

Chapter 35

4. Note that Snell's law (the law of refraction) leads to $\theta_1 = \theta_2$ when $n_1 = n_2$. The graph indicates that $\theta_2 = 30^\circ$ (which is what the problem gives as the value of θ_1) occurs at $n_2 = 1.5$. Thus, $n_1 = 1.5$, and the speed with which light propagates in that medium is

$$v = \frac{c}{n_1} = \frac{2.998 \times 10^8 \text{ m/s}}{1.5} = 2.0 \times 10^8 \text{ m/s}.$$

13. (a) We choose a horizontal x axis with its origin at the left edge of the plastic. Between $x = 0$ and $x = L_2$ the phase difference is that given by Eq. 35-11 (with L in that equation replaced with L_2). Between $x = L_2$ and $x = L_1$ the phase difference is given by an expression similar to Eq. 35-11 but with L replaced with $L_1 - L_2$ and n_2 replaced with 1 (since the top ray in Fig. 35-35 is now traveling through air, which has index of refraction approximately equal to 1). Thus, combining these phase differences with $\lambda = 0.600 \mu\text{m}$, we have

$$\begin{aligned} \frac{L_2}{\lambda}(n_2 - n_1) + \frac{L_1 - L_2}{\lambda}(1 - n_1) &= \frac{3.50 \mu\text{m}}{0.600 \mu\text{m}}(1.60 - 1.40) + \frac{4.00 \mu\text{m} - 3.50 \mu\text{m}}{0.600 \mu\text{m}}(1 - 1.40) \\ &= 0.833. \end{aligned}$$

(b) Since the answer in part (a) is closer to an integer than to a half-integer, the interference is more nearly constructive than destructive.

23. Initially, source A leads source B by 90° , which is equivalent to $1/4$ wavelength. However, source A also lags behind source B since r_A is longer than r_B by 100 m, which is $100\text{m}/400\text{m} = 1/4$ wavelength. So the net phase difference between A and B at the detector is zero.

30. In adding these with the phasor method (as opposed to, say, trig identities), we may set $t = 0$ and add them as vectors:

$$\begin{aligned} y_h &= 10\cos 0^\circ + 8.0\cos 30^\circ = 16.9 \\ y_v &= 10\sin 0^\circ + 8.0\sin 30^\circ = 4.0 \end{aligned}$$

so that

$$y_R = \sqrt{y_h^2 + y_v^2} = 17.4$$

$$\beta = \tan^{-1}\left(\frac{y_v}{y_h}\right) = 13.3^\circ.$$

Thus,

$$y = y_1 + y_2 = y_R \sin(\omega t + \beta) = 17.4 \sin(\omega t + 13.3^\circ).$$

Quoting the answer to two significant figures, we have $y \approx 17 \sin(\omega t + 13^\circ)$.

34. (a) Referring to Figure 35-10(a) makes clear that

$$\theta = \tan^{-1}(y/D) = \tan^{-1}(0.205/4) = 2.93^\circ.$$

Thus, the phase difference at point P is $\phi = d \sin \theta / \lambda = 0.397$ wavelengths, which means it is between the central maximum (zero wavelength difference) and the first minimum ($\frac{1}{2}$ wavelength difference). Note that the above computation could have been simplified somewhat by avoiding the explicit use of the tangent and sine functions and making use of the small-angle approximation ($\tan \theta \approx \sin \theta$).

(b) From Eq. 35-22, we get (with $\phi = (0.397)(2\pi) = 2.495$ rad)

$$I = 4I_0 \cos^2(\phi/2) = 0.404 I_0$$

at point P and

$$I_{\text{center}} = 4I_0 \cos^2(0) = 4 I_0$$

at the center. Thus, $I / I_{\text{center}} = 0.404 / 4 = 0.101$.

