Direct Current Circuits

A direct current is established when the established electric field is constant in a circuit. The potential difference is provided by a battery which uses chemical energy to energize the charges for them to move through the circuit.

As the charges move through the circuit, energy transfers occur, the charges transfer their mechanical energy to internal energy of the circuit and the resistors present in their path. The amount of energy given by the charges is regain by the charges when they enter the battery and where chemical energy is t transfer to the charges a mechanical energy. This transfer of energy at the battery is called a Source of emf. ε The emf is the work done per unit charge. Its units: **J/Q=volt.**

When only looking at the battery we find its potential difference as follows: $\boxed{\Delta V = \varepsilon - Ir}$ r << R
where r = resistance to the current flow in existence inside the battery.
However, the value of this resistance r is smaller than the resistance R of a resistor that can be installed in a circuit connected to the battery. For this type of circuit and applying Ohm's Law

we have: $\underline{\Delta}V = IR$ consequently : $\varepsilon =$ $IR + Ir = I(R + r)$ $I = \dfrac{\varepsilon}{R + r}$

Therefore the Total power output = $I\varepsilon = I^2 R + I^2 r$

Where $I^2 R$ = energy delivered to the load resistor in an amount of time.

          Where $I^2 r$ = (power), energy delivered to the internal resistor in an amount of time.
r = is very small; this energy is absorbed as an internal energy absorption in an amount of time, which increases the temperature of the battery while performing the conversion of chemical energy into mechanical energy delivered to the charges (the electrons).

When we have 2 Resistors with resistance's R_1 and R_2 connected to
a battery, this can be accomplished by connecting the 2 resistors
in two possible ways: (1) Connection in Series.
            (2) Connection in Parallel.

(1) Connection in Series: There is a potential difference across each resistor.
Hence we have: $\underline{\Delta}V_1 = I\,R_1$ and $\underline{\Delta}V_2 = I\,R_2$
        Consequently we will have: $\Delta V = \Delta V_1 + \Delta V_2 = IR_1 + IR_2 = I(R_1 + R_2)$

Since r << R we will only consider the potential difference of the battery as the source of potential difference of the circuit.
The summation of the resistance's can be replaced by one equivalent resistor R_{eq}
Hence $R_{eq} = R_1 + R_2$
When " n " resistors are connected in series we have: $R_{eq} = \sum_{i=1}^{n} R_i$ $\boxed{\Delta V = IR_{eq}}$

(2) Connection in Parallel: There is a potential difference across each resistor, but their value is the same.

$$\Delta V = \Delta V_1 = \Delta V_2$$

The potential difference must be the same across each group of resistors in parallel.

The resistors and the battery are connected on both
sides to a junction. That is, there is a junction on
the left side where 2 resistors and the battery connect
and there is also a junction on the right side.
Consider the left side of the battery to be the positive side (+), hence the current flows from the battery to the left point junction. At this junction the current follows Kirchhoff's Rule: "The sum of the of the currents entering a junction, must equal the sum of the currents leaving that junction."

"Junction: A point in the circuit where the current can split."

This occurrence implies a conservation of __charge__.

Hence the current I is split into two I_1 and I_2 each one flowing through R_1 and R_2 respectively.

Therefore we have: $I = I_1 + I_2$ then $\Delta V_1 = I_1 R_1$ and $\Delta V_2 = I_2 R_2$ and
            Since $\Delta V = \Delta V_1 = \Delta V_2$ then $I_1 = \Delta V / R_1$ and $I_2 = \Delta V / R_2$ and

$$I = I_1 + I_2 = \boxed{\frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} =} \Delta V(\frac{1}{R_1} + \frac{1}{R_2}) = \boxed{\Delta V(\frac{1}{R_{eq}}) = \frac{\Delta V}{R_{eq}}}$$

From Kirchhoff's Rule 2 we have: An algebraic sum of the changes in potential around any closed circuit loop must be zero.
This implies for the algebraic sum some potential differences are positive others are considered negative.

