Chapter 42

9. (a) 6 protons, since $Z = 6$ for carbon (see Appendix F).

(b) 8 neutrons, since $A - Z = 14 - 6 = 8$ (see Eq. 42-1).

16. The binding energy is given by

$$\Delta E_{\text{be}} = \left[Z m_H + (A - Z) m_n - M_{\text{Eu}} \right] c^2,$$

where Z is the atomic number (number of protons), A is the mass number (number of nucleons), m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M_{Eu} is the mass of a ${}^{152}_{63}\text{Eu}$ atom. In principle, nuclear masses should be used, but the mass of the Z electrons included in ZM_H is canceled by the mass of the Z electrons included in M_{Eu}, so the result is the same. First, we calculate the mass difference in atomic mass units:

$$\Delta m = (63)(1.007825 \text{ u}) + (152 - 63)(1.008665 \text{ u}) - (151.921742 \text{ u}) = 1.342418 \text{ u}.$$

Since 1 u is equivalent to 931.494013 MeV,

$$\Delta E_{\text{be}} = (1.342418 \text{ u})(931.494013 \text{ MeV/u}) = 1250.454 \text{ MeV}.$$

Since there are 152 nucleons, the binding energy per nucleon is

$$\Delta E_{\text{ben}} = E/A = (1250.454 \text{ MeV})/152 = 8.23 \text{ MeV}.$$

26. By the definition of half-life, the same has reduced to $\frac{1}{2}$ its initial amount after 140 d. Thus, reducing it to $\frac{1}{4} = \left(\frac{1}{2}\right)^2$ of its initial number requires that two half-lives have passed: $t = 2T_{1/2} = 280$ d.

33. We note that 3.82 days is 330048 s, and that a becquerel is a disintegration per second (see Section 42-3). From Eq. 34-19, we have

$$\frac{N}{\mathcal{V}} = \frac{R}{\mathcal{V}} \frac{T_{1/2}}{\ln 2} = \left(1.55 \times 10^5 \frac{\text{Bq}}{\text{m}^3}\right) \frac{330048 \text{ s}}{\ln 2} = 7.4 \times 10^{10} \frac{\text{atoms}}{\text{m}^3}$$

where we have divided by volume v. We estimate v (the volume breathed in 48 h = 2880 min) as follows:

$$\left(2\,\frac{\text{liters}}{\text{breath}}\right)\left(\frac{1\,\text{m}^3}{1000\,\text{L}}\right)\left(40\,\frac{\text{breaths}}{\text{min}}\right)\left(2880\,\text{min}\right)$$

which yields $v \approx 200\ \text{m}^3$. Thus, the order of magnitude of N is

$$\left(\frac{N}{\mathcal{V}}\right)(\mathcal{V}) \approx \left(7\times10^{10}\,\frac{\text{atoms}}{\text{m}^3}\right)\left(200\,\text{m}^3\right) \approx 1\times10^{13}\,\text{atoms.}$$

38. With $T_{1/2} = 3.0\ \text{h} = 1.08\times10^4\ \text{s},$ the decay constant is (using Eq. 42-18)

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.08\times10^4\ \text{s}} = 6.42\times10^{-5}\,/\,\text{s}\,.$$

Thus, the number of isotope parents injected is

$$N = \frac{R}{\lambda} = \frac{(8.60\times10^{-6}\,\text{Ci})(3.7\times10^{10}\,\text{Bq/Ci})}{6.42\times10^{-5}\ /\,\text{s}} = 4.96\times10^{9}\,.$$

42. Adapting Eq. 42-21, we have

$$N_{\text{Kr}} = \frac{M_{\text{sam}}}{M_{\text{Kr}}}\,N_A = \left(\frac{20\times10^{-9}\,\text{g}}{92\,\text{g/mol}}\right)\left(6.02\times10^{23}\,\text{atoms/mol}\right) = 1.3\times10^{14}\,\text{atoms.}$$

Consequently, Eq. 42-20 leads to

$$R = \frac{N\ln 2}{T_{1/2}} = \frac{\left(1.3\times10^{14}\right)\ln 2}{1.84\,\text{s}} = 4.9\times10^{13}\ \text{Bq.}$$

