Chapter 30

2. Using Faraday's law, the induced emf is

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d\left(BA\right)}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(\pi r^2\right)}{dt} = -2\pi r B\frac{dr}{dt}\\ &= -2\pi\left(0.12\,\text{m}\right)\left(0.800\,\text{T}\right)\left(-0.750\,\text{m/s}\right)\\ &= 0.452\,\text{V}.\end{aligned}$$

3. **THINK** Changing the current in the solenoid changes the flux, and therefore, induces a current in the coil.

EXPRESS Using Faraday's law, the total induced emf is given by

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NA\left(\frac{dB}{dt}\right) = -NA\frac{d}{dt}(\mu_0 n i) = -N\mu_0 n A\frac{di}{dt} = -N\mu_0 n(\pi r^2)\frac{di}{dt}$$

By Ohm's law, the induced current in the coil is $i_{\text{ind}} = |\varepsilon|/R,$ where R is the resistance of the coil.

ANALYZE Substituting the values given, we obtain

$$\begin{aligned}\varepsilon &= -N\mu_0 n(\pi r^2)\frac{di}{dt} = -(120)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(22000/\text{m})\,\pi\left(0.016\,\text{m}\right)^2\left(\frac{1.5\text{ A}}{0.025\text{ s}}\right)\\ &= 0.16\,\text{V}.\end{aligned}$$

Ohm's law then yields $i_{\text{ind}} = \dfrac{|\varepsilon|}{R} = \dfrac{0.016\text{ V}}{5.3\,\Omega} = 0.030\text{ A}.$

LEARN The direction of the induced current can be deduced from Lenz's law, which states that the direction of the induced current is such that the magnetic field which it produces opposes the change in flux that induces the current.

4. (a) We use $\varepsilon = -d\Phi_B/dt = -\pi r^2 dB/dt$. For $0 < t < 2.0$ s:

$$\varepsilon = -\pi r^2\frac{dB}{dt} = -\pi\left(0.12\,\text{m}\right)^2\left(\frac{0.5\,\text{T}}{2.0\,\text{s}}\right) = -1.1\times10^{-2}\text{ V}.$$

(b) For 2.0 s < t < 4.0 s: $\varepsilon \propto dB/dt = 0$.

(c) For 4.0 s < t < 6.0 s:

$$\varepsilon = -\pi r^2 \frac{dB}{dt} = -\pi(0.12\,\text{m})^2\left(\frac{-0.5\,\text{T}}{6.0\,\text{s} - 4.0\,\text{s}}\right) = 1.1\times10^{-2}\ \text{V}.$$

7. (a) The magnitude of the emf is

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}\left(6.0t^2 + 7.0t\right) = 12t + 7.0 = 12(2.0) + 7.0 = 31\,\text{mV}.$$

(b) Appealing to Lenz's law (especially Fig. 30-5(a)) we see that the current flow in the loop is clockwise. Thus, the current is to the left through R.

11. (a) It should be emphasized that the result, given in terms of $\sin(2\pi ft)$, could as easily be given in terms of $\cos(2\pi ft)$ or even $\cos(2\pi ft + \phi)$ where ϕ is a phase constant as discussed in Chapter 15. The angular position θ of the rotating coil is measured from some reference line (or plane), and which line one chooses will affect whether the magnetic flux should be written as $BA\cos\theta$, $BA\sin\theta$ or $BA\cos(\theta + \phi)$. Here our choice is such that $\Phi_B = BA\cos\theta$. Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ (equivalent to $\theta = 2\pi ft$) if θ is understood to be in radians (and ω would be the angular velocity). Since the area of the rectangular coil is $A=ab$, Faraday's law leads to

$$\varepsilon = -N\frac{d(BA\cos\theta)}{dt} = -NBA\frac{d\cos(2\pi ft)}{dt} = N\,Bab2\pi f\sin(2\pi ft)$$

which is the desired result, shown in the problem statement. The second way this is written ($\varepsilon_0 \sin(2\pi ft)$) is meant to emphasize that the voltage output is sinusoidal (in its time dependence) and has an amplitude of $\varepsilon_0 = 2\pi f NabB$.

