Chapter 28

2. The force associated with the magnetic field must point in the $\hat{\mathrm{j}}$ direction in order to cancel the force of gravity in the $-\hat{\mathrm{j}}$ direction. By the right-hand rule, $\vec{B}$ points in the $-\hat{\mathrm{k}}$ direction (since $\hat{\mathrm{i}}\times\left(-\hat{\mathrm{k}}\right)=\hat{\mathrm{j}}$). Note that the charge is positive; also note that we need to assume $B_y = 0$. The magnitude $|B_z|$ is given by Eq. 28-3 (with $\phi = 90°$). Therefore, with $m = 1.0\times10^{-2}$ kg, $v = 2.0\times10^{4}$ m/s, and $q = 8.0\times10^{-5}$ C, we find

$$\vec{B} = B_z\hat{\mathrm{k}} = -\left(\frac{mg}{qv}\right)\hat{\mathrm{k}} = (-0.061\ \mathrm{T})\hat{\mathrm{k}}\ .$$

3. (a) The force on the electron is

$$\begin{aligned}\vec{F}_B &= q\vec{v}\times\vec{B} = q\left(v_x\hat{\mathrm{i}}+v_y\hat{\mathrm{j}}\right)\times\left(B_x\hat{\mathrm{i}}+B_y\vec{j}\right) = q\left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}} \\ &= \left(-1.6\times10^{-19}\,\mathrm{C}\right)\left[\left(2.0\times10^{6}\,\mathrm{m/s}\right)\left(-0.15\,\mathrm{T}\right) - \left(3.0\times10^{6}\,\mathrm{m/s}\right)\left(0.030\,\mathrm{T}\right)\right] \\ &= \left(6.2\times10^{-14}\,\mathrm{N}\right)\hat{\mathrm{k}}.\end{aligned}$$

Thus, the magnitude of $\vec{F}_B$ is 6.2×10^{14} N, and $\vec{F}_B$ points in the positive z direction.

(b) This amounts to repeating the above computation with a change in the sign in the charge. Thus, $\vec{F}_B$ has the same magnitude but points in the negative z direction, namely, $\vec{F}_B = -\left(6.2\times10^{-14}\,\mathrm{N}\right)\hat{\mathrm{k}}.$

5. Using Eq. 28-2 and Eq. 3-30, we obtain

$$\vec{F} = q\left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}} = q\left(v_x\left(3B_x\right) - v_yB_x\right)\hat{\mathrm{k}}$$

where we use the fact that $B_y = 3B_x$. Since the force (at the instant considered) is $F_z\,\hat{\mathrm{k}}$ where $F_z = 6.4 \times 10^{-19}$ N, then we are led to the condition

$$q\left(3v_x - v_y\right)B_x = F_z \quad\Rightarrow\quad B_x = \frac{F_z}{q\left(3v_x - v_y\right)}.$$

Substituting $v_x = 2.0$ m/s, $v_y = 4.0$ m/s, and $q = -1.6 \times 10^{-19}$ C, we obtain

$$B_x = \frac{F_z}{q(3v_x - v_y)} = \frac{6.4\times10^{-19}\ \text{N}}{(-1.6\times10^{-19}\,\text{C})[3(2.0\ \text{m/s}) - 4.0\ \text{m}]} = -2.0\ \text{T}.$$

8. Letting $\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) = 0$, we get

$$vB\sin\phi = E\ .$$

We note that (for given values of the fields) this gives a minimum value for speed whenever the sin ϕ factor is at its maximum value (which is 1, corresponding to ϕ = 90°). So

$$v_{\min} = \frac{E}{B} = \frac{1.50\times10^{3}\ \text{V/m}}{0.400\ \text{T}} = 3.75\times10^{3}\ \text{m/s}\,.$$

23. From Eq. 28-16, we find

$$B = \frac{m_e v}{er} = \frac{9.11\times10^{-31}\text{kg}\quad 1.30\times10^{6}\ \text{m/s}}{1.60\times10^{-19}\text{C}\quad 0.350\ \text{m}} = 2.11\times10^{-5}\ \text{T}.$$

41. (a) The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force *mg* on the wire. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by $F_B = iLB$, where L is the length of the wire. Thus,

$$iLB = mg \quad\Rightarrow\quad i = \frac{mg}{LB} = \frac{0.0130\,\text{kg}\quad 9.8\,\text{m/s}^2}{0.620\,\text{m}\quad 0.440\,\text{T}} = 0.467\,\text{A}.$$

(b) Applying the right-hand rule reveals that the current must be from left to right.