50. (a) The disintegration energy for uranium-235 “decaying” into thorium-232 is

$$\begin{aligned}Q_3 &= \left(m_{^{235}\text{U}} - m_{^{232}\text{Th}} - m_{^{3}\text{He}}\right)c^2 = \left(235.0439\,\text{u} - 232.0381\,\text{u} - 3.0160\,\text{u}\right)\left(931.5\,\text{MeV/u}\right)\\ &= -9.50\,\text{MeV.}\end{aligned}$$

(b) Similarly, the disintegration energy for uranium-235 decaying into thorium-231 is

$$\begin{aligned}Q_4 &= \left(m_{^{235}\text{U}} - m_{^{231}\text{Th}} - m_{^{4}\text{He}}\right)c^2 = \left(235.0439\,\text{u} - 231.0363\,\text{u} - 4.0026\,\text{u}\right)\left(931.5\,\text{MeV/u}\right)\\ &= 4.66\,\text{MeV.}\end{aligned}$$

(c) Finally, the considered transmutation of uranium-235 into thorium-230 has a Q-value of

$$Q_5 = \left(m_{^{235}\text{U}} - m_{^{230}\text{Th}} - m_{^{5}\text{He}}\right)c^2 = \left(235.0439\,\text{u} - 230.0331\,\text{u} - 5.0122\,\text{u}\right)\left(931.5\,\text{MeV/u}\right)$$
$$= -1.30\,\text{MeV}.$$

Only the second decay process (the α decay) is spontaneous, as it releases energy.

54. Assuming the neutrino has negligible mass, then

$$\Delta mc^2 = \left(\mathbf{m}_{\text{Ti}} - \mathbf{m}_{\text{V}} - m_e\right)c^2.$$

Now, since vanadium has 23 electrons (see Appendix F and/or G) and titanium has 22 electrons, we can add and subtract $22m_e$ to the above expression and obtain

$$\Delta mc^2 = \left(\mathbf{m}_{\text{Ti}} + 22m_e - \mathbf{m}_{\text{V}} - 23m_e\right)c^2 = \left(m_{\text{Ti}} - m_{\text{V}}\right)c^2.$$

We note that our final expression for Δmc^2 involves the *atomic* masses, and that this assumes (due to the way they are usually tabulated) the atoms are in the ground states (which is certainly not the case here, as we discuss below). The question now is: do we set $Q = -\Delta mc^2$ as in Sample Problem —"Q value in a beta decay, suing masses?" The answer is "no." The atom is left in an excited (high energy) state due to the fact that an electron was captured from the lowest shell (where the absolute value of the energy, E_K, is quite large for large Z). To a very good approximation, the energy of the K-shell electron in Vanadium is equal to that in Titanium (where there is now a "vacancy" that must be filled by a readjustment of the whole electron cloud), and we write $Q = -\Delta mc^2 - E_K$ so that Eq. 42-26 still holds. Thus,

$$Q = \left(m_{\text{V}} - m_{\text{Ti}}\right)c^2 - E_{K.}$$

67. The absorbed dose is

$$\text{absorbed dose} = \frac{2.00\times10^{-3}\ \text{J}}{4.00\ \text{kg}} = 5.00\times10^{-4}\ \text{J/kg} = 5.00\times10^{-4}\ \text{Gy}$$

where $1\ \text{J/kg} = 1\,\text{Gy}$. With $\text{RBE} = 5$, the dose equivalent is

$$\text{dose equivalent} = \text{RBE}\cdot(5.00\times10^{-4}\ \text{Gy}) = 5(5.00\times10^{-4}\ \text{Gy}) = 2.50\times10^{-3}\ \text{Sv}$$
$$= 2.50\ \text{mSv}.$$

68. (a) Using Eq. 42-32, the energy absorbed is

$$\left(2.4\times10^{-4}\,\text{Gy}\right)\left(75\,\text{kg}\right) = 18\,\text{mJ}.$$

(b) The dose equivalent is

$$\left(2.4\times10^{-4}\,\text{Gy}\right)\left(12\right)=2.9\times10^{-3}\,\text{Sv}\,.$$

(c) Using Eq. 42-33, we have $2.9\times10^{-3}\,\text{Sv}=0.29\text{ rem}\,.$