Chapter 25

3. (a) The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 A/d$, where A is the area of each plate and d is the plate separation. Since the plates are circular, the plate area is $A = \pi R^2$, where R is the radius of a plate. Thus,

$$C = \frac{\varepsilon_0 \pi R^2}{d} = \frac{\left(8.85\times10^{-12}\ \text{F/m}\right)\pi\left(8.2\times10^{-2}\,\text{m}\right)^2}{1.3\times10^{-3}\text{m}} = 1.44\times10^{-10}\,\text{F} = 144\ \text{pF}.$$

(b) The charge on the positive plate is given by $q = CV$, where V is the potential difference across the plates. Thus,

$$q = (1.44 \times 10^{-10}\ \text{F})(120\ \text{V}) = 1.73 \times 10^{-8}\ \text{C} = 17.3\ \text{nC}.$$

5. Assuming conservation of volume, we find the radius of the combined spheres, then use $C = 4\pi\varepsilon_0 R$ to find the capacitance. When the drops combine, the volume is doubled. It is then $V = 2(4\pi/3)R^3$. The new radius R' is given by

$$\frac{4p}{3}\left(R'\right)^3 = 2\frac{4p}{3}R^3 \quad \Rightarrow \quad R' = 2^{1/3}R.$$

The new capacitance is

$$C' = 4p\varepsilon_0 R' = 4p\varepsilon_0 2^{1/3} R = 5.04p\varepsilon_0 R.$$

With R = 2.00 mm, we obtain $C = 5.04\pi\left(8.85\times10^{-12}\ \text{F/m}\right)\left(2.00\times10^{-3}\text{m}\right) = 2.80\times10^{-13}\,\text{F}$.

8. The equivalent capacitance is given by $C_{\text{eq}} = q/V$, where q is the total charge on all the capacitors and V is the potential difference across any one of them. For N identical capacitors in parallel, $C_{\text{eq}} = NC$, where C is the capacitance of one of them. Thus, $NC = q/V$ and

$$N = \frac{q}{VC} = \frac{1.00\text{C}}{(110\text{V})\left(1.00\times10^{-6}\,\text{F}\right)} = 9.09\times10^3.$$

17. (a) and (b) The original potential difference V_1 across C_1 is

$$V_1 = \frac{C_{\text{eq}}V}{C_1 + C_2} = \frac{(3.16\,\mu\text{F})(100.0\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 21.1\text{V}.$$

Thus ΔV_1 = 100.0 V – 21.1 V = 78.9 V and

$$\Delta q_1 = C_1 \Delta V_1 = (10.0\ \mu\text{F})(78.9\ \text{V}) = 7.89 \times 10^{-4}\ \text{C}.$$

31. The total energy is the sum of the energies stored in the individual capacitors. Since they are connected in parallel, the potential difference V across the capacitors is the same and the total energy is

$$U = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}\left(2.0\times10^{-6}\ \text{F} + 4.0\times10^{-6}\ \text{F}\right)(300\ \text{V})^2 = 0.27\ \text{J}.$$

39. (a) They each store the same charge, so the maximum voltage is across the smallest capacitor. With 100 V across 10 μF, then the voltage across the 20 μF capacitor is 50 V and the voltage across the 25 μF capacitor is 40 V. Therefore, the voltage across the arrangement is 190 V.

(b) Using Eq. 25-21 or Eq. 25-22, we sum the energies on the capacitors and obtain $U_{\text{total}} = 0.095$ J.

40. If the original capacitance is given by $C = \varepsilon_0 A/d$, then the new capacitance is $C' = \varepsilon_0 \kappa A/2d$. Thus $C'/C = \kappa/2$ or

$$\kappa = 2C'/C = 2(2.6\ \text{pF}/1.3\ \text{pF}) = 4.0.$$

43. The capacitance with the dielectric in place is given by $C = \kappa C_0$, where C_0 is the capacitance before the dielectric is inserted. The energy stored is given by $U = \frac{1}{2}CV^2 = \frac{1}{2}\kappa C_0 V^2$, so

$$\kappa = \frac{2U}{C_0 V^2} = \frac{2(7.4\times10^{-6}\ \text{J})}{(7.4\times10^{-12}\ \text{F})(652\ \text{V})^2} = 4.7.$$

According to Table 25-1, you should use Pyrex.

76. One way to approach this is to note that since they are identical, the voltage is evenly divided between them. That is, the voltage across each capacitor is $V = (10/n)$ volt. With $C = 2.0 \times 10^{-6}$ F, the electric energy stored by each capacitor is $\frac{1}{2}CV^2$. The total energy stored by the capacitors is n times that value, and the problem requires the total be equal to 25×10^{-6} J. Thus,

$$\frac{n}{2}(2.0 \times 10^{-6})\left(\frac{10}{n}\right)^2 = 25 \times 10^{-6},$$

which leads to $n = 4$.