Chapter 40

2. For a given quantum number n there are n possible values of ℓ, ranging from 0 to $n-1$. For each ℓ the number of possible electron states is $N_\ell = 2(2\ell + 1)$. Thus the total number of possible electron states for a given n is

$$N_n = \sum_{\ell=0}^{n-1} N_\ell = 2\sum_{\ell=0}^{n-1} \left(2\ell+1\right) = 2n^2.$$

Thus, in this problem, the total number of electron states is $N_n = 2n^2 = 2(5)^2 = 50$.

6. For a given quantum number ℓ there are $(2\ell + 1)$ different values of m_ℓ. For each given m_ℓ the electron can also have two different spin orientations. Thus, the total number of electron states for a given ℓ is given by $N_\ell = 2(2\ell + 1)$.

(a) Now $\ell = 3$, so $N_\ell = 2(2 \times 3 + 1) = 14$.

(b) In this case, $\ell = 1$, which means $N_\ell = 2(2 \times 1 + 1) = 6$.

(c) Here $\ell = 1$, so $N_\ell = 2(2 \times 1 + 1) = 6$.

(d) Now $\ell = 0$, so $N_\ell = 2(2 \times 0 + 1) = 2$.

28. For a given value of the principal quantum number n, there are n possible values of the orbital quantum number ℓ, ranging from 0 to $n - 1$. For any value of ℓ, there are $2\ell+1$ possible values of the magnetic quantum number m_ℓ, ranging from $-\ell$ to $+\ell$. Finally, for each set of values of ℓ and m_ℓ, there are two states, one corresponding to the spin quantum number $m_s = -\frac{1}{2}$ and the other corresponding to $m_s = +\frac{1}{2}$. Hence, the total number of states with principal quantum number n is

$$N = 2\sum_{\ell=0}^{n-1}(2\ell+1).$$

Now

$$\sum_{\ell=0}^{n-1}2\ell = 2\sum_{\ell=0}^{n-1}\ell = 2\frac{n}{2}(n-1) = n(n-1),$$

since there are n terms in the sum and the average term is $(n - 1)/2$. Furthermore,

$$\sum_{\ell=0}^{n-1} 1 = n \ .$$

Thus $N = 2\ n(n-1) + n\ = 2n^2$.

30. When a helium atom is in its ground state, both of its electrons are in the 1*s* state. Thus, for each of the electrons, n = 1, ℓ = 0, and m_ℓ = 0. One of the electrons is spin up $\left(m_s = +\frac{1}{2}\right)$ while the other is spin down $\left(m_s = -\frac{1}{2}\right)$. Thus,

(a) the quantum numbers (n, ℓ, m_ℓ, m_s) for the spin-up electron are (1,0,0,+1/2), and

(b) the quantum numbers (n, ℓ, m_ℓ, m_s) for the spin-down electron are (1,0,0,–1/2).

52. The energy of the laser pulse is

$$E_p = P\Delta t = (2.80\times10^6\ \text{J/s})(0.500\times10^{-6}\ \text{s}) = 1.400\ \text{J}\ .$$

Since the energy carried by each photon is

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\,\text{m/s})}{424\times10^{-9}\,\text{m}} = 4.69\times10^{-19}\,\text{J}\,,$$

the number of photons emitted in each pulse is

$$N = \frac{E_p}{E} = \frac{1.400\text{J}}{4.69\times10^{-19}\ \text{J}} = 3.0\times10^{18}\ \text{photons.}$$

With each atom undergoing stimulated emission only once, the number of atoms contributed to the pulse is also 3.0×10^{18}.

55. (a) If t is the time interval over which the pulse is emitted, the length of the pulse is

$$L = ct = (3.00 \times 10^8\ \text{m/s})(1.20 \times 10^{-11}\ \text{s}) = 3.60 \times 10^{-3}\ \text{m}.$$

(b) If E_p is the energy of the pulse, E is the energy of a single photon in the pulse, and N is the number of photons in the pulse, then $E_p = NE$. The energy of the pulse is

$$E_p = (0.150\ \text{J})/(1.602 \times 10^{-19}\ \text{J/eV}) = 9.36 \times 10^{17}\ \text{eV}$$

and the energy of a single photon is E = (1240 eV·nm)/(694.4 nm) = 1.786 eV. Hence,

$$N = \frac{E_p}{E} = \frac{9.36 \times 10^{17}\ \text{eV}}{1.786\ \text{eV}} = 5.24 \times 10^{17}\ \text{photons.}$$

65. (a) Using hc = 1240 eV·nm,

$$\Delta E = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = (1240\,\text{eV}\cdot\text{nm})\left(\frac{1}{588.995\,\text{nm}} - \frac{1}{589.592\,\text{nm}}\right) = 2.13\,\text{meV}\ .$$

(b) From $\Delta E = 2\mu_B B$ (see Fig. 40-10 and Eq. 40-18), we get

$$B = \frac{\Delta E}{2\mu_B} = \frac{2.13 \times 10^{-3}\,\text{eV}}{2(5.788 \times 10^{-5}\,\text{eV/T})} = 18\,\text{T}\ .$$

71. The principal quantum number n must be greater than 3. The magnetic quantum number m_ℓ can have any of the values – 3, – 2, – 1, 0, +1, +2, or +3. The spin quantum number can have either of the values $-\frac{1}{2}$ or $+\frac{1}{2}$.