This occurrence implies a conservation of __energy__.

Observe that upon calculating the values of the current I and or the resistance's, power quantities or energy delivered in a given amount of time can be calculated and consequently one can determine the cost of this delivery of energy.

A group of resistors connect in parallel has an equivalent resistance which is less than the smallest value of resistance in the group.

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \bullet \bullet \bullet + \frac{1}{R_n}$$

Note: that many circuits which contain several resistors connected can be reduced to an equivalent single loop circuit making use of successive step by step combinations of groups of resistors in series or in parallel.   This process requires the applications of Kirchhoff's Rules and in each section of the circuit the current must be assigned a direction. This assignment is arbitrary but there assignment is maintained when writing out the equations and doing the calculations, when the
            calculations are completed, the correct direction of the currents are determined.

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Consider the circuit where the battery, a resistor, a capacitor, and a switch are connected in series.

The capacitor is considered to be initially uncharged. When the switch is closed, the battery begins to charge the capacitor. As the capacitor charges, the current is changing The charging process continues until the capacitor is charged to its maximum. $Q = C\varepsilon$ $\varepsilon = \max .voltage$

The current in the circuit is zero when the capacitor reaches its maximum charge.
Hence   q = the charge of the capacitor varies from zero to Q in the following form:

$$q = Q (1 - e^{-t/RC}) \qquad\qquad \tau = RC = timecons \tan t$$

When $t = \tau = RC$ then q = Q (1 - e^{-1}) = Q(0.632) hence q is at 63.2% of the
maximum Q value.

Note: After 10 * (time constant) the capacitor is charged at 99.9% of its total charge.

When the battery is removed from the circuit where only a resistor, a capacitor, and a switch are present. Before the switch closes the potential difference across the charged capacitor is Q/C
When the switch closes, the capacitor discharges thought the circuit
The value of q at time t=0 is equal to the maximum Q, but as t increases, the value of q decreases exponentially.

$$q = Q\, e^{-t/RC} \qquad\qquad When \qquad t = \tau = RC \qquad\qquad q = Q(0.368)$$

That is q is 36.8 % of the value of Q.

The Magnetic Field

Most people know about magnets, There are two types of magnets, the ones found in nature where the material has an element known as magnetita; and the other magnets man made.

Magnetic fields are present in the magnets, they can also exist as induced magnetic fields around lines carrying electric current.

The magnets have 2 poles, one assigned as "N" north pole and the "S" south pole. Magnetic field lines are designed to help us visualize the direction of the magnetic field. The magnetic field lines are closed lines. The direction of the lines in the outside of the magnet is from the positive pole to the negative pole.

The center of the earth is a magnet that behaves as a cylinder and its magnetic south pole is located at the geographic north pole of the planet. The north magnetic pole is located at the geographic south pole. Research is currently being to the reversal or change of the direction of the magnetic field lines, consequently the change of the position of the magnetic poles with respect to the geographic magnetic poles.

The magnetic field **B** is a vector at any location, and it is always tangent to the magnetic field lines.
The earth magnetic field has effects on the planet, from the migration charateristics of birds, to clay figures. The direction of the earth magnetic field and its intensity are set when clay figures cool of after being inside an oven at high temperatures.

Particles in the Electric Field

Particles in the Magnetic Field

Charges can be inserted into the electric field with velocity **v** equals to or different than zero.

When electric charges q are inserted into the magnetic field, they need to be moving at a velocity **v** in order to experience the magnetic force **F$_m$**.

$$F_m. = q \, v \, B \sin(theta)$$

The charges experience an electric force **F$_e$**;

the direction of the force depends on the charge inserted; but it will be either in the direction of the electric field lines or in the opposite direction of the electric field lines when q is a negative charge.

$$F_e. = q \, E$$

F$_e$.= Magnitude of the electric force

$$\mathbf{F_e.} = q \, \mathbf{E}$$

F$_m$.=The magnitude of the magnetic force.
q = Electric charge
v = is the magnitude of the velocity vector
B = the magnitude of the magenetic field
   vector
theta = the angle between the velocity vector
   and the magnetic field vector.

$$\mathbf{F_m.} = q \, \mathbf{v} \times \mathbf{B}$$

Note: **F$_m$** the magnetic force vector is perpendicular to the plane made by the velocity vector **v**
           and the magnetic field vector **B**. Since F$_m$ is a force, its units are Newtons.

