Chapter 27

1. (a) Let i be the current in the circuit and take it to be positive if it is to the left in R_1. We use Kirchhoff's loop rule: $\varepsilon_1 - iR_2 - iR_1 - \varepsilon_2 = 0$. We solve for i:

$$i = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{12\ \text{V} - 6.0\ \text{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\ \text{A}.$$

A positive value is obtained, so the current is counterclockwise around the circuit.

If i is the current in a resistor R, then the power dissipated by that resistor is given by $P = i^2 R$.

(b) For R_1, $P_1 = i^2 R_1 = (0.50\ \text{A})^2(4.0\ \Omega) = 1.0$ W,

(c) and for R_2, $P_2 = i^2 R_2 = (0.50\ \text{A})^2\,(8.0\ \Omega) = 2.0$ W.

If i is the current in a battery with emf ε, then the battery supplies energy at the rate $P = i\varepsilon$ provided the current and emf are in the same direction. The battery absorbs energy at the rate $P = i\varepsilon$ if the current and emf are in opposite directions.

(d) For ε_1, $P_1 = i\varepsilon_1 = (0.50\ \text{A})(12\ \text{V}) = 6.0$ W

(e) and for ε_2, $P_2 = i\varepsilon_2 = (0.50\ \text{A})(6.0\ \text{V}) = 3.0$ W.

(f) In battery 1 the current is in the same direction as the emf. Therefore, this battery supplies energy to the circuit; the battery is discharging.

(g) The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

5. The chemical energy of the battery is reduced by $\Delta E = q\varepsilon$, where q is the charge that passes through in time $\Delta t = 6.0$ min, and ε is the emf of the battery. If i is the current, then $q = i\,\Delta t$ and

$$\Delta E = i\varepsilon\,\Delta t = (5.0\ \text{A})(6.0\ \text{V})\,(6.0\ \text{min})\,(60\ \text{s/min}) = 1.1 \times 10^4\ \text{J}.$$

We note the conversion of time from minutes to seconds.

6. (a) The cost is $(100\ \text{W} \cdot 8.0\ \text{h}/2.0\ \text{W} \cdot \text{h})\,(\$0.80) = \$3.2 \times 10^2$.

(b) The cost is (100 W · 8.0 h/10^3 W · h) ($0.06) = $0.048 = 4.8 cents.

7. (a) The energy transferred is

$$U = Pt = \frac{\varepsilon^2 t}{r+R} = \frac{(2.0\ \text{V})^2(2.0\,\text{min})(60\,\text{s}/\text{min})}{1.0\,\Omega + 5.0\,\Omega} = 80\ \text{J}.$$

(b) The amount of thermal energy generated is

$$U' = i^2 Rt = \left(\frac{\varepsilon}{r+R}\right)^2 Rt = \left(\frac{2.0\ \text{V}}{1.0\,\Omega + 5.0\,\Omega}\right)^2 (5.0\,\Omega)(2.0\ \text{min})(60\ \text{s}/\text{min}) = 67\ \text{J}.$$

(c) The difference between U and U', which is equal to 13 J, is the thermal energy that is generated in the battery due to its internal resistance.

10. (a) We solve $i = (\varepsilon_2 - \varepsilon_1)/(r_1 + r_2 + R)$ for R:

$$R = \frac{\varepsilon_2 - \varepsilon_1}{i} - r_1 - r_2 = \frac{3.0\ \text{V} - 2.0\ \text{V}}{1.0\times 10^{-3}\ \text{A}} - 3.0\,\Omega - 3.0\,\Omega = 9.9\times 10^2\ \Omega.$$

(b) $P = i^2R = (1.0 \times 10^{-3}\ \text{A})^2(9.9 \times 10^2\ \Omega) = 9.9 \times 10^{-4}$ W.

