Chapter 39

2. (a) The ground-state energy is

$$E_1 = \left(\frac{h^2}{8m_e L^2}\right)n^2 = \left(\frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)^2}{8(9.11\times10^{-31}\,\text{kg})\left(200\times10^{-12}\,\text{m}\right)^2}\right)(1)^2 = 1.51\times10^{-18}\,\text{J}$$
$$= 9.42\,\text{eV}.$$

(b) With $m_p = 1.67 \times 10^{-27}$ kg, we obtain

$$E_1 = \left(\frac{h^2}{8m_p L^2}\right)n^2 = \left(\frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)^2}{8(1.67\times10^{-27}\,\text{kg})\left(200\times10^{-12}\,\text{m}\right)^2}\right)(1)^2 = 8.225\times10^{-22}\,\text{J}$$
$$= 5.13\times10^{-3}\,\text{eV}.$$

6. With $m = m_p = 1.67 \times 10^{-27}$ kg, we obtain

$$E_1 = \left(\frac{h^2}{8mL^2}\right)n^2 = \left(\frac{\left(6.63\times10^{-34}\,\text{J.s}\right)^2}{8(1.67\times10^{-27}\,\text{kg})\left(100\times10^{12}\,\text{m}\right)^2}\right)(1)^2 = 3.29\times10^{-21}\,\text{J} = 0.0206\,\text{eV}.$$

Alternatively, we can use the mc^2 value for a proton from Table 37-3 (938×10^6 eV) and $hc = 1240$ eV $\cdot$ nm by writing Eq. 39-4 as

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(hc)^2}{8\left(m_pc^2\right)L^2}.$$

This alternative approach is perhaps easier to plug into, but it is recommended that both approaches be tried to find which is most convenient.

13. The position of maximum probability density corresponds to the center of the well: $x = L/2 = (200\text{ pm})/2 = 100\text{ pm}.$

(a) The probability of detection at x is given by Eq. 39-11:

$$p(x) = \psi_n^2(x)dx = \left[\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)\right]^2 dx = \frac{2}{L}\sin^2\left(\frac{n\pi}{L}x\right)dx$$

For $n=3$, $L=200$ pm, and $dx=2.00$ pm (width of the probe), the probability of detection at $x=L/2=100$ pm is

$$p(x=L/2)=\frac{2}{L}\sin^2\left(\frac{3\pi}{L}\cdot\frac{L}{2}\right)dx=\frac{2}{L}\sin^2\left(\frac{3\pi}{2}\right)dx=\frac{2}{L}dx=\frac{2}{200\text{ pm}}\left(2.00\text{ pm}\right)=0.020\,.$$

(b) With $N=1000$ independent insertions, the number of times we expect the electron to be detected is $n=Np=(1000)(0.020)=20$.

17. According to Fig. 39-9, the electron's initial energy is 106 eV. After the additional energy is absorbed, the total energy of the electron is 106 eV + 400 eV = 506 eV. Since it is in the region $x > L$, its potential energy is 450 eV (see Section 39-5), so its kinetic energy must be 506 eV – 450 eV = 56 eV.

31. The energy E of the photon emitted when a hydrogen atom jumps from a state with principal quantum number n to a state with principal quantum number n' is given by

$$E=A\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)$$

where A = 13.6 eV. The frequency f of the electromagnetic wave is given by $f = E/h$ and the wavelength is given by $\lambda = c/f$. Thus,

$$\frac{1}{\lambda}=\frac{f}{c}=\frac{E}{hc}=\frac{A}{hc}\left(\frac{1}{n'^2}-\frac{1}{n^2}\right).$$

The shortest wavelength occurs at the series limit, for which $n = \infty$. For the Balmer series, $n'=2$ and the shortest wavelength is $\lambda_B = 4hc/A$. For the Lyman series, $n'=1$ and the shortest wavelength is $\lambda_L = hc/A$. The ratio is $\lambda_B/\lambda_L = 4.0$.

32. The difference between the energy absorbed and the energy emitted is

$$E_{\text{photon absorbed}}-E_{\text{photon emitted}}=\frac{hc}{\lambda_{\text{absorbed}}}-\frac{hc}{\lambda_{\text{emitted}}}\,.$$

Thus, using hc = 1240 eV · nm, the net energy absorbed is

$$hc\Delta\left(\frac{1}{\lambda}\right)=\left(1240\,\text{eV}\cdot\text{nm}\right)\left(\frac{1}{375\,\text{nm}}-\frac{1}{580\,\text{nm}}\right)=1.17\,\text{eV}\,.$$

33. (a) Since energy is conserved, the energy E of the photon is given by $E = E_i - E_f$, where E_i is the initial energy of the hydrogen atom and E_f is the final energy. The electron energy is given by (– 13.6 eV)/n^2, where n is the principal quantum number. Thus,

$$E = E_3 - E_1 = \frac{-13.6\,\text{eV}}{(3)^2} - \frac{-13.6\,\text{eV}}{(1)^2} = 12.1\,\text{eV} .$$

(b) The photon momentum is given by

$$p = \frac{E}{c} = \frac{(12.1\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{3.00\times10^{8}\ \text{m/s}} = 6.45\times10^{-27}\ \text{kg}\cdot\text{m/s} .$$

(c) Using hc = 1240 eV · nm, the wavelength is $\lambda = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{12.1\,\text{eV}} = 102\,\text{nm}.$

38. From Eq. 39-6, $\Delta E = hf = (4.14\times10^{-15}\ \text{eV}\cdot\text{s})(6.2\times10^{14}\ \text{Hz}) = 2.6\,\text{eV}$.

62. (a) The “home-base” energy level for the Balmer series is n = 2. Thus the transition with the least energetic photon is the one from the n = 3 level to the n = 2 level. The energy difference for this transition is

$$\Delta E = E_3 - E_2 = -(13.6\,\text{eV})\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 1.889\,\text{eV} .$$

Using hc = 1240 eV · nm, the corresponding wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.889\,\text{eV}} = 658\,\text{nm} .$$

(b) For the series limit, the energy difference is

$$\Delta E = E_\infty - E_2 = -(13.6\,\text{eV})\left(\frac{1}{\infty^2} - \frac{1}{2^2}\right) = 3.40\,\text{eV} .$$

The corresponding wavelength is then $\lambda = \frac{hc}{\Delta E} = \frac{1240\,\text{eV}\cdot\text{nm}}{3.40\,\text{eV}} = 366\,\text{nm}$.

63. (a) The allowed values of ℓ for a given n are 0, 1, 2, ..., n – 1. Thus there are n different values of ℓ .

(b) The allowed values of m_ℓ for a given ℓ are $-\ell$, $-\ell$ + 1, ..., ℓ . Thus there are 2ℓ + 1 different values of m_ℓ .

(c) According to part (a) above, for a given n there are n different values of ℓ. Also, each of these ℓ's can have $2\ell + 1$ different values of m_ℓ [see part (b) above]. Thus, the total number of m_ℓ's is

$$\sum_{\ell=0}^{n-1}(2\ell+1) = n^2.$$