Chapter 15

1. (a) During simple harmonic motion, the speed is (momentarily) zero when the object is at a “turning point” (that is, when $x = +x_m$ or $x = –x_m$). Consider that it starts at $x = +x_m$ and we are told that t = 0.25 second elapses until the object reaches $x = –x_m$. To execute a full cycle of the motion (which takes a period T to complete), the object which started at $x = +x_m$, must return to $x = +x_m$ (which, by symmetry, will occur 0.25 second *after* it was at $x = –x_m$). Thus, $T = 2t$ = 0.50 s.

(b) Frequency is simply the reciprocal of the period: $f = 1/T$ = 2.0 Hz.

(c) The 36 cm distance between $x = +x_m$ and $x = –x_m$ is $2x_m$. Thus, x_m = 36/2 = 18 cm.

3. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = \left(2\pi\left(6.60\text{ Hz}\right)\right)^2\left(0.0220\text{ m}\right) = 37.8\text{ m/s}^2.$$

7. The magnitude of the maximum acceleration is given by $a_m = \omega^2 x_m$, where ω is the angular frequency and x_m is the amplitude.

(a) The angular frequency for which the maximum acceleration is g is given by $\omega = \sqrt{g/x_m}$, and the corresponding frequency is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = \frac{1}{2\pi}\sqrt{\frac{9.8\text{ m/s}^2}{1.0\times10^{-6}\text{ m}}} = 498\text{ Hz}.$$

(b) For frequencies greater than 498 Hz, the acceleration exceeds g for some part of the motion.

8. We note (from the graph in the text) that x_m = 6.00 cm. Also the value at t = 0 is x_o = – 2.00 cm. Then Eq. 15-3 leads to

$$\phi = \cos^{-1}(-2.00/6.00) = +1.91\text{ rad or } -4.37\text{ rad}.$$

The other “root” (+4.37 rad) can be rejected on the grounds that it would lead to a positive slope at t = 0.

14. Equation 15-12 gives the angular velocity:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \text{ N/m}}{2.00 \text{ kg}}} = 7.07 \text{rad/s.}$$

Energy methods (discussed in Section 15-4) provide one method of solution. Here, we use trigonometric techniques based on Eq. 15-3 and Eq. 15-6.

(a) Dividing Eq. 15-6 by Eq. 15-3, we obtain

$$\frac{v}{x} = -\omega \tan(\omega t + \phi)$$

so that the phase ($\omega t + \phi$) is found from

$$\omega t + \phi = \tan^{-1}\left(\frac{-v}{\omega x}\right) = \tan^{-1}\left(\frac{-3.415 \text{ m/s}}{(7.07 \text{ rad/s})(0.129 \text{ m})}\right).$$

With the calculator in radians mode, this gives the phase equal to –1.31 rad. Plugging this back into Eq. 15-3 leads to $0.129 \text{ m} = x_m \cos(-1.31) \;\; \Rightarrow \;\; x_m = 0.500 \text{ m}.$

(b) Since $\omega t + \phi = -1.31$ rad at $t = 1.00$ s, we can use the above value of ω to solve for the phase constant ϕ. We obtain $\phi = -8.38$ rad (though this, as well as the previous result, can have 2π or 4π (and so on) added to it without changing the physics of the situation). With this value of ϕ, we find $x_o = x_m \cos \phi = -0.251$ m.

(c) And we obtain $v_o = -x_m\omega \sin\phi = 3.06$ m/s.

29. When the block is at the end of its path and is momentarily stopped, its displacement is equal to the amplitude and all the energy is potential in nature. If the spring potential energy is taken to be zero when the block is at its equilibrium position, then

$$E = \frac{1}{2}kx_m^2 = \frac{1}{2}\left(1.3\times10^2 \text{ N/m}\right)(0.024 \text{ m})^2 = 3.7\times10^{-2} \text{ J.}$$

40. We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = d$, the unknown. For a meter stick of mass m, the rotational inertia about its center of mass is $I_{cm} = mL^2/12$ where $L = 1.0$ m. Thus, for $T = 2.5$ s, we obtain

$$T = 2\pi\sqrt{\frac{mL^2/12 + md^2}{mgd}} = 2\pi\sqrt{\frac{L^2}{12gd} + \frac{d}{g}}.$$

Squaring both sides and solving for d leads to the quadratic formula:

$$d = \frac{g(T/2\pi)^2 \pm \sqrt{d^2(T/2\pi)^4 - L^2/3}}{2}.$$

Choosing the plus sign leads to an impossible value for d ($d = 1.5 > L$). If we choose the minus sign, we obtain a physically meaningful result: $d = 0.056$ m.

$$1\times10^{13}\ \text{Hz} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow k = \left(2\pi\times10^{13}\right)^2\left(1.8\times10^{-25}\right) \approx 7\times10^2\ \text{N/m}.$$