15. Let the emf be V. Then $V = iR = i'(R + R')$, where $i = 5.0$ A, $i' = 4.0$ A, and $R' = 2.0\ \Omega$. We solve for R:

$$R = \frac{i'R'}{i - i'} = \frac{(4.0\ \text{A})(2.0\,\Omega)}{5.0\ \text{A} - 4.0\ \text{A}} = 8.0\ \Omega.$$

18. The currents i_1, i_2 and i_3 are obtained from Eqs. 27-18 through 27-20:

$$i_1 = \frac{\varepsilon_1(R_2+R_3) - \varepsilon_2 R_3}{R_1R_2 + R_2R_3 + R_1R_3} = \frac{(4.0\text{V})(10\ \Omega + 5.0\,\Omega) - (1.0\,\text{V})(5.0\ \Omega)}{(10\ \Omega)(10\ \Omega) + (10\ \Omega)(5.0\ \Omega) + (10\,\Omega)(5.0\ \Omega)} = 0.275\ \text{A},$$

$$i_2 = \frac{\varepsilon_1 R_3 - \varepsilon_2(R_1+R_2)}{R_1R_2 + R_2R_3 + R_1R_3} = \frac{(4.0\ \text{V})(5.0\,\Omega) - (1.0\ \text{V})(10\,\Omega + 5.0\ \Omega)}{(10\,\Omega)(10\ \Omega) + (10\,\Omega)(5.0\,\Omega) + (10\,\Omega)(5.0\ \Omega)} = 0.025\ \text{A},$$

$$i_3 = i_2 - i_1 = 0.025\text{A} - 0.275\text{A} = -0.250\text{A}\ .$$

$V_d - V_c$ can now be calculated by taking various paths. Two examples: from $V_d - i_2R_2 = V_c$ we get

$$V_d - V_c = i_2R_2 = (0.0250\ \text{A})\ (10\ \Omega) = +0.25\ \text{V};$$

from $V_d + i_3R_3 + \varepsilon_2 = V_c$ we get

$$V_d - V_c = i_3R_3 - \varepsilon_2 = -(-0.250\text{ A})(5.0\ \Omega) - 1.0\text{ V} = +0.25\text{ V}.$$

22. (a) R_{eq} (FH) = (10.0 Ω)(10.0 Ω)(5.00 Ω)/[(10.0 Ω)(10.0 Ω) + 2(10.0 Ω)(5.00 Ω)] = 2.50 Ω.

(b) R_{eq} (FG) = (5.00 Ω) R/(R + 5.00 Ω), where

$$R = 5.00\ \Omega + (5.00\ \Omega)(10.0\ \Omega)/(5.00\ \Omega + 10.0\ \Omega) = 8.33\ \Omega.$$

So R_{eq} (FG) = (5.00 Ω)(8.33 Ω)/(5.00 Ω + 8.33 Ω) = 3.13 Ω.

26. The part of R_0 connected in parallel with R is given by $R_1 = R_0x/L$, where L = 10 cm. The voltage difference across R is then $V_R = \varepsilon R'/R_{eq}$, where $R' = RR_1/(R + R_1)$ and

$$R_{eq} = R_0(1 - x/L) + R'.$$

Thus,

$$P_R = \frac{V_R^2}{R} = \frac{1}{R}\left(\frac{\varepsilon RR_1/(R+R_1)}{R_0(1-x/L)+RR_1/(R+R_1)}\right)^2 = \frac{100R(\varepsilon x/R_0)^2}{\left(100\,R/R_0+10x-x^2\right)^2},$$

where x is measured in cm.

64. (a) The potential difference V across the plates of a capacitor is related to the charge q on the positive plate by $V = q/C$, where C is capacitance. Since the charge on a discharging capacitor is given by $q = q_0\,e^{-t/\tau}$, this means $V = V_0\,e^{-t/\tau}$ where V_0 is the initial potential difference. We solve for the time constant τ by dividing by V_0 and taking the natural logarithm:

$$\tau = -\frac{t}{\ln(V/V_0)} = -\frac{10.0\text{ s}}{\ln\left[(1.00\text{ V})/(100\text{ V})\right]} = 2.17\text{ s}.$$

(b) At t = 17.0 s, t/τ = (17.0 s)/(2.17 s) = 7.83, so

$$V = V_0e^{-t/\tau} = (100\text{ V})e^{-7.83} = 3.96\times10^{-2}\text{ V}.$$