = \sin x \cos x + \sin x \cos x \\ = 2 \sin x \cos x$$

$$\cos(2x) = \cos x \cos x - \sin x \sin x$$

$$= \cos^2 x - \sin^2 x$$

$$\cos(2x) = 1 - \sin^2 x - \sin^2 x = 1 - 2 \sin^2 x$$

Project- Many solutions



$X = 25 + 80n$

To sell 400 items $X = 35 + 80n$

We charge \$25, \$35 and multiples of \$40 and account to six regions