Chapter 22

2. (a) We note that the electric field points leftward at both points. Using $\vec{F} = q_0\vec{E}$, and orienting our x axis rightward (so $\hat{\text{i}}$ points right in the figure), we find

$$\vec{F} = \left(+1.6\times10^{-19}\,\text{C}\right)\left(-40\,\frac{\text{N}}{\text{C}}\,\hat{\text{i}}\right) = (-6.4\times10^{-18}\,\text{N})\,\hat{\text{i}}$$

which means the magnitude of the force on the proton is 6.4×10^{-18} N and its direction $(-\hat{\text{i}})$ is leftward.

(b) As the discussion in Section 22-2 makes clear, the field strength is proportional to the "crowdedness" of the field lines. It is seen that the lines are twice as crowded at A than at B, so we conclude that $E_A = 2E_B$. Thus, $E_B = 20$ N/C.

4. With $x_1 = 6.00$ cm and $x_2 = 21.00$ cm, the point midway between the two charges is located at $x = 13.5$ cm. The values of the charge are

$$q_1 = -q_2 = -\,2.00 \times 10^{-7}\ \text{C},$$

and the magnitudes and directions of the individual fields are given by:

$$\vec{E}_1 = -\frac{|q_1|}{4\pi\varepsilon_0(x-x_1)^2}\,\hat{\text{i}} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\,|-2.00\times10^{-7}\,\text{C}|}{\left(0.135\ \text{m}-0.060\ \text{m}\right)^2}\,\hat{\text{i}} = -(3.196\times10^5\ \text{N/C})\hat{\text{i}}$$

$$\vec{E}_2 = -\frac{q_2}{4\pi\varepsilon_0(x-x_2)^2}\,\hat{\text{i}} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\,(2.00\times10^{-7}\,\text{C})}{\left(0.135\ \text{m}-0.210\ \text{m}\right)^2}\,\hat{\text{i}} = -(3.196\times10^5\ \text{N/C})\hat{\text{i}}$$

Thus, the net electric field is $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = -(6.39\times10^5\ \text{N/C})\hat{\text{i}}$.

6. We find the charge magnitude $|q|$ from $E = |q|/4\pi\varepsilon_0 r^2$:

$$q = 4\pi\varepsilon_0 E r^2 = \frac{\left(1.00\,\text{N/C}\right)\left(1.00\,\text{m}\right)^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.11\times10^{-10}\,\text{C}.$$

14. (a) The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 22-3, where the absolute value signs for q_2 are unnecessary since this charge is positive. Whether we add the magnitudes or subtract them depends on whether $\vec{E}_1$ is in the same, or opposite,

direction as $\vec{E}_2$. At points left of q_1 (on the $-x$ axis) the fields point in opposite directions, but there is no possibility of cancellation (zero net field) since $|\vec{E}_1|$ is everywhere bigger than $|\vec{E}_2|$ in this region. In the region between the charges ($0 < x < L$) both fields point leftward and there is no possibility of cancellation. At points to the right of q_2 (where $x > L$), $\vec{E}_1$ points leftward and $\vec{E}_2$ points rightward so the net field in this range is

$$\vec{E}_{\text{net}} = \left(|\vec{E}_2| - |\vec{E}_1|\right)\hat{\text{i}}\,.$$

Although $|q_1| > q_2$ there is the possibility of $\vec{E}_{\text{net}} = 0$ since these points are closer to q_2 than to q_1. Thus, we look for the zero net field point in the $x > L$ region:

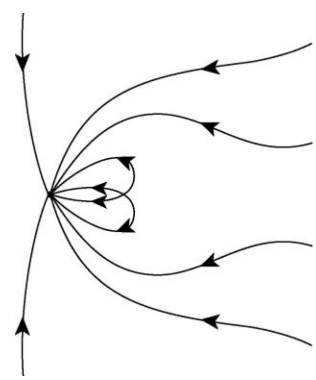
$$|\vec{E}_1| = |\vec{E}_2| \quad \Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(x-L)^2}$$

which leads to

$$\frac{x-L}{x} = \sqrt{\frac{q_2}{|q_1|}} = \sqrt{\frac{2}{5}}.$$

Thus, we obtain $x = \dfrac{L}{1-\sqrt{2/5}} \approx 2.72L$.

(b) A sketch of the field lines is shown in the figure below:



42. (a) $F_e = Ee = (3.0 \times 10^6 \text{ N/C})(1.6 \times 10^{-19} \text{ C}) = 4.8 \times 10^{-13}$ N.

(b) $F_i = Eq_{\text{ion}} = Ee = (3.0 \times 10^6 \text{ N/C})(1.6 \times 10^{-19} \text{ C}) = 4.8 \times 10^{-13}$ N.

43. **THINK** The acceleration of the electron is given by Newton's second law: $F = ma$, where F is the electrostatic force.

EXPRESS The magnitude of the force acting on the electron is $F = eE$, where E is the magnitude of the electric field at its location. Using Newton's second law, the acceleration of the electron is

$$a = \frac{F}{m} = \frac{eE}{m}.$$

ANALYZE With $e = 1.6 \times 10^{-19}$ C, $E = 2.00 \times 10^4$ N/C, and $m = 9.11 \times 10^{-31}$ kg, we find the acceleration to be

$$a = \frac{eE}{m} = \frac{(1.60 \times 10^{-19}\ \text{C})(2.00 \times 10^4\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}} = 3.51 \times 10^{15}\ \text{m/s}^2.$$

LEARN In vector notation, $\vec{a} = \vec{F}/m = -e\vec{E}/m$, so $\vec{a}$ is in the opposite direction of $\vec{E}$. The magnitude of electron's acceleration is proportional to the field strength E: the greater the value of E, the greater the acceleration.

47. **THINK** The acceleration of the proton is given by Newton's second law: $F = ma$, where F is the electrostatic force.