39. For constructive interference, we use Eq. 35-36:

$$2n_2 L = (m + 1/2)\lambda.$$

For the smallest value of L , let $m = 0$:

$$L_0 = \frac{\lambda/2}{2n_2} = \frac{624 \text{ nm}}{4(1.33)} = 117 \text{ nm} = 0.117 \mu\text{m}.$$

(b) For the second smallest value, we set $m = 1$ and obtain

$$L_1 = \frac{(1 + 1/2)\lambda}{2n_2} = \frac{3\lambda}{2n_2} = 3L_0 = 3(0.1173 \mu\text{m}) = 0.352 \mu\text{m}.$$

55. The index of refraction of oil is greater than that of the air, but smaller than that of the water. Let the indices of refraction of the air, oil, and water be n_1 , n_2 , and n_3 , respectively. Since $n_1 < n_2$ and $n_2 < n_3$, there is a phase change of π rad from both surfaces. Since the second wave travels an additional distance of $2L$, the phase difference is

$$\phi = \frac{2\pi}{\lambda_2}(2L)$$

where $\lambda_2 = \lambda / n_2$ is the wavelength in the oil. The condition for constructive interference is

$$\frac{2\pi}{\lambda_2}(2L) = 2m\pi,$$

or

$$2L = m \frac{\lambda}{n_2}, \quad m = 0, 1, 2, \dots$$

(a) For $m = 1, 2, \dots$, maximum reflection occurs for wavelengths

$$\lambda = \frac{2n_2L}{m} = \frac{2(1.20)(460 \text{ nm})}{m} = 1104 \text{ nm}, 552 \text{ nm}, 368 \text{ nm}, \dots$$

We note that only the 552 nm wavelength falls within the visible light range.

(b) Maximum transmission into the water occurs for wavelengths for which reflection is a minimum. The condition for such destructive interference is given by

$$2L = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2} \Rightarrow \lambda = \frac{4n_2L}{2m+1}$$

which yields $\lambda = 2208 \text{ nm}, 736 \text{ nm}, 442 \text{ nm} \dots$ for the different values of m . We note that only the 442-nm wavelength (blue) is in the visible range, though we might expect some red contribution since the 736 nm is very close to the visible range.

Note: A light ray reflected by a material changes phase by π rad (or 180°) if the refractive index of the material is greater than that of the medium in which the light is traveling. Otherwise, there is no phase change. Note that refraction at an interface does not cause a phase shift.

75. Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a π rad phase change while the wave reflected from the upper surface does not. At a place where the thickness of the wedge is d , the condition for a maximum in intensity is $2d = (m + \frac{1}{2})\lambda$, where λ is the wavelength in air and m is an integer. Therefore,

$$d = (2m + 1)\lambda/4.$$

As the geometry of Fig. 35-45 shows, $d = R - \sqrt{R^2 - r^2}$, where R is the radius of curvature of the lens and r is the radius of a Newton's ring. Thus, $(2m + 1)\lambda/4 = R - \sqrt{R^2 - r^2}$. First, we rearrange the terms so the equation becomes

$$\sqrt{R^2 - r^2} = R - \frac{(2m + 1)\lambda}{4}.$$

Next, we square both sides, rearrange to solve for r^2 , then take the square root. We get

$$r = \sqrt{\frac{(2m + 1)R\lambda}{2} - \frac{(2m + 1)^2\lambda^2}{16}}.$$

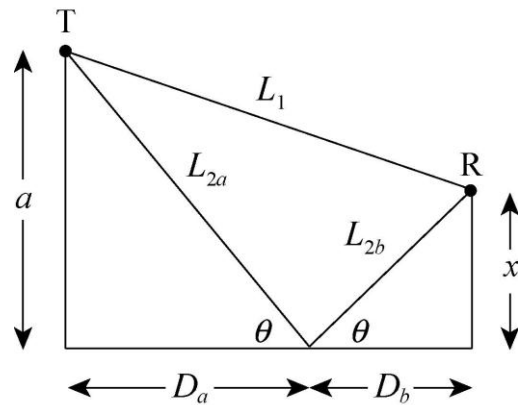
If R is much larger than a wavelength, the first term dominates the second and

