# Chapter 18

4. (a) Let the reading on the Celsius scale be $x$ and the reading on the Fahrenheit scale be $y$. Then $y = \frac{9}{5}x + 32$. For $x$ = –71°C, this gives $y$ = –96°F.

(b) The relationship between $y$ and $x$ may be inverted to yield $x = \frac{5}{9}(y - 32)$. Thus, for $y$ = 134 we find $x \approx 56.7$ on the Celsius scale.

9. The new diameter is

$$D = D_0(1 + \alpha_{Al}\Delta T) = (2.725\,\text{cm})[1 + (23\times10^{-6}/\text{C°})(100.0\text{°C} - 0.000\text{°C})] = 2.731\,\text{cm}.$$

15. After the change in temperature the diameter of the steel rod is $D_s = D_{s0} + \alpha_s D_{s0}\,\Delta T$ and the diameter of the brass ring is $D_b = D_{b0} + \alpha_b D_{b0}\,\Delta T$, where $D_{s0}$ and $D_{b0}$ are the original diameters, $\alpha_s$ and $\alpha_b$ are the coefficients of linear expansion, and $\Delta T$ is the change in temperature. The rod just fits through the ring if $D_s = D_b$. This means

$$D_{s0} + \alpha_s D_{s0}\,\Delta T = D_{b0} + \alpha_b D_{b0}\,\Delta T.$$

Therefore,

$$\Delta T = \frac{D_{s0} - D_{b0}}{\alpha_b D_{b0} - \alpha_s D_{s0}} = \frac{3.000\,\text{cm} - 2.992\,\text{cm}}{(19.00\times10^{-6}/\text{C°})(2.992\,\text{cm}) - (11.00\times10^{-6}/\text{C°})(3.000\,\text{cm})}$$
$$= 335.0\,\text{°C}.$$

The temperature is $T$ = (25.00°C + 335.0 °C) = 360.0°C.

29. The power consumed by the system is

$$P = \left(\frac{1}{20\%}\right)\frac{cm\Delta T}{t} = \left(\frac{1}{20\%}\right)\frac{(4.18\,\text{J/g}\cdot\text{°C})(200\times10^3\,\text{cm}^3)(1\,\text{g/cm}^3)(40\text{°C} - 20\text{°C})}{(1.0\,\text{h})(3600\,\text{s/h})}$$
$$= 2.3\times10^4\ \text{W}.$$

The area needed is then $A = \dfrac{2.3\times10^4\ \text{W}}{700\,\text{W/m}^2} = 33\,\text{m}^2.$

35. We denote the ice with subscript $I$ and the coffee with $c$, respectively. Let the final temperature be $T_f$. The heat absorbed by the ice is

$$Q_I = \lambda_F m_I + m_I c_w\,(T_f - 0\text{°C}),$$

and the heat given away by the coffee is $|Q_c| = m_w c_w\,(T_I - T_f)$. Setting $Q_I = |Q_c|$, we solve for $T_f$:

$$T_f = \frac{m_w c_w T_I - \lambda_F m_I}{(m_I + m_c)c_w} = \frac{(130\,\text{g})(4190\,\text{J/kg}\cdot\text{C°})\,(80.0\text{°C}) - (333\times10^3\,\text{J/g})(12.0\,\text{g})}{(12.0\,\text{g}+130\,\text{g})(4190\,\text{J/kg}\cdot\text{C°})}$$
$$= 66.5\text{°C}.$$

Note that we work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. Therefore, the temperature of the coffee will cool by $|\Delta T| = 80.0\text{°C} - 66.5\text{°C} = 13.5\text{C°}$.