EXPRESS The magnitude of the force acting on the proton is $F = eE$, where E is the magnitude of the electric field. According to Newton's second law, the acceleration of the proton is $a = F/m = eE/m$, where m is the mass of the proton. Thus,

$$a = \frac{F}{m} = \frac{eE}{m}.$$

We assume that the proton starts from rest ($v_0 = 0$) and apply the kinematic equation $v^2 = v_0^2 + 2ax$ (or else $x = \frac{1}{2}at^2$ and $v = at$). Thus, the speed of the proton after having traveling a distance x is $v = \sqrt{2ax}$.

ANALYZE (a) With $e = 1.6 \times 10^{-19}$ C, $E = 2.00 \times 10^4$ N/C, and $m = 1.67 \times 10^{-27}$ kg, we find the acceleration to be

$$a = \frac{eE}{m} = \frac{(1.60 \times 10^{-19}\ \text{C})(2.00 \times 10^4\ \text{N/C})}{1.67 \times 10^{-27}\ \text{kg}} = 1.92 \times 10^{12}\ \text{m/s}^2.$$

(b) With $x = 1.00\ \text{cm} = 1.0 \times 10^{-2}$ m, the speed of the proton is

$$v = \sqrt{2ax} = \sqrt{2\left(1.92\times10^{12}\ \text{m/s}^2\right)\left(0.0100\,\text{m}\right)} = 1.96\times10^{5}\ \text{m/s}.$$

LEARN The time it takes for the proton to attain the final speed is

$$t = \frac{v}{a} = \frac{1.96\times10^{5}\ \text{m/s}}{1.92\times10^{12}\ \text{m/s}^2} = 1.02\times10^{-7}\ \text{s}.$$

The distance the proton travels can be written as

$$x = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{eE}{m}\right)t^2 .$$

52. (a) Due to the fact that the electron is negatively charged, then (as a consequence of Eq. 22-28 and Newton's second law) the field $\vec{E}$ pointing in the same direction as the velocity leads to deceleration. Thus, with $t = 1.5\times10^{-9}$ s, we find

$$v = v_0 - |a|t = v_0 - \frac{eE}{m}t = 4.0\times10^{4}\ \text{m/s} - \frac{(1.6\times10^{-19}\ \text{C})(50\ \text{N/C})}{9.11\times10^{-31}\ \text{kg}}(1.5\times10^{-9}\ \text{s})$$
$$= 2.7\times10^{4}\ \text{m/s}\,.$$

(b) The displacement is equal to the distance since the electron does not change its direction of motion. The field is uniform, which implies the acceleration is constant. Thus,

$$d = \frac{v+v_0}{2}t = 5.0\times10^{-5}\,\text{m}.$$

53. We take the positive direction to be to the right in the figure. The acceleration of the proton is $a_p = eE/m_p$ and the acceleration of the electron is $a_e = -eE/m_e$, where E is the magnitude of the electric field, m_p is the mass of the proton, and m_e is the mass of the electron. We take the origin to be at the initial position of the proton. Then, the coordinate of the proton at time t is $x = \frac{1}{2}a_p t^2$ and the coordinate of the electron is $x = L + \frac{1}{2}a_e t^2$. They pass each other when their coordinates are the same, or

$$\frac{1}{2}a_p t^2 = L + \frac{1}{2}a_e t^2 .$$

This means $t^2 = 2L/(a_p - a_e)$ and

$$x=\frac{a_p}{a_p-a_e}L=\frac{eE/m_p}{\left(eE/m_p\right)+\left(eE/m_e\right)}L=\left(\frac{m_e}{m_e+m_p}\right)L$$
$$=\left(\frac{9.11\times10^{-31}\text{kg}}{9.11\times10^{-31}\text{kg}+1.67\times10^{-27}\text{kg}}\right)(0.050\text{ m})$$
$$=2.7\times10^{-5}\text{m.}$$

54. Due to the fact that the electron is negatively charged, then (as a consequence of Eq. 22-28 and Newton's second law) the field $\vec{E}$ pointing in the $+y$ direction (which we will call "upward") leads to a downward acceleration. This is exactly like a projectile motion problem as treated in Chapter 4 (but with g replaced with $a = eE/m = 8.78 \times 10^{11}$ m/s^2). Thus, Eq. 4-21 gives

$$t=\frac{x}{v_0\cos\theta_0}=\frac{3.00\text{ m}}{(2.00\times10^6\text{ m/s})\cos40.0°}=1.96\times10^{-6}\text{ s}.$$

This leads (using Eq. 4-23) to

$$v_y=v_0\sin\theta_0-at=(2.00\times10^6\text{ m/s})\sin40.0°-(8.78\times10^{11}\text{ m/s}^2)(1.96\times10^{-6}\text{ s})$$
$$=-4.34\times10^5\text{ m/s}.$$

Since the x component of velocity does not change, then the final velocity is

$$\vec{v} = (1.53\times10^6\text{ m/s})\,\hat{\text{i}} - (4.34\times10^5\text{ m/s})\,\hat{\text{j}}.$$

55. (a) We use $\Delta x = v_{\text{avg}}t = vt/2$:

$$v=\frac{2\Delta x}{t}=\frac{2\left(2.0\times10^{-2}\text{ m}\right)}{1.5\times10^{-8}\text{s}}=2.7\times10^6\text{ m/s}.$$

(b) We use $\Delta x=\frac{1}{2}at^2$ and $E = F/e = ma/e$:

$$E=\frac{ma}{e}=\frac{2\Delta xm}{et^2}=\frac{2\left(2.0\times10^{-2}\text{m}\right)\left(9.11\times10^{-31}\text{kg}\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(1.5\times10^{-8}\text{s}\right)^2}=1.0\times10^3\text{ N/C}.$$

56. (a) Equation 22-33 leads to $\tau=pE\sin0°=0.$

(b) With $\theta=90°$, the equation gives

$$\tau=pE=\left(2\left(1.6\times10^{-19}\text{ C}\right)\left(0.78\times10^{-9}\text{ m}\right)\right)\left(3.4\times10^6\text{ N/C}\right)=8.5\times10^{-22}\text{ N}\cdot\text{m}.$$

(c) Now the equation gives $\tau = pE\sin 180° = 0.$

57. **THINK** The potential energy of the electric dipole placed in an electric field depends on its orientation relative to the electric field.