In the MKS system we find:

The units of the magnetic field vector **B**: Tesla (T)   1 Tesla = 1 Weber / m^2 wb = weber

$$\boxed{F_{m.} = q\ v\ B\ \sin(theta)}$$

$$\boxed{1N = 1C\frac{m}{s}T = 1C\frac{m}{s}\frac{wb}{m^2} = 1\frac{C}{s}\frac{wb}{m}}$$

$$\boxed{1T = \frac{wb}{m^2}}$$

A = amp: a unit of current

Therefore: $1N = 1A\dfrac{wb}{m}$ then $1wb = 1\dfrac{Nm}{A}$

$$\boxed{1T = \frac{wb}{m^2} = 1\frac{\dfrac{Nm}{A}}{m^2} = 1\frac{N}{mA}}$$

Note: Observe clearly the difference between 1 Tesla and 1 Weber.

In the CGS system: The units of the magnetic field vector **B:** The gauss (G)

$$1\ T = 10^4\ G$$

	Magnetic fields from magnets	in nature
Types of magnetic		man made
fields	Magnetic fields induced by the presence of the motion of electric charges.	

The current on a wire inside a magnetic field:
The direction of the electric current follows the motion of the positive charges in the wire.
To observe the effects of the magnetic force on the motion of those electric charges moving through
the wire one follows the right hand rule

Consider: n A l = total number of carrier through the wire
n = number of carriers per unit volume
A = cross sectional area of the wire
l = length of the wire

Then the total magnetic force experience by the wire:

F$_m$ = (q v$_d$ B sin (theta)) (n A l)

v$_d$ = drift velocity of the charges in the wire
n q v$_d$ A = I current

F$_m$ = B I l sin (theta)
When the wire is perpendicular to the magnetic field
then theta = 90° then sin (theta) = 1

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When a closed **loop wire** is placed inside a magnetic field, as a the result of the magnetic force and the direction of the charges in the wire, the wire experiences a torque.

Consider a rectangular loop; with the bottom and top two parallel sides of length = a

Then the torque = τ $\tau = F_m \dfrac{a}{2} + F_m \dfrac{a}{2} = F_m a = BIla$

Note: l a = Area A then the torque = B I (Area A) $\tau = BI(AreaA)$

Example: Consider a high tension wire of length l and cross sectional area A. When the wire is placed from west to east and carries a current A amps in the west east direction, the earth magnetic field will produce a magnetic field force on the wire given by the following conditions:

(a) the earth magnetic field follows the south to north direction.
(b) The magnitude of the earth magnetic field is 0.500 x 10^{-4} T
(c) Since the current, that is, the positive charges are moving from west to east then using the the right hand rule one finds that the magnetic field force on the wire points upwards.
(d) By knowing the length of the wire, its cross sectional area, and substance, example copper, one can find the volume density of the wire and consequently calculate the mass and weight of that wire. The gravitational force points downward.

When a circular, rectangular, or triangular loop is placed in the presence of a magnetic field, and current flows through the loop, one finds that the torque is given by: $\boxed{\tau = BI(A)\sin\theta}$

Where theta = is the angle between the area normal vector to the loop defined by the plane of the loop and the magnetic field vector.