(b) We solve

$$\varepsilon_0 = 150\ \text{V} = 2\pi f NabB$$

when f = 60.0 rev/s and B = 0.500 T. The three unknowns are N, a, and b which occur in a product; thus, we obtain $Nab = 0.796\ \text{m}^2$.

29. (a) Equation 30-8 leads to

$$\varepsilon = BLv = (0.350\ \text{T})(0.250\ \text{m})(0.55\ \text{m/s}) = 0.0481\ \text{V}.$$

(b) By Ohm's law, the induced current is

$$i = 0.0481 \text{ V}/18.0\ \Omega = 0.00267 \text{ A}.$$

By Lenz's law, the current is clockwise in Fig. 30-52.

(c) Equation 26-27 leads to $P = i^2R = 0.000129$ W.

32. Noting that $|\Delta B| = B$, we find the thermal energy is

$$P_{\text{thermal}}\Delta t = \frac{\varepsilon^2 \Delta t}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right)^2 \Delta t = \frac{1}{R}\left(-A\frac{\Delta B}{\Delta t}\right)^2 \Delta t = \frac{A^2B^2}{R\Delta t}$$
$$= \frac{(2.00\times10^{-4}\,\text{m}^2)^2(17.0\times10^{-6}\,\text{T})^2}{(5.21\times10^{-6}\,\Omega)(2.96\times10^{-3}\,\text{s})} = 7.50\times10^{-10}\,\text{J}.$$

44. Since $\varepsilon = -L(di/dt)$, we may obtain the desired induced emf by setting

$$\frac{di}{dt} = -\frac{\varepsilon}{L} = -\frac{60\,\text{V}}{12\,\text{H}} = -5.0\,\text{A/s},$$

or $|di/dt| = 5.0\,\text{A/s}$. We might, for example, uniformly reduce the current from 2.0 A to zero in 40 ms.

51. The current in the circuit is given by $i = i_0 e^{-t/\tau_L}$, where i_0 is the current at time $t = 0$ and τ_L is the inductive time constant (L/R). We solve for τ_L. Dividing by i_0 and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{i}{i_0}\right) = -\frac{t}{\tau_L}.$$

This yields

$$\tau_L = -\frac{t}{\ln(i/i_0)} = -\frac{1.0\,\text{s}}{\ln\left((10\times10^{-3}\,\text{A})/(1.0\,\text{A})\right)} = 0.217\,\text{s}.$$

Therefore, $R = L/\tau_L = 10$ H/0.217 s = 46 Ω.

53. **THINK** The inductor in the *RL* circuit initially acts to oppose changes in current through it.

EXPRESS If the battery is switched into the circuit at $t = 0$, then the current at a later time t is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$.

(a) We want to find the time at which $i = 0.800\varepsilon/R$. This means

$$0.800 = 1 - e^{-t/\tau_L} \;\Rightarrow\; e^{-t/\tau_L} = 0.200.$$

Taking the natural logarithm of both sides, we obtain

$$-(t/\tau_L) = \ln(0.200) = -1.609.$$

Thus,

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30\times10^{-6}\,\text{H})}{1.20\times10^{3}\,\Omega} = 8.45\times10^{-9}\,\text{s}.$$

(b) At $t = 1.0\,\tau_L$ the current in the circuit is

$$i = \frac{\varepsilon}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\text{V}}{1.20\times10^{3}\Omega}\right)(1 - e^{-1.0}) = 7.37\times10^{-3}\ \text{A}.$$

LEARN At $t = 0$, the current in the circuit is zero. However, after a very long time, the inductor acts like an ordinary connecting wire, so the current is

$$i_0 = \frac{\varepsilon}{R} = \frac{14.0\,\text{V}}{1.20\times10^{3}\Omega} = 0.0117\ \text{A}.$$

The current as a function of t/τ_L is plotted to the right.