EXPRESS The magnitude of the electric dipole moment is $p = qd$, where q is the magnitude of the charge, and d is the separation between the two charges. When placed in an electric field, the potential energy of the dipole is given by Eq. 22-38:

$$U(\theta) = -\vec{p}\cdot\vec{E} = -pE\cos\theta\,.$$

Therefore, if the initial angle between $\vec{p}$ and $\vec{E}$ is θ_0 and the final angle is θ, then the change in potential energy would be

$$\Delta U = U(\theta) - U_0(\theta) = -pE\left(\cos\theta - \cos\theta_0\right).$$

ANALYZE (a) With $q = 1.50\times10^{-9}$ C and $d = 6.20\times10^{-6}$ m, we find the magnitude of the dipole moment to be

$$p = qd = \left(1.50\times10^{-9}\text{ C}\right)\left(6.20\times10^{-6}\text{ m}\right) = 9.30\times10^{-15}\text{ C}\cdot\text{m}.$$

(b) The initial and the final angles are $\theta_0 = 0$ (parallel) and $\theta = 180°$ (anti-parallel), so we find ΔU to be

$$\Delta U = U\left(180°\right) - U\left(0\right) = 2pE = 2\left(9.30\times10^{-15}\text{C}\cdot\text{m}\right)\left(1100\text{ N/C}\right) = 2.05\times10^{-11}\text{J}.$$

LEARN The potential energy is a maximum ($U_{\text{max}} = +pE$) when the dipole is oriented antiparallel to $\vec{E}$, and is a minimum ($U_{\text{min}} = -pE$) when it is parallel to $\vec{E}$.

58. Examining the lowest value on the graph, we have (using Eq. 22-38)

$$U = -\vec{p}\cdot\vec{E} = -1.00\times10^{-28}\text{ J}.$$

If E = 20 N/C, we find $p = 5.0\times10^{-28}$ C·m.

59. Following the solution to part (c) of Sample Problem 22.05 — “Torque and energy of an electric dipole in an electric field,” we find

$$\begin{aligned} W &= U\left(\theta_0+\pi\right) - U\left(\theta_0\right) = -pE\left(\cos\left(\theta_0+\pi\right) - \cos\left(\theta_0\right)\right) = 2pE\cos\theta_0 \\ &= 2(3.02\times10^{-25}\text{C}\cdot\text{m})(46.0\text{ N/C})\cos 64.0° \\ &= 1.22\times10^{-23}\text{ J}. \end{aligned}$$

60. Using Eq. 22-35, considering θ as a variable, we note that it reaches its maximum value when θ = –90°: $\tau_{max} = pE$. Thus, with E = 40 N/C and $\tau_{max} = 100 \times 10^{-28}$ N·m (determined from the graph), we obtain the dipole moment: $p = 2.5 \times 10^{-28}$ C·m.

61. Equation 22-35 $(\tau = -pE\sin\theta)$ captures the sense as well as the magnitude of the effect. That is, this is a restoring torque, trying to bring the tilted dipole back to its aligned equilibrium position. If the amplitude of the motion is small, we may replace sin θ with θ in radians. Thus, $\tau \approx -pE\theta$. Since this exhibits a simple negative proportionality to the angle of rotation, the dipole oscillates in simple harmonic motion, like a torsional pendulum with torsion constant $\kappa = pE$. The angular frequency ω is given by

$$\omega^2 = \frac{\kappa}{I} = \frac{pE}{I}$$

where I is the rotational inertia of the dipole. The frequency of oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}.$$

62. (a) We combine Eq. 22-28 (in absolute value) with Newton's second law:

$$a = \frac{|q|E}{m} = \left(\frac{1.60\times10^{-19}\,\text{C}}{9.11\times10^{-31}\,\text{kg}}\right)\left(1.40\times10^{6}\,\frac{\text{N}}{\text{C}}\right) = 2.46\times10^{17}\ \text{m/s}^2\,.$$

(b) With $v = \dfrac{c}{10} = 3.00\times10^{7}\ \text{m/s}$, we use Eq. 2-11 to find

$$t = \frac{v - v_o}{a} = \frac{3.00\times10^{7}\ \text{m/s}}{2.46\times10^{17}\ \text{m/s}^2} = 1.22\times10^{-10}\ \text{s}.$$

(c) Equation 2-16 gives

$$\Delta x = \frac{v^2 - v_o^2}{2a} = \frac{\left(3.00\times10^{7}\ \text{m/s}\right)^2}{2\left(2.46\times10^{17}\ \text{m/s}^2\right)} = 1.83\times10^{-3}\ \text{m}.$$

63. (a) Using the density of water (ρ = 1000 kg/m^3), the weight mg of the spherical drop (of radius $r = 6.0 \times 10^{-7}$ m) is

$$W = \rho V g = \left(1000\,\text{kg/m}^3\right)\left(\frac{4\pi}{3}\left(6.0\times10^{-7}\ \text{m}\right)^3\right)\left(9.8\,\text{m/s}^2\right) = 8.87\times10^{-15}\,\text{N}.$$

(b) Vertical equilibrium of forces leads to $mg = qE = neE$, which we solve for n, the number of excess electrons:

$$n = \frac{mg}{eE} = \frac{8.87\times10^{-15}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(462\,\text{N/C}\right)} = 120.$$

64. The two closest charges produce fields at the midpoint that cancel each other out. Thus, the only significant contribution is from the furthest charge, which is a distance $r = \sqrt{3}d/2$ away from that midpoint. Plugging this into Eq. 22-3 immediately gives the result:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0(\sqrt{3}d/2)^2} = \frac{4}{3}\frac{Q}{4\pi\varepsilon_0 d^2}\ .$$