A coil with N = number of loops:will experience a torque given by: $\boxed{\tau = BI(A)N\sin\theta}$

Defining: $\mu = magneticmoment = IAN =$ Magnetic moment vector of the coil

This vector always points in the direction of the area normal vector, that is, it is always perpendicular to the plane of the loop

Then the magnitude of torque vector is given by: $\boxed{\tau = \mu B \sin\theta}$

While the torque vector is given by: $\tau = \mu$ X **B**

The torque vector is perpendicular to the plane defined by the magnetic moment vector and the magnetic field vector.

Applications to electric motors.

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Charged particles in a magnetic field

The charged particles entering the magnetic field experience the magnetic field force vector given by:

F$_m$ = q **v** X **B**   with magnitude $\boxed{F_m = qvB\sin\theta}$ $\theta =$ the angle between the vector velocity and the magnetic field vector.

The path followed by the charge particle describes a circular path and its center location depends on the type of charge of the particle.

F$_m$ = The magnetic field force is always directed toward the center of a circular path
This direction implies the presence of the centripetal force. Since the particle maintains the circular path, the magnitude of the centripetal force is equal to the magnetic force.

When the particle motion is perpendicular to the **B** vector,then the angle theta= 90° hence sin(theta) = 1, then the particle will move in a circular path of constant radius.

F$_m$ = q v B = m v^2 / r   where v^2/r = centripetal acceleration. Hence the radius of curvature can be determined.

When **v** and **B** are not exactly 90° separated, and the charge motion originates inside B, then a helical path is followed by the particle.

Then   q B r = m v   or   q B r sin(theta) = m v   also   r = (mv) / (q B)

The period of revolution in the circular path is independent of the value of the radius: $T = \dfrac{2\pi r}{v}$

Therefore: $\boxed{T = \dfrac{2\pi mv}{vBq} =}$ and $\boxed{T = \dfrac{2\pi an}{qB}}$

Magnetic fields around a wire with a current

When there is a current in a wire, a magnetic field is inudced and it is located around the wire,
Its direction is determined by using the right hand rule where the thum points to the direction of the current hence the close hand fingers points to the direction of the magnetic field vector.
The magnitude of this magnetic field vector is inversely proportional to the distance from the center of the wire, and the magnitude of the magnetic field vector at a distance r from a long straight

current-carrying conductor is given: $B = \dfrac{\mu_o I}{2\pi r}$ Ampere Circuital Law

Permeability of free space = $\mu_o = 4\pi x10^{-7} T.m/A$

I = current in the wire
r = the radial distance from the wire

When two wires conducting current in the same directions.   d = distance between the wire
Determine the magnetic field B_2 produced by one wire at a distance d from it. This magnetic field
B_2 is experienced by the charges moving through the other wire #1, hence that wire experiences a
magnetic force F_1.

$$B_2 = \frac{\mu_o I_2}{2\pi d}$$ $$F_1 = B_2 I_1 l$$ $$F_1 = (\frac{\mu_o I_2}{2\pi d}) I_1 l$$

F_1 / l = force per unit length on the wire can be determined.

The magnetif field vector B_2 is tangent to the circle around wire #2 at a distance d. (use the right
hand close rule to determine the direction of the B_2 field).

The magnetic force F_1 experienced by wire #1 is directed towards wire #2. (use the right hand rule).

Therefore the magnitude of the force per unit length on each of two parallel wires separated by a
distance d and carryibg currents I_1 and I_2

$$\frac{F}{l} = \frac{\mu_o I_1 I_2}{2\pi d}$$

When considering a current loop of radius R, the magnitude of the magnetic field vector at the center

of the loop of radius R: $$B = \frac{\mu_o I}{2R}$$

When the coil has N turns, the magnetic field at the center of the coil: $$B = N\frac{\mu_o I}{2R}$$

When considering the magnitude of the **magnetic field inside a solenoid with N turns** or

n turns of conductor per unit length, and at the center $$B = \mu_o nI$$

n = N / l   N = number of turns   l = length of the solenoid.

Induced Volteages

Faraday demonstrated the production of a current by changing the magnetic field.