45. Over a cycle, the internal energy is the same at the beginning and end, so the heat $Q$ absorbed equals the work done: $Q = W$. Over the portion of the cycle from $A$ to $B$ the pressure $p$ is a linear function of the volume $V$, and we may write $p = a + bV$. The work done over this portion of the cycle is

$$W_{AB} = \int_{V_A}^{V_B} p\,dV = \int_{V_A}^{V_B} (a + bV)\,dV = a(V_B - V_A) + \frac{1}{2}b\left(V_B^2 - V_A^2\right).$$

The $BC$ portion of the cycle is at constant pressure, and the work done by the gas is

$$W_{BC} = p_B \Delta V_{BC} = p_B (V_C - V_B).$$

The $CA$ portion of the cycle is at constant volume, so no work is done. The total work done by the gas is

$$W = W_{AB} + W_{BC} + W_{CA}.$$

The pressure function can be written as

$$p = \frac{10}{3}\ \text{Pa} + \left(\frac{20}{3}\ \text{Pa/m}^3\right)V,$$

where the coefficients $a$ and $b$ were chosen so that $p = 10$ Pa when $V = 1.0\ \text{m}^3$ and $p = 30$ Pa when $V = 4.0\ \text{m}^3$. Therefore, the work done going from $A$ to $B$ is

$$\begin{aligned} W_{AB} &= a(V_B - V_A) + \frac{1}{2}b\left(V_B^2 - V_A^2\right) \\ &= \left(\frac{10}{3}\,\text{Pa}\right)\left(4.0\ \text{m}^3 - 1.0\ \text{m}^3\right) + \frac{1}{2}\left(\frac{20}{3}\ \text{Pa/m}^3\right)\left[(4.0\ \text{m}^3)^2 - (1.0\ \text{m}^3)^2\right] \\ &= 10\ \text{J} + 50\ \text{J} = 60\ \text{J}. \end{aligned}$$

Similarly, with $p_B = p_C = 30\ \text{Pa}$, $V_C = 1.0\ \text{m}^3$, and $V_B = 4.0\ \text{m}^3$, we have

$$W_{BC} = p_B(V_C - V_B) = (30 \text{ Pa})(1.0 \text{ m}^3 - 4.0 \text{ m}^3) = -90 \text{ J}.$$

Adding up all contributions, we find the total work done by the gas to be

$$W = W_{AB} + W_{BC} + W_{CA} = 60 \text{ J} - 90 \text{ J} + 0 = -30 \text{ J} .$$

Thus, the total heat absorbed is $Q = W = -30$ J. This means the gas loses 30 J of energy in the form of heat. Notice that in calculating the work done by the gas, we always start with Eq. 18-25: $W = \int p dV$ . For an isobaric process where $p$ = constant, $W = p\Delta V$ , and for an isochoric process where $V$ = constant, $W = 0$.

57. (a) We use

$$P_{\text{cond}} = kA\frac{T_H - T_C}{L}$$

with the conductivity of glass given in Table 18-6 as 1.0 W/m·K. We choose to use the Celsius scale for the temperature: a temperature difference of

$$T_H - T_C = 72°\text{F} - (-20°\text{F}) = 92\ °\text{F}$$

is equivalent to $\frac{5}{9}(92) = 51.1\text{C}°$ . This, in turn, is equal to 51.1 K since a change in Kelvin temperature is entirely equivalent to a Celsius change. Thus,

$$\frac{P_{\text{cond}}}{A} = k\frac{T_H - T_C}{L} = (1.0\,\text{W/m}\cdot\text{K})\left(\frac{51.1°\text{C}}{3.0\times10^{-3}\text{m}}\right) = 1.7\times10^4\ \text{W/m}^2.$$

(b) The energy now passes in succession through 3 layers, one of air and two of glass. The heat transfer rate $P$ is the same in each layer and is given by

$$P_{\text{cond}} = \frac{A(T_H - T_C)}{\sum L/k}$$

where the sum in the denominator is over the layers. If $L_g$ is the thickness of a glass layer, $L_a$ is the thickness of the air layer, $k_g$ is the thermal conductivity of glass, and $k_a$ is the thermal conductivity of air, then the denominator is

$$\sum\frac{L}{k} = \frac{2L_g}{k_g} + \frac{L_a}{k_a} = \frac{2L_g k_a + L_a k_g}{k_a k_g}.$$

Therefore, the heat conducted per unit area occurs at the following rate:

$$\frac{P_{\text{cond}}}{A}=\frac{(T_H-T_C)k_ak_g}{2L_gk_a+L_ak_g}=\frac{(51.1°\text{C})(0.026\,\text{W/m}\cdot\text{K})(1.0\,\text{W/m}\cdot\text{K})}{2(3.0\times10^{-3}\text{m})(0.026\,\text{W/m}\cdot\text{K})+(0.075\,\text{m})(1.0\,\text{W/m}\cdot\text{K})}$$
$$=18\,\text{W/m}^2.$$