65. First, we need a formula for the field due to the arc. We use the notation λ for the charge density, $\lambda = Q/L$. Sample Problem 22.03 — "Electric field of a charged circular rod," illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle θ) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}\,.$$

Now, the arc length is $L = r\theta$ with θ expressed in radians. Thus, using R instead of r, we obtain

$$E_{\text{arc}} = \frac{2(Q/L)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2(Q/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta}\ .$$

Thus, the problem requires $E_{\text{arc}} = \frac{1}{2}\,E_{\text{particle}}$, where E_{particle} is given by Eq. 22-3. Hence,

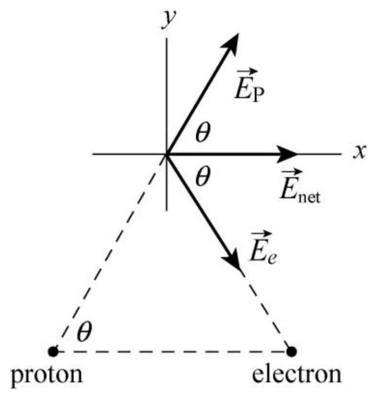
$$\frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta} = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R^2} \quad\Rightarrow\quad \sin\frac{\theta}{2} = \frac{\theta}{4}$$

where we note, again, that the angle is in radians. The approximate solution to this equation is $\theta = 3.791$ rad $\approx 217°$.

66. We denote the electron with subscript e and the proton with p. From the figure below we see that

$$\left|\vec{E}_e\right| = \left|\vec{E}_p\right| = \frac{e}{4\pi\varepsilon_0 d^2}$$

where $d = 2.0\times10^{-6}$ m. We note that the components along the y axis cancel during the vector summation. With $k = 1/4\pi\varepsilon_0$ and $\theta = 60°$, the magnitude of the net electric field is obtained as follows:



$$\left|\vec{E}_{net}\right| = E_x = 2E_e\cos\theta = 2\left(\frac{e}{4\pi\varepsilon_0 d^2}\right)\cos\theta = 2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(1.6\times10^{-19}\ \text{C}\right)}{\left(2.0\times10^{-6}\ \text{m}\right)^2}\cos 60^\circ$$

$$= 3.6\times10^2\ \text{N/C}.$$

67. A small section of the distribution that has charge dq is $\lambda\, dx$, where $\lambda = 9.0 \times 10^{-9}$ C/m. Its contribution to the field at $x_P = 4.0$ m is

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0\left(x - x_P\right)^2}$$

pointing in the $+x$ direction. Thus, we have

$$\vec{E} = \int_0^{3.0\,\text{m}} \frac{\lambda\, dx}{4\pi\varepsilon_0\left(x - x_P\right)^2}\,\hat{\text{i}}$$

which becomes, using the substitution $u = x - x_P$,

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\int_{-4.0\,\text{m}}^{-1.0\,\text{m}} \frac{du}{u^2}\,\hat{\text{i}} = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{-1}{-1.0\,\text{m}} - \frac{-1}{-4.0\,\text{m}}\right)\hat{\text{i}}$$

which yields 61 N/C in the $+x$ direction.

68. Most of the individual fields, caused by diametrically opposite charges, will cancel, except for the pair that lie on the x axis passing through the center. This pair of charges produces a field pointing to the right

$$\vec{E} = \frac{3q}{4\pi\varepsilon_0 d^2}\,\hat{\text{i}} = \frac{3e}{4\pi\varepsilon_0 d^2}\,\hat{\text{i}} = \frac{3\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)}{\left(0.020\text{m}\right)^2}\,\hat{\text{i}} = (1.08\times10^{-5}\ \text{N/C})\hat{\text{i}}\,.$$

69. (a) From symmetry, we see the net field component along the x axis is zero; the net field component along the y axis points upward. With $\theta = 60°$,

$$E_{\text{net},y} = 2\frac{Q\sin\theta}{4\pi\varepsilon_0 a^2} \;.$$

Since sin(60°) = $\sqrt{3}/2$, we can write this as $E_{\text{net}} = kQ\sqrt{3}/a^2$ (using the notation of the constant k defined in Eq. 21-5). Numerically, this gives roughly 47 N/C.

(b) From symmetry, we see in this case that the net field component along the y axis is zero; the net field component along the x axis points rightward. With $\theta = 60°$,

$$E_{\text{net},x} = 2\frac{Q\cos\theta}{4\pi\varepsilon_0 a^2}\,.$$

Since cos(60°) = 1/2, we can write this as $E_{\text{net}} = kQ/a^2$ (using the notation of Eq. 21-5). Thus, $E_{\text{net}} \approx 27$ N/C.

70. Our approach (based on Eq. 22-29) consists of several steps. The first is to find an *approximate* value of e by taking differences between all the given data. The smallest difference is between the fifth and sixth values:

$$18.08 \times 10^{-19}\text{ C} - 16.48 \times 10^{-19}\text{ C} = 1.60 \times 10^{-19}\text{ C}$$

which we denote e_{approx}. The goal at this point is to assign integers n using this approximate value of e:

datum1	$\frac{6.563\times10^{-19}\text{C}}{e_{\text{approx}}} = 4.10 \Rightarrow n_1 = 4$	datum6	$\frac{18.08\times10^{-19}\text{C}}{e_{\text{appeox}}} = 11.30 \Rightarrow n_6 = 11$
datum2	$\frac{8.204\times10^{-19}\text{C}}{e_{\text{approx}}} = 5.13 \Rightarrow n_2 = 5$	datum7	$\frac{19.71\times10^{-19}\text{C}}{e_{\text{approx}}} = 12.32 \Rightarrow n_7 = 12$
datum3	$\frac{11.50\times10^{-19}\text{C}}{e_{\text{approx}}} = 7.19 \Rightarrow n_3 = 7$	datum8	$\frac{22.89\times10^{-19}\text{C}}{e_{\text{approx}}} = 14.31 \Rightarrow n_8 = 14$
datum4	$\frac{13.13\times10^{-19}\text{C}}{e_{\text{approx}}} = 8.21 \Rightarrow n_4 = 8$	datum9	$\frac{26.13\times10^{-19}\text{C}}{e_{\text{approx}}} = 16.33 \Rightarrow n_9 = 16$
datum5	$\frac{16.48\times10^{-19}\text{C}}{e_{\text{approx}}} = 10.30 \Rightarrow n_5 = 10$		