When a circular loop conductor with one side with a primary coil wrap around in 1/4 of the circular ring while the secondary location has also a coil wrap around it, and the primary coil is connected. to a battery, while the secondary coil is connected to an ammeter, then current is induced in the secondary as long as there is magnetic field change in the primary coil.

The changing magnetic flux in the presence of a closed wire creates an electric field in it and current flow occurs.

Magnetic flux = $\quad \Phi_B = B_\perp \bullet A = BA\cos\theta$

$\theta = \quad$ is the angle between the B vector and the normal to the surface.

Faraday's Law of Induction: when a coil (circular or any other shape) is connecfed to an ammeter, and a magnet moves back and forth closer to the coil, then as the result of coil crossing the magnetic field lines (magnetic flux changes), a current is induced in the coil.

If the circuit contains N loops and the magnetic flux through each loop changes by the amount $\quad \Delta\Phi_B$

during an interval $\underline{\triangle}$t, then the induced emf in the circuit during time $\underline{\triangle}$t is given by: $\quad \varepsilon = -N\dfrac{\Delta\Phi_B}{\Delta t}$

Hence the average emf induced in a circuit is proportional to the rate of change of the magnetic flux through the circuit.

Lenz' Law:
The - sign indicates the polarity of the induced emf; hence the emf is responsible for the current flow in the loop(s) hence inducing a magnetic field around the wire of the loop(s) and in the same direction of the original magnetic field flux; hence it is to oppose the change in the original flux through the circuit.

 The motional emf induced in a conductor of length " l " moving with speed v, perpendicular to the magnetic field yields:

$$\varepsilon = Blv = IR \quad \text{hence} \quad I = \frac{Blv}{R}$$

Note: If the moving conductor is part of a complete circuit of resistance, R, a current will be induced in the circuit, then

$$|\varepsilon| = \frac{\Delta\Phi_B}{\Delta t} = Bl\frac{\Delta x}{\Delta t} \qquad \text{with} \qquad \boxed{\frac{\Delta x}{\Delta t} = v}$$

with $\quad \boxed{\Delta A = l\Delta x}$
$\underline{\triangle}$A= area

Motors and generators

A conducting loop of N turns and cross sectional area A rotates with a constant angular velocity in a magnetic field, then an emf is induced in the loop and it will vary sinusoidally in time.

$\varepsilon = NBA\omega\sin(\omega t)$ A = the area of the loop
The max emf occurs when: $\quad \omega t = 90^o \quad$ then $\quad \varepsilon_{max} = NBA\omega$

where $\quad \omega = 2\pi f$ f = frquency

$$\boxed{\varepsilon_{max} = I_{max}R}$$

This is a conversion process from mechanical energy to electrical energy.

A solenoid, coil, coaxial cable, toroid or any other conducting device of this type is characterized by a parameter called: Inductance, L. In this devices, any change in the current in the circuit will produce a self-induced emf which will be proportional to the rate of change of the current.

$$\varepsilon_L = -L\frac{\Delta I}{\Delta t}$$ The units of the inductance is the " Henry " H. $$1H = 1\frac{Vs}{A}$$

V = Volts   s = Seconds   A = amps   H = Henry

For a given current, the inductance of a coil can be calculated if the magnetic flux through the coil is known.

$$L = \frac{N\Phi_B}{I}$$ $$L = \frac{\mu_o N^2 A}{l}$$

A RL circuit

The potential drop across a resistor is $\Delta V_L = IR$

Note: At the instant the switch is closed due to the fact that IR=0, then the emf of the battery is equal to the emf generated in the coil.

$$\varepsilon_L = -L\frac{\Delta I}{\Delta t}$$

with $\tau = \dfrac{L}{R}$ the time constant, we find that the current in the circuit is given:

$$I = \frac{\varepsilon}{R}(1 - e^{-t/\tau})$$

Energy stored in an inductor: $$PE_L = \frac{1}{2}LI^2$$

Energy stored in a capacitor: $$PE_C = \frac{1}{2}C(\Delta V)^2$$

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