

## Chapter 33

5. If  $f$  is the frequency and  $\lambda$  is the wavelength of an electromagnetic wave, then  $f\lambda = c$ . The frequency is the same as the frequency of oscillation of the current in the  $LC$  circuit of the generator. That is,  $f = 1/2\pi\sqrt{LC}$ , where  $C$  is the capacitance and  $L$  is the inductance. Thus

$$\frac{\lambda}{2\pi\sqrt{LC}} = c.$$

The solution for  $L$  is

$$L = \frac{\lambda^2}{4\pi^2 C c^2} = \frac{(550 \times 10^{-9} \text{ m})^2}{4\pi^2 (17 \times 10^{-12} \text{ F})(2.998 \times 10^8 \text{ m/s})^2} = 5.00 \times 10^{-21} \text{ H}.$$

This is exceedingly small.

6. The emitted wavelength is

$$\lambda = \frac{c}{f} = 2\pi c \sqrt{LC} = 2\pi (2.998 \times 10^8 \text{ m/s}) \sqrt{(0.253 \times 10^{-6} \text{ H})(25.0 \times 10^{-12} \text{ F})} = 4.74 \text{ m}.$$

33. Let  $I_0$  be the intensity of the unpolarized light that is incident on the first polarizing sheet. The transmitted intensity is  $I_1 = \frac{1}{2} I_0$ , and the direction of polarization of the transmitted light is  $\theta_1 = 40^\circ$  counterclockwise from the  $y$  axis in the diagram. The polarizing direction of the second sheet is  $\theta_2 = 20^\circ$  clockwise from the  $y$  axis, so the angle between the direction of polarization that is incident on that sheet and the polarizing direction of the sheet is  $40^\circ + 20^\circ = 60^\circ$ . The transmitted intensity is

$$I_2 = I_1 \cos^2 60^\circ = \frac{1}{2} I_0 \cos^2 60^\circ,$$

and the direction of polarization of the transmitted light is  $20^\circ$  clockwise from the  $y$  axis. The polarizing direction of the third sheet is  $\theta_3 = 40^\circ$  counterclockwise from the  $y$  axis. Consequently, the angle between the direction of polarization of the light incident on that sheet and the polarizing direction of the sheet is  $20^\circ + 40^\circ = 60^\circ$ . The transmitted intensity is

$$I_3 = I_2 \cos^2 60^\circ = \frac{1}{2} I_0 \cos^4 60^\circ = 3.1 \times 10^{-2} I_0.$$

Thus, 3.1% of the light's initial intensity is transmitted.

34. In this case, we replace  $I_0 \cos^2 70^\circ$  by  $\frac{1}{2} I_0$  as the intensity of the light after passing through the first polarizer. Therefore,

$$I_f = \frac{1}{2} I_0 \cos^2 (90^\circ - 70^\circ) = \frac{1}{2} (43 \text{ W/m}^2) (\cos^2 20^\circ) = 19 \text{ W/m}^2.$$

45. Note that the normal to the refracting surface is vertical in the diagram. The angle of refraction is  $\theta_2 = 90^\circ$  and the angle of incidence is given by  $\tan \theta_1 = L/D$ , where  $D$  is the height of the tank and  $L$  is its width. Thus

$$\theta_1 = \tan^{-1} \left( \frac{L}{D} \right) = \tan^{-1} \left( \frac{1.10 \text{ m}}{0.850 \text{ m}} \right) = 52.31^\circ.$$

The law of refraction yields

$$n_1 = n_2 \frac{\sin \theta_2}{\sin \theta_1} = (1.00) \left( \frac{\sin 90^\circ}{\sin 52.31^\circ} \right) = 1.26,$$

where the index of refraction of air was taken to be unity.

47. The law of refraction states

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

We take medium 1 to be the vacuum, with  $n_1 = 1$  and  $\theta_1 = 32.0^\circ$ . Medium 2 is the glass, with  $\theta_2 = 21.0^\circ$ . We solve for  $n_2$ :

$$n_2 = n_1 \frac{\sin \theta_1}{\sin \theta_2} = (1.00) \left( \frac{\sin 32.0^\circ}{\sin 21.0^\circ} \right) = 1.48.$$

57. Reference to Fig. 33-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point  $a$  to point  $f$  in that figure) is related to the tangent of the angle of incidence. Thus, the diameter  $D$  of the circle in question is

$$D = 2h \tan \theta_c = 2h \tan \left[ \sin^{-1} \left( \frac{1}{n_w} \right) \right] = 2(80.0 \text{ cm}) \tan \left[ \sin^{-1} \left( \frac{1}{1.33} \right) \right] = 182 \text{ cm}.$$

58. The critical angle is  $\theta_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.8}\right) = 34^\circ$ .

62. (a) Reference to Fig. 33-24 may help in the visualization of why there appears to be a “circle of light” (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point  $a$  to point  $f$  in that figure) is related to the tangent of the angle of incidence. The diameter of the circle in question is given by  $d = 2h \tan \theta_c$ . For water  $n = 1.33$ , so Eq. 33-47 gives  $\sin \theta_c = 1/1.33$ , or  $\theta_c = 48.75^\circ$ . Thus,

$$d = 2h \tan \theta_c = 2(2.00 \text{ m})(\tan 48.75^\circ) = 4.56 \text{ m}.$$

(b) The diameter  $d$  of the circle will increase if the fish descends (increasing  $h$ ).

65. When examining Fig. 33-61, it is important to note that the angle (measured from the central axis) for the light ray in air,  $\theta$ , is not the angle for the ray in the glass core, which we denote  $\theta'$ . The law of refraction leads to

$$\sin \theta' = \frac{1}{n_1} \sin \theta$$

assuming  $n_{\text{air}} = 1$ . The angle of incidence for the light ray striking the coating is the complement of  $\theta'$ , which we denote as  $\theta'_{\text{comp}}$ , and recall that

$$\sin \theta'_{\text{comp}} = \cos \theta' = \sqrt{1 - \sin^2 \theta'}.$$

In the critical case,  $\theta'_{\text{comp}}$  must equal  $\theta_c$  specified by Eq. 33-47. Therefore,

$$\frac{n_2}{n_1} = \sin \theta'_{\text{comp}} = \sqrt{1 - \sin^2 \theta'} = \sqrt{1 - \left(\frac{1}{n_1} \sin \theta\right)^2}$$

which leads to the result:  $\sin \theta = \sqrt{n_1^2 - n_2^2}$ . With  $n_1 = 1.58$  and  $n_2 = 1.53$ , we obtain

$$\theta = \sin^{-1}(1.58^2 - 1.53^2) = 23.2^\circ.$$