Next, we construct a new data set (e_1, e_2, e_3, …) by dividing the given data by the respective exact integers n_i (for i = 1, 2, 3, …):

$$(e_1, e_2, e_3, \ldots) = \left(\frac{6.563\times10^{-19}\,\text{C}}{n_1}, \frac{8.204\times10^{-19}\,\text{C}}{n_2}, \frac{11.50\times10^{-19}\,\text{C}}{n_3}, \ldots \right)$$

which gives (carrying a few more figures than are significant)

$$\left(1.64075\times10^{-19}\,\text{C},\ 1.6408\times10^{-19}\,\text{C},\ 1.64286\times10^{-19}\,\text{C}, \ldots\right)$$

as the new data set (our experimental values for e). We compute the average and standard deviation of this set, obtaining

$$e_{\text{exptal}} = e_{\text{avg}} \pm \Delta e = (1.641 \pm 0.004)\times 10^{-19}\,\text{C}$$

which does not agree (to within one standard deviation) with the modern accepted value for e. The lower bound on this spread is $e_{\text{avg}} - \Delta e = 1.637 \times 10^{-19}$ C, which is still about 2% too high.

71. Studying Sample Problem 22.03 — "Electric field of a charged circular rod," we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 r} \sin\theta \Big|_{-\theta}^{\theta}$$

along the symmetry axis, where $\lambda = q/\ell = q/r\theta$ with θ in radians. Here ℓ is the length of the arc, given as $\ell = 4.0\,\text{m}$. Therefore, the angle is $\theta = \ell/r = 4.0/2.0 = 2.0\,\text{rad}$. Thus, with $q = 20 \times 10^{-9}$ C, we obtain

$$\left|\vec{E}\right| = \frac{(q/\ell)}{4\pi\varepsilon_0 r} \sin\theta \Big|_{-1.0\text{ rad}}^{1.0\text{ rad}} = 38\text{ N/C}.$$

72. The electric field at a point on the axis of a uniformly charged ring, a distance z from the ring center, is given by

$$E = \frac{qz}{4\pi\varepsilon_0 \left(z^2 + R^2\right)^{3/2}}$$

where q is the charge on the ring and R is the radius of the ring (see Eq. 22-16). For q positive, the field points upward at points above the ring and downward at points below the ring. We take the positive direction to be upward. Then, the force acting on an electron on the axis is

$$F = -\frac{eqz}{4\pi\varepsilon_0 \left(z^2 + R^2\right)^{3/2}}.$$

For small amplitude oscillations $z \ll R$ and z can be neglected in the denominator. Thus,

$$F = -\frac{eqz}{4\pi\varepsilon_0 R^3}.$$

The force is a restoring force: it pulls the electron toward the equilibrium point $z = 0$. Furthermore, the magnitude of the force is proportional to z, just as if the electron were attached to a spring with spring constant $k = eq/4\pi\varepsilon_0 R^3$. The electron moves in simple harmonic motion with an angular frequency given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{eq}{4\pi\varepsilon_0 m R^3}}$$

where m is the mass of the electron.

73. **THINK** We have a positive charge in the xy plane. From the electric fields it produces at two different locations, we can determine the position and the magnitude of the charge.

EXPRESS Let the charge be placed at (x_0, y_0). In Cartesian coordinates, the electric field at a point (x, y) can be written as

$$\vec{E} = E_x\hat{\mathrm{i}} + E_y\hat{\mathrm{j}} = \frac{q}{4\pi\varepsilon_0}\frac{(x-x_0)\hat{\mathrm{i}} + (y-y_0)\hat{\mathrm{j}}}{\left[(x-x_0)^2 + (y-y_0)^2\right]^{3/2}}.$$

The ratio of the field components is

$$\frac{E_y}{E_x} = \frac{y-y_0}{x-x_0}.$$

ANALYZE (a) The fact that the second measurement at the location (2.0 cm, 0) gives $\vec{E} = (100\ \mathrm{N/C})\hat{\mathrm{i}}$ indicates that $y_0 = 0$, that is, the charge must be somewhere on the x axis. Thus, the above expression can be simplified to

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{(x-x_0)\hat{\mathrm{i}} + y\hat{\mathrm{j}}}{\left[(x-x_0)^2 + y^2\right]^{3/2}}.$$

On the other hand, the field at (3.0 cm, 3.0 cm) is $\vec{E} = (7.2\ \mathrm{N/C})(4.0\hat{\mathrm{i}} + 3.0\hat{\mathrm{j}})$, which gives $E_y / E_x = 3/4$. Thus, we have

$$\frac{3}{4} = \frac{3.0\ \mathrm{cm}}{3.0\ \mathrm{cm} - x_0}$$

which implies $x_0 = -1.0$ cm.

(b) As shown above, the y coordinate is $y_0 = 0$.

(c) To calculate the magnitude of the charge, we note that the field magnitude measured at (2.0 cm, 0) (which is $r = 0.030$ m from the charge) is

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 100\ \text{N/C}.$$

Therefore,

$$q = 4\pi\varepsilon_0 |\vec{E}| r^2 = \frac{(100\ \text{N/C})(0.030\ \text{m})^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.0\times10^{-11}\text{C}.$$

LEARN Alternatively, we may calculate q by noting that at (3.0 cm, 3.00 cm)

$$E_x = 28.8\ \text{N/C} = \frac{q}{4\pi\varepsilon_0}\frac{0.040\ \text{m}}{\left[(0.040\ \text{m})^2 + (0.030\ \text{m})^2\right]^{3/2}} = \frac{q}{4\pi\varepsilon_0}\left(320/\text{m}^2\right).$$

This gives

$$q = \frac{28.8\ \text{N/C}}{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(320/\text{m}^2)} = 1.0\times10^{-11}\ \text{C},$$

in agreement with that calculated above.

