Chapter 16

5. (a) The motion from maximum displacement to zero is one-fourth of a cycle. One-fourth of a period is 0.170 s, so the period is T = 4(0.170 s) = 0.680 s.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{0.680\,\text{s}} = 1.47\,\text{Hz}.$$

(c) A sinusoidal wave travels one wavelength in one period:

$$v = \frac{\lambda}{T} = \frac{1.40\,\text{m}}{0.680\,\text{s}} = 2.06\,\text{m/s}.$$

12. With length in centimeters and time in seconds, we have

$$u = \frac{du}{dt} = 225\pi \sin(\pi x - 15\pi t)\ .$$

Squaring this and adding it to the square of $15\pi y$, we have

$$u^2 + (15\pi y)^2 = (225\pi)^2 [\sin^2(\pi x - 15\pi t) + \cos^2(\pi x - 15\pi t)]$$

so that

$$u = \sqrt{(225\pi)^2 - (15\pi y)^2} = 15\pi\sqrt{15^2 - y^2}.$$

Therefore, where y = 12, u must be ± 135π. Consequently, the *speed* there is 424 cm/s = 4.24 m/s.

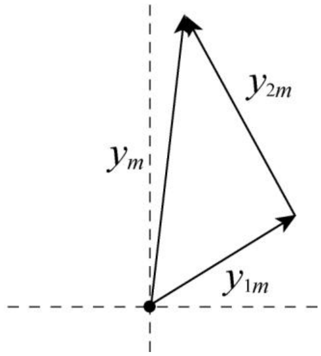
16. We use $v = \sqrt{\tau/\mu} \propto \sqrt{\tau}$ to obtain

$$\tau_2 = \tau_1\left(\frac{v_2}{v_1}\right)^2 = 120\,\text{N}\left(\frac{180\,\text{m/s}}{170\,\text{m/s}}\right)^2 = 135\text{N}.$$

29. The wave $y(x,t) = (2.00\text{ mm})[(20\text{ m}^{-1})x - (4.0\text{ s}^{-1})t]^{1/2}$ is of the form $h(kx - \omega t)$ with angular wave number $k = 20\text{ m}^{-1}$ and angular frequency $\omega = 4.0$ rad/s. Thus, the speed of the wave is

$$v = \omega/k = (4.0\text{ rad/s})/(20\text{ m}^{-1}) = 0.20\text{ m/s}.$$

35. The phasor diagram is shown below: y_{1m} and y_{2m} represent the original waves and y_m represents the resultant wave.



The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle. The Pythagorean theorem gives

$$y_m^2 = y_{1m}^2 + y_{2m}^2 = (3.0\,\text{cm})^2 + (4.0\,\text{cm})^2 = (25\,\text{cm})^2 .$$

Thus y_m = 5.0 cm.

Note: When adding two waves, it is convenient to represent each wave with a phasor, which is a vector whose magnitude is equal to the amplitude of the wave. The same result, however, could also be obtained as follows: Writing the two waves as $y_1 = 3\sin(kx - \omega t)$ and $y_2 = 4\sin(kx - \omega t + \pi/2) = 4\cos(kx - \omega t)$, we have, after a little algebra,

$$\begin{aligned} y = y_1 + y_2 &= 3\sin(kx - \omega t) + 4\cos(kx - \omega t) = 5\left[\frac{3}{5}\sin(kx - \omega t) + \frac{4}{5}\cos(kx - \omega t)\right] \\ &= 5\sin(kx - \omega t + \phi) \end{aligned}$$

where $\phi = \tan^{-1}(4/3)$. In deducing the phase ϕ, we set $\cos\phi = 3/5$ and $\sin\phi = 4/5$, and use the relation $\cos\phi\sin\theta + \sin\phi\cos\theta = \sin(\theta + \phi)$.

44. (a) The wave speed is given by

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{7.00\,\text{N}}{2.00 \times 10^{-3}\,\text{kg}/1.25\,\text{m}}} = 66.1\,\text{m/s}.$$

(b) The wavelength of the wave with the lowest resonant frequency f_1 is $\lambda_1 = 2L$, where L = 125 cm. Thus,

$$f_1 = \frac{v}{\lambda_1} = \frac{66.1\text{ m/s}}{2(1.25\text{ m})} = 26.4\text{ Hz}.$$

47. The harmonics are integer multiples of the fundamental, which implies that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency. Thus,

$$f_1 = (390\text{ Hz} - 325\text{ Hz}) = 65\text{ Hz}.$$

This further implies that the next higher resonance above 195 Hz should be (195 Hz + 65 Hz) = 260 Hz.

49. (a) Equation 16-26 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150\,\text{N}}{7.20\times10^{-3}\text{ kg/m}}} = 144.34\text{ m/s} \approx 1.44\times10^{2}\text{ m/s}.$$

(b) From the figure, we find the wavelength of the standing wave to be

$$\lambda = (2/3)(90.0\text{ cm}) = 60.0\text{ cm}.$$

(c) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.44\times10^{2}\text{ m/s}}{0.600\,\text{m}} = 241\,\text{Hz}.$$

55. Recalling the discussion in section 16-12, we observe that this problem presents us with a standing wave condition with amplitude 12 cm. The angular wave number and frequency are noted by comparing the given waves with the form $y = y_m \sin(kx \pm \omega t)$. The anti-node moves through 12 cm in simple harmonic motion, just as a mass on a vertical spring would move from its upper turning point to its lower turning point, which occurs during a half-period. Since the period T is related to the angular frequency by Eq. 15-5, we have

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.00\pi} = 0.500\text{ s}.$$

Thus, in a time of $t = \frac{1}{2}T = 0.250$ s, the wave moves a distance $\Delta x = vt$ where the speed of the wave is $v = \omega / k = 1.00$ m/s. Therefore, $\Delta x = (1.00\text{ m/s})(0.250\text{ s}) = 0.250$ m.

56. The nodes are located from vanishing of the spatial factor $\sin 5\pi x = 0$ for which the solutions are

$$5\pi x = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad x = 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \cdots$$

(a) The smallest value of x that corresponds to a node is $x = 0$.

(b) The second smallest value of x that corresponds to a node is $x = 0.20$ m.

(c) The third smallest value of x that corresponds to a node is $x = 0.40$ m.

(d) Every point (except at a node) is in simple harmonic motion of frequency $f = \omega/2\pi = 40\pi/2\pi = 20$ Hz. Therefore, the period of oscillation is $T = 1/f = 0.050$ s.

(e) Comparing the given function with Eq. 16-58 through Eq. 16-60, we obtain

$$y_1 = 0.020\sin(5\pi x - 40\pi t) \quad \text{and} \quad y_2 = 0.020\sin(5\pi x + 40\pi t)$$

for the two traveling waves. Thus, we infer from these that the speed is $v = \omega/k = 40\pi/5\pi = 8.0$ m/s.

(f) And we see the amplitude is $y_m = 0.020$ m.

(g) The derivative of the given function with respect to time is

$$u = \frac{\partial y}{\partial t} = -(0.040)(40\pi)\sin(5\pi x)\sin(40\pi t)$$

which vanishes (for all x) at times such as $\sin(40\pi t) = 0$. Thus,

$$40\pi t = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad t = 0, \frac{1}{40}, \frac{2}{40}, \frac{3}{40}, \cdots$$

Thus, the first time in which all points on the string have zero transverse velocity is when $t = 0$ s.

(h) The second time in which all points on the string have zero transverse velocity is when $t = 1/40$ s $= 0.025$ s.

(i) The third time in which all points on the string have zero transverse velocity is when $t = 2/40$ s $= 0.050$ s.