45. The magnetic force on the wire is

$$\begin{aligned}\vec{F}_B &= i\vec{L}\times\vec{B} = iL\hat{\text{i}}\times\ B_y\hat{\text{j}} + B_z\hat{\text{k}}\ = iL\ -B_z\hat{\text{j}} + B_y\hat{\text{k}} \\ &=\ 0.500\text{A}\quad 0.500\text{m}\ \left[-\ 0.0100\text{T}\ \hat{\text{j}} +\ 0.00300\text{T}\ \hat{\text{k}}\right] \\ &=\ -2.50\times10^{-3}\hat{\text{j}} + 0.750\times10^{-3}\hat{\text{k}}\ \ \text{N}.\end{aligned}$$

49. The applied field has two components: $B_x > 0$ and $B_z > 0$. Considering each straight segment of the rectangular coil, we note that Eq. 28-26 produces a nonzero force only for the component of $\vec{B}$ that is perpendicular to that segment; we also note that the equation is effectively multiplied by N = 20 due to the fact that this is a 20-turn coil. Since we wish

to compute the torque about the hinge line, we can ignore the force acting on the straight segment of the coil that lies along the y axis (forces acting at the axis of rotation produce no torque about that axis). The top and bottom straight segments experience forces due to Eq. 28-26 (caused by the B_z component), but these forces are (by the right-hand rule) in the $\pm y$ directions and are thus unable to produce a torque about the y axis. Consequently, the torque derives completely from the force exerted on the straight segment located at $x = 0.050$ m, which has length $L = 0.10$ m and is shown in Figure 28-44 carrying current in the $-y$ direction. Now, the B_z component will produce a force on this straight segment which points in the $-x$ direction (back towards the hinge) and thus will exert no torque about the hinge. However, the B_x component (which is equal to $B\cos\theta$ where $B = 0.50$ T and $\theta = 30°$) produces a force equal to $NiLB_x$ that points (by the right-hand rule) in the $+z$ direction. Since the action of this force is perpendicular to the plane of the coil, and is located a distance x away from the hinge, then the torque has magnitude

$$\tau = (NiLB_x)\,x = NiLxB\cos\theta = (20)(0.10\text{ A})(0.10\text{ m})(0.050\text{ m})(0.50\text{ T})\cos 30°$$
$$= 0.0043\text{ N}\cdot\text{m}.$$

Since $\vec{\tau} = \vec{r} \times \vec{F}$, the direction of the torque is $-y$. In unit-vector notation, the torque is $\vec{\tau} = (-4.3\times10^{-3}\text{ N}\cdot\text{m})\hat{\text{j}}$.

An alternative way to do this problem is through the use of Eq. 28-37. We do not show those details here, but note that the magnetic moment vector (a necessary part of Eq. 28-37) has magnitude

$$|\vec{\mu}| = NiA = (20)(0.10\text{ A})(0.0050\text{ m}^2)$$

and points in the $-z$ direction. At this point, Eq. 3-30 may be used to obtain the result for the torque vector.

51. We use Eq. 28-37 where $\vec{\mu}$ is the magnetic dipole moment of the wire loop and $\vec{B}$ is the magnetic field, as well as Newton's second law. Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity mg, acting downward from the center of mass, the normal force of the incline F_N, acting perpendicularly to the incline through the center of mass, and the force of friction f, acting up the incline at the point of contact. We take the x axis to be positive down the incline. Then the x component of Newton's second law for the center of mass yields

$$mg\sin\theta - f = ma.$$

For purposes of calculating the torque, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B\sin\theta$, and the force of friction produces a torque with magnitude fr, where r is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction, and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B \sin\theta = I\alpha.$$

Since we want the current that holds the cylinder in place, we set $a = 0$ and $\alpha = 0$, and use one equation to eliminate f from the other. The result is $mgr = \mu B$. The loop is rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $\mu = NiA = Ni(2rL)$. Thus, $mgr = 2NirLB$ and

$$i = \frac{mg}{2NLB} = \frac{(0.250\,\text{kg})(9.8\,\text{m/s}^2)}{2(10.0)(0.100\,\text{m})(0.500\,\text{T})} = 2.45\,\text{A}.$$