74. (a) Let $E = \sigma/2\varepsilon_0 = 3\times10^6$ N/C. With $\sigma = |q|/A$, this leads to

$$|q| = \pi R^2\sigma = 2\pi\varepsilon_0 R^2 E = \frac{R^2 E}{2k} = \frac{\left(2.5\times10^{-2}\,\text{m}\right)^2\left(3.0\times10^6\ \text{N/C}\right)}{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)} = 1.0\times10^{-7}\ \text{C},$$

where $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$.

(b) Setting up a simple proportionality (with the areas), the number of atoms is estimated to be

$$n = \frac{\pi\left(2.5\times10^{-2}\,\text{m}\right)^2}{0.015\times10^{-18}\,\text{m}^2} = 1.3\times10^{17}.$$

(c) The fraction is

$$\frac{q}{Ne} = \frac{1.0\times10^{-7}\,\text{C}}{\left(1.3\times10^{17}\right)\left(1.6\times10^{-19}\,\text{C}\right)} \approx 5.0\times10^{-6}.$$

75. On the one hand, the conclusion (that $Q = +1.00\ \mu$C) is clear from symmetry. If a more in-depth justification is desired, one should use Eq. 22-3 for the electric field magnitudes of the three charges (each at the same distance $r = a/\sqrt{3}$ from C) and then find field components along suitably chosen axes, requiring each component-sum to be

zero. If the y axis is vertical, then (assuming $Q > 0$) the component-sum along that axis leads to $2kq\sin 30°/r^2 = kQ/r^2$ where q refers to either of the charges at the bottom corners. This yields $Q = 2q \sin 30° = q$ and thus to the conclusion mentioned above.

76. Equation 22-38 gives $U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$. We note that $\theta_i = 110°$ and $\theta_f = 70.0°$. Therefore,

$$\Delta U = -pE\left(\cos 70.0° - \cos 110°\right) = -3.28\times 10^{-21}\ \text{J}.$$

77. (a) Since the two charges in question are of the same sign, the point $x = 2.0$ mm should be located in between them (so that the field vectors point in the opposite direction). Let the coordinate of the second particle be x' ($x' > 0$). Then, the magnitude of the field due to the charge $-q_1$ evaluated at x is given by $E = q_1/4\pi\varepsilon_0 x^2$, while that due to the second charge $-4q_1$ is $E' = 4q_1/4\pi\varepsilon_0(x' - x)^2$. We set the net field equal to zero:

$$\vec{E}_{\text{net}} = 0 \ \Rightarrow\ E = E'$$

so that

$$\frac{q_1}{4\pi\varepsilon_0 x^2} = \frac{4q_1}{4\pi\varepsilon_0\left(x' - x\right)^2}.$$

Thus, we obtain $x' = 3x = 3(2.0 \text{ mm}) = 6.0$ mm.

(b) In this case, with the second charge now positive, the electric field vectors produced by both charges are in the negative x direction, when evaluated at $x = 2.0$ mm. Therefore, the net field points in the negative x direction, or 180°, measured counterclockwise from the $+x$ axis.

78. Let q_1 denote the charge at $y = d$ and q_2 denote the charge at $y = -d$. The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 22-3, where the absolute value signs for q are unnecessary since these charges are both positive. The distance from q_1 to a point on the x axis is the same as the distance from q_2 to a point on the x axis: $r = \sqrt{x^2 + d^2}$. By symmetry, the y component of the net field along the x axis is zero. The x component of the net field, evaluated at points on the positive x axis, is

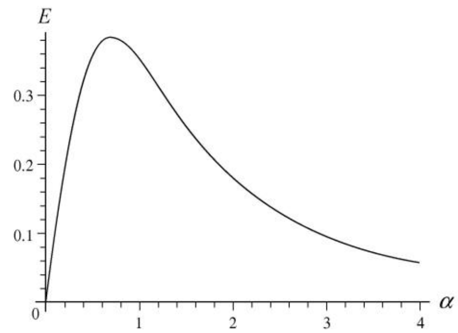
$$E_x = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{x^2 + d^2}\right)\left(\frac{x}{\sqrt{x^2 + d^2}}\right)$$

where the last factor is $\cos\theta = x/r$ with θ being the angle for each individual field as measured from the x axis.

(a) If we simplify the above expression, and plug in $x = \alpha d$, we obtain

$$E_x = \frac{q}{2\pi\varepsilon_0 d^2}\frac{\alpha}{(\alpha^2+1)^{3/2}}.$$

(b) The graph of $E = E_x$ versus α is shown below. For the purposes of graphing, we set d = 1 m and $q = 5.56 \times 10^{-11}$ C.



(c) From the graph, we estimate E_{max} occurs at about α = 0.71. More accurate computation shows that the maximum occurs at $\alpha = 1/\sqrt{2}$.

(d) The graph suggests that "half-height" points occur at $\alpha \approx 0.2$ and $\alpha \approx 2.0$. Further numerical exploration leads to the values: $\alpha = 0.2047$ and $\alpha = 1.9864$.

79. We consider pairs of diametrically opposed charges. The net field due to just the charges in the one o'clock (–q) and seven o'clock (–7q) positions is clearly equivalent to that of a single –6q charge sitting at the seven o'clock position. Similarly, the net field due to just the charges in the six o'clock (–6q) and twelve o'clock (–12q) positions is the same as that due to a single –6q charge sitting at the twelve o'clock position. Continuing with this line of reasoning, we see that there are six equal-magnitude electric field vectors pointing at the seven o'clock, eight o'clock, … twelve o'clock positions. Thus, the resultant field of all of these points, by symmetry, is directed toward the position midway between seven and twelve o'clock. Therefore, $\vec{E}_{\text{resultant}}$ points toward the nine-thirty position.