$$r = \sqrt{\frac{(2m + 1)R\lambda}{2}}, \quad m = 0, 1, 2, \dots$$

Note: Similarly, one may show that the radii of the dark fringes are given by

$$r = \sqrt{mR\lambda}.$$

89. The wave that goes directly to the receiver travels a distance L_1 and the reflected wave travels a distance L_2 . Since the index of refraction of water is greater than that of air this last wave suffers a phase change on reflection of half a wavelength. To obtain constructive interference at the receiver, the difference $L_2 - L_1$ must be an odd multiple of a half wavelength. Consider the diagram on the right. The right triangle on the left, formed by the vertical line from the water to the transmitter T, the ray incident on the water, and the water line, gives $D_a = a / \tan \theta$. The right triangle on the right, formed by the vertical line from the water to the receiver R, the reflected ray, and the water line leads to $D_b = x / \tan \theta$. Since $D_a + D_b = D$,



$$\tan \theta = \frac{a + x}{D}.$$

We use the identity $\sin^2 \theta = \tan^2 \theta / (1 + \tan^2 \theta)$ to show that

$$\sin \theta = (a + x) / \sqrt{D^2 + (a + x)^2}.$$

This means

$$L_{2a} = \frac{a}{\sin \theta} = \frac{a \sqrt{D^2 + (a + x)^2}}{a + x}$$

and

$$L_{2b} = \frac{x}{\sin \theta} = \frac{x \sqrt{D^2 + (a + x)^2}}{a + x}.$$

Therefore,

$$L_2 = L_{2a} + L_{2b} = \frac{(a + x) \sqrt{D^2 + (a + x)^2}}{a + x} = \sqrt{D^2 + (a + x)^2}.$$

Using the binomial theorem, with D^2 large and $a^2 + x^2$ small, we approximate this expression: $L_2 \approx D + (a + x)^2 / 2D$. The distance traveled by the direct wave is

$L_1 = \sqrt{D^2 + (a - x)^2}$. Using the binomial theorem, we approximate this expression: $L_1 \approx D + (a - x)^2 / 2D$. Thus,

$$L_2 - L_1 \approx D + \frac{a^2 + 2ax + x^2}{2D} - D - \frac{a^2 - 2ax + x^2}{2D} = \frac{2ax}{D}.$$

Setting this equal to $(m + \frac{1}{2})\lambda$, where m is zero or a positive integer, we find $x = (m + \frac{1}{2})(D/2a)\lambda$.

94. A light ray traveling directly along the central axis reaches the end in time

$$t_{\text{direct}} = \frac{L}{v_1} = \frac{n_1 L}{c}.$$

For the ray taking the critical zig-zag path, only its velocity component along the core axis direction contributes to reaching the other end of the fiber. That component is $v_1 \cos \theta'$, so the time of travel for this ray is

$$t_{\text{zig zag}} = \frac{L}{v_1 \cos \theta'} = \frac{n_1 L}{c \sqrt{1 - (\sin \theta / n_1)^2}}$$

using results from the previous solution. Plugging in $\sin \theta = \sqrt{n_1^2 - n_2^2}$ and simplifying, we obtain

$$t_{\text{zig zag}} = \frac{n_1 L}{c(n_2 / n_1)} = \frac{n_1^2 L}{n_2 c}.$$

The difference is

$$\Delta t = t_{\text{zig zag}} - t_{\text{direct}} = \frac{n_1^2 L}{n_2 c} - \frac{n_1 L}{c} = \frac{n_1 L}{c} \left(\frac{n_1}{n_2} - 1 \right).$$

With $n_1 = 1.58$, $n_2 = 1.53$, and $L = 300$ m, we obtain

$$\Delta t = \frac{n_1 L}{c} \left(\frac{n_1}{n_2} - 1 \right) = \frac{(1.58)(300 \text{ m})}{3.0 \times 10^8 \text{ m/s}} \left(\frac{1.58}{1.53} - 1 \right) = 5.16 \times 10^{-8} \text{ s} = 51.6 \text{ ns}.$$