Chapter 13

1. The gravitational force between the two parts is

$$F = \frac{Gm(M-m)}{r^2} = \frac{G}{r^2}\left(mM - m^2\right)$$

which we differentiate with respect to *m* and set equal to zero:

$$\frac{dF}{dm} = 0 = \frac{G}{r^2}(M - 2m) \Rightarrow M = 2m\,.$$

This leads to the result $m/M = 1/2$.

3. The magnitude of the force of one particle on the other is given by $F = Gm_1m_2/r^2$, where m_1 and m_2 are the masses, r is their separation, and G is the universal gravitational constant. We solve for r:

$$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)(5.2\text{kg})(2.4\text{kg})}{2.3\times10^{-12}\,\text{N}}} = 19\,\text{m}\,.$$

9. Both the Sun and the Earth exert a gravitational pull on the space probe. The net force can be calculated by using superposition principle. At the point where the two forces balance, we have $GM_em/r_1^2 = GM_sm/r_2^2$, where M_e is the mass of Earth, M_s is the mass of the Sun, m is the mass of the space probe, r_1 is the distance from the center of Earth to the probe, and r_2 is the distance from the center of the Sun to the probe. We substitute $r_2 = d - r_1$, where d is the distance from the center of Earth to the center of the Sun, to find

$$\frac{M_e}{r_1^2} = \frac{M_s}{(d-r_1)^2}\,.$$

Using the values for M_e, M_s, and d given in Appendix C, we take the positive square root of both sides to solve for r_1. A little algebra yields

$$r_1 = \frac{d}{1+\sqrt{M_s/M_e}} = \frac{1.50\times10^{11}\text{ m}}{1+\sqrt{(1.99\times10^{30}\text{ kg})/(5.98\times10^{24}\text{ kg})}} = 2.60\times10^{8}\text{ m}.$$

Note: The fact that $r_1 = \; d$ indicates that the probe is much closer to the Earth than the Sun.

17. (a) The gravitational acceleration at the surface of the Moon is g_{moon} = 1.67 m/s^2 (see Appendix C). The ratio of weights (for a given mass) is the ratio of *g*-values, so

$$W_{\text{moon}} = (100\text{ N})(1.67/9.8) = 17\text{ N}.$$

(b) For the force on that object caused by Earth's gravity to equal 17 N, then the free-fall acceleration at its location must be a_g = 1.67 m/s^2. Thus,

$$a_g = \frac{Gm_E}{r^2} \Rightarrow r = \sqrt{\frac{Gm_E}{a_g}} = 1.5 \times 10^7\text{ m}$$

so the object would need to be a distance of r/R_E = 2.4 "radii" from Earth's center.