80. The magnitude of the dipole moment is given by $p = qd$, where q is the positive charge in the dipole and d is the separation of the charges. For the dipole described in the problem,

$$p = \left(1.60\times10^{-19}\text{ C}\right)\left(4.30\times10^{-9}\text{ m}\right) = 6.88\times10^{-28}\text{ C}\cdot\text{m}.$$

The dipole moment is a vector that points from the negative toward the positive charge.

81. (a) Since $\vec{E}$ points down and we need an upward electric force (to cancel the downward pull of gravity), then we require the charge of the sphere to be negative. The magnitude of the charge is found by working with the absolute value of Eq. 22-28:

$$|q| = \frac{F}{E} = \frac{mg}{E} = \frac{4.4\text{N}}{150\ \text{N/C}} = 0.029\text{C},$$

or $q = -0.029$ C.

(b) The feasibility of this experiment may be studied by using Eq. 22-3 (using k for $1/4\pi\varepsilon_0$). We have $E = k|q|/r^2$ with

$$\rho_{\text{sulfur}}\left(\frac{4}{3}\pi r^3\right) = m_{\text{sphere}}$$

Since the mass of the sphere is 4.4/9.8 ≈ 0.45 kg and the density of sulfur is about 2.1×10^3 kg/m^3 (see Appendix F), then we obtain

$$r = \left(\frac{3m_{\text{sphere}}}{4\pi\rho_{\text{sulfur}}}\right)^{1/3} = 0.037\,\text{m} \Rightarrow E = k\frac{|q|}{r^2} \approx 2\times10^{11}\ \text{N/C}$$

which is much too large a field to maintain in air.

82. We interpret the linear charge density, $\lambda = |Q|/L$, to indicate a positive quantity (so we can relate it to the magnitude of the field). Sample Problem 22.03 — "Electric field of a charged circular rod" illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle θ) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}.$$

Now, the arc length is $L = r\theta$ with θ is expressed in radians. Thus, using R instead of r, we obtain

$$E_{\text{arc}} = \frac{2(|Q|/L)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2(|Q|/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2|Q|\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta}.$$

With $|Q| = 6.25\times10^{-12}$ C, $\theta = 2.40$ rad $= 137.5°$, and $R = 9.00\times10^{-2}$ m, the magnitude of the electric field is $E = 5.39$ N/C.

83. **THINK** The potential energy of the electric dipole placed in an electric field depends on its orientation relative to the electric field. The field causes a torque that tends to align the dipole with the field.

EXPRESS When placed in an electric field $\vec{E}$, the potential energy of the dipole $\vec{p}$ is given by Eq. 22-38:

$$U(\theta) = -\vec{p}\cdot\vec{E} = -pE\cos\theta.$$

The torque caused by the electric field is (see Eq. 22-34) $\vec{\tau} = \vec{p} \times \vec{E}$.

ANALYZE (a) From Eq. 22-38 (and the facts that $\hat{\text{i}} \cdot \hat{\text{i}} = 1$ and $\hat{\text{j}} \cdot \hat{\text{i}} = 0$), the potential energy is

$$U = -\vec{p} \cdot \vec{E} = -\left[\left(3.00\hat{\text{i}} + 4.00\hat{\text{j}}\right)\left(1.24\times10^{-30}\,\text{C}\cdot\text{m}\right)\right] \cdot \left[\left(4000\,\text{N/C}\right)\hat{\text{i}}\right]$$
$$= -1.49\times10^{-26}\ \text{J}.$$

(b) From Eq. 22-34 (and the facts that $\hat{\text{i}} \times \hat{\text{i}} = 0$ and $\hat{\text{j}} \times \hat{\text{i}} = -\hat{\text{k}}$), the torque is

$$\vec{\tau} = \vec{p} \times \vec{E} = \left[\left(3.00\hat{\text{i}} + 4.00\hat{\text{j}}\right)\left(1.24\times10^{-30}\,\text{C}\cdot\text{m}\right)\right] \times \left[\left(4000\,\text{N/C}\right)\hat{\text{i}}\right] = \left(-1.98\times10^{-26}\,\text{N}\cdot\text{m}\right)\hat{\text{k}}.$$

(c) The work done is

$$W = \Delta U = \Delta\left(-\vec{p} \cdot \vec{E}\right) = \left(\vec{p}_i - \vec{p}_f\right) \cdot \vec{E}$$
$$= \left(3.00\hat{\text{i}} + 4.00\hat{\text{j}}\right) - \left(-4.00\hat{\text{i}} + 3.00\hat{\text{j}}\right) \left(1.24\times10^{-30}\,\text{C}\cdot\text{m}\right) \cdot \left(4000\,\text{N/C}\right)\hat{\text{i}}$$
$$= 3.47\times10^{-26}\ \text{J}.$$

LEARN The work done by the agent is equal to the change in the potential energy of the dipole.