79. Let $V_i$ and $V_f$ be the initial and final volumes, respectively. With $p=aV^2$, the work done by the gas is

$$W=\int_{V_i}^{V_f}p\,dV=\int_{V_i}^{V_f}aV^2dV=\frac{1}{3}a\left(V_f^3-V_i^3\right).$$

With $a=10\text{ N/m}^8$, $V_i=1.0\text{ m}^3$ and $V_f=2.0\text{ m}^3$, we obtain

$$W=\frac{1}{3}a\left(V_f^3-V_i^3\right)=\frac{1}{3}\left(10\text{ N/m}^8\right)\left[(2.0\text{ m}^3)^3-(1.0\text{ m}^3)^3\right]=23\text{ J}.$$

Note: In this problem, the initial and final pressures are

$$p_i=aV_i^2=(10\text{ N/m}^8)(1.0\text{ m}^3)^2=10\text{ N/m}^2=10\text{ Pa}$$
$$p_f=aV_f^2=(10\text{ N/m}^8)(2.0\text{ m}^3)^2=40\text{ N/m}^2=40\text{ Pa}.$$

In this case, since $p\sim V^2$, the work done would be proportional to $V^3$ after volume integration.

86. If the window is $L_1$ high and $L_2$ wide at the lower temperature and $L_1+\Delta L_1$ high and $L_2+\Delta L_2$ wide at the higher temperature, then its area changes from $A_1=L_1L_2$ to

$$A_2=(L_1+\Delta L_1)(L_2+\Delta L_2)\approx L_1L_2+L_1\,\Delta L_2+L_2\,\Delta L_1$$

where the term $\Delta L_1\,\Delta L_2$ has been omitted because it is much smaller than the other terms, if the changes in the lengths are small. Consequently, the change in area is

$$\Delta A=A_2-A_1=L_1\,\Delta L_2+L_2\,\Delta L_1.$$

If $\Delta T$ is the change in temperature then $\Delta L_1=\alpha L_1\,\Delta T$ and $\Delta L_2=\alpha L_2\,\Delta T$, where $\alpha$ is the coefficient of linear expansion. Thus

$$\Delta A=\alpha(L_1L_2+L_1L_2)\,\Delta T=2\alpha L_1L_2\Delta T$$
$$=2\left(9\times10^{-6}/\text{C°}\right)(30\,\text{cm})\,(20\,\text{cm})\,(30°\text{C})$$
$$=0.32\,\text{cm}^2.$$

95. The net work may be computed as a sum of works (for the individual processes involved) or as the "area" (with appropriate ± sign) inside the figure (representing the cycle). In this solution, we take the former approach (sum over the processes) and will need the following fact related to processes represented in *pV* diagrams:

$$\text{for a straight line: Work} = \frac{p_i + p_f}{2}\Delta V$$

which is easily verified using the definition Eq. 18-25. The cycle represented by the "triangle" *BC* consists of three processes:

• "tilted" straight line from (1.0 m$^3$, 40 Pa) to (4.0 m$^3$, 10 Pa), with

$$\text{Work} = \frac{40\,\text{Pa} + 10\,\text{Pa}}{2}\left(4.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 75\,\text{J}$$

• horizontal line from (4.0 m$^3$, 10 Pa) to (1.0 m$^3$, 10 Pa), with

$$\text{Work} = (10\,\text{Pa})\left(1.0\,\text{m}^3 - 4.0\,\text{m}^3\right) = -30\,\text{J}$$

• vertical line from (1.0 m$^3$, 10 Pa) to (1.0 m$^3$, 40 Pa), with

$$\text{Work} = \frac{10\,\text{Pa} + 40\,\text{Pa}}{2}\left(1.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 0$$

(a) and (b) Thus, the total work during the *BC* cycle is (75 – 30) J = 45 J. During the *BA* cycle, the "tilted" part is the same as before, and the main difference is that the horizontal portion is at higher pressure, with Work = (40 Pa)(–3.0 m$^3$) = –120 J. Therefore, the total work during the *BA* cycle is (75 – 120) J = – 45 J.