84. (a) The electric field is upward in the diagram and the charge is negative, so the force of the field on it is downward. The magnitude of the acceleration is $a = eE/m$, where E is the magnitude of the field and m is the mass of the electron. Its numerical value is

$$a = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(2.00\times10^{3}\ \text{N/C}\right)}{9.11\times10^{-31}\,\text{kg}} = 3.51\times10^{14}\ \text{m/s}^2\,.$$

We put the origin of a coordinate system at the initial position of the electron. We take the x axis to be horizontal and positive to the right; take the y axis to be vertical and positive toward the top of the page. The kinematic equations are

$$x = v_0 t\cos\theta,\quad y = v_0 t\sin\theta - \frac{1}{2}at^2,\qquad \text{and}\qquad v_y = v_0\sin\theta - at.$$

First, we find the greatest y coordinate attained by the electron. If it is less than d, the electron does not hit the upper plate. If it is greater than d, it will hit the upper plate if the corresponding x coordinate is less than L. The greatest y coordinate occurs when $v_y = 0$. This means $v_0 \sin\theta - at = 0$ or $t = (v_0/a)\sin\theta$ and

$$y_{\max} = \frac{v_0^2 \sin^2\theta}{a} - \frac{1}{2} a \frac{v_0^2 \sin^2\theta}{a^2} = \frac{1}{2} \frac{v_0^2 \sin^2\theta}{a} = \frac{\left(6.00\times10^6 \text{ m/s}\right)^2 \sin^2 45°}{2\left(3.51\times10^{14} \text{ m/s}^2\right)}$$
$$= 2.56\times10^{-2}\,\text{m}.$$

Since this is greater than d = 2.00 cm, the electron might hit the upper plate.

(b) Now, we find the x coordinate of the position of the electron when $y = d$. Since

$$v_0 \sin\theta = \left(6.00\times10^6 \text{ m/s}\right)\sin 45° = 4.24\times10^6 \text{ m/s}$$

and

$$2ad = 2\left(3.51\times10^{14} \text{ m/s}^2\right)\left(0.0200\,\text{m}\right) = 1.40\times10^{13} \text{ m}^2/\text{s}^2$$

the solution to $d = v_0 t \sin\theta - \frac{1}{2}at^2$ is

$$t = \frac{v_0 \sin\theta - \sqrt{v_0^2 \sin^2\theta - 2ad}}{a} = \frac{(4.24\times10^6 \text{ m/s}) - \sqrt{\left(4.24\times10^6 \text{ m/s}\right)^2 - 1.40\times10^{13} \text{ m}^2/\text{s}^2}}{3.51\times10^{14} \text{ m/s}^2}$$
$$= 6.43\times10^{-9}\,\text{s}.$$

The negative root was used because we want the *earliest* time for which $y = d$. The x coordinate is

$$x = v_0 t\cos\theta = \left(6.00\times10^6 \text{ m/s}\right)\left(6.43\times10^{-9}\,\text{s}\right)\cos 45° = 2.72\times10^{-2}\,\text{m}.$$

This is less than L so the electron hits the upper plate at x = 2.72 cm.

85. (a) If we subtract each value from the next larger value in the table, we find a set of numbers that are suggestive of a basic unit of charge: 1.64×10^{-19}, 3.3×10^{-19}, 1.63×10^{-19}, 3.35×10^{-19}, 1.6×10^{-19}, 1.63×10^{-19}, 3.18×10^{-19}, 3.24×10^{-19}, where the SI unit Coulomb is understood. These values are either close to a common $e \approx 1.6\times10^{-19}\,\text{C}$ value or are double that. Taking this, then, as a crude approximation to our experimental e we divide it into all the values in the original data set and round to the nearest integer, obtaining n = 4, 5, 7, 8,10, 11, 12, 14, and 16.

(b) When we perform a least squares fit of the original data set versus these values for n we obtain the linear equation:

$$q = 7.18 \times 10^{-21} + 1.633 \times 10^{-19} n\,.$$

If we dismiss the constant term as unphysical (representing, say, systematic errors in our measurements) then we obtain $e = 1.63 \times 10^{-19}$ when we set $n = 1$ in this equation.

86. (a) From symmetry, we see the net force component along the y axis is zero.

(b) The net force component along the *x* axis points rightward. With $\theta = 60°$,

$$F_3 = 2\frac{q_3 q_1 \cos\theta}{4\pi\varepsilon_0 a^2}.$$

Since cos(60°) =1/2, we can write this as

$$F_3 = \frac{kq_3 q_1}{a^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-12}\,\text{C})(2.00\times10^{-12}\,\text{C})}{(0.0950\ \text{m})^2} = 9.96\times10^{-12}\,\text{N}.$$

87. (a) For point *A*, we have (in SI units)

$$\vec{E}_A = \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} + \frac{q_2}{4\pi\varepsilon_0 r_2^2}\right](-\hat{\text{i}}) = \frac{(8.99\times10^9)(1.00\times10^{-12}\,\text{C})}{\left(5.00\times10^{-2}\right)^2}(-\hat{\text{i}}) + \frac{(8.99\times10^9)\,|-2.00\times10^{-12}\,\text{C}|}{\left(2\times5.00\times10^{-2}\right)^2}(+\hat{\text{i}})$$

$$= (-1.80\,\text{N/C})\hat{\text{i}}.$$

(b) Similar considerations leads to

$$\vec{E}_B = \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} + \frac{|q_2|}{4\pi\varepsilon_0 r_2^2}\right]\hat{\text{i}} = \frac{\left(8.99\times10^9\right)\left(1.00\times10^{-12}\,\text{C}\right)}{\left(0.500\times5.00\times10^{-2}\right)^2}\hat{\text{i}} + \frac{\left(8.99\times10^9\right)\,|-2.00\times10^{-12}\,\text{C}|}{\left(0.500\times5.00\times10^{-2}\right)^2}\hat{\text{i}}$$

$$= (43.2\,\text{N/C})\hat{\text{i}}.$$

(c) For point C, we have

$$\vec{E}_C = \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} - \frac{|q_2|}{4\pi\varepsilon_0 r_2^2}\right]\hat{\text{i}} = \frac{\left(8.99\times10^9\right)\left(1.00\times10^{-12}\,\text{C}\right)}{\left(2.00\times5.00\times10^{-2}\right)^2}\hat{\text{i}} - \frac{\left(8.99\times10^9\right)\,|-2.00\times10^{-12}\,\text{C}|}{\left(5.00\times10^{-2}\right)^2}\hat{\text{i}}$$

$$= -(6.29\,\text{N/C})\hat{\text{i}}.$$

(d) The field lines are shown to the right. Note that there are twice as many field lines "going into" the negative charge $-2q$ as compared to that flowing out from the positive charge $+q$.