# Chapter 11

27. Let $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$ be the position vector of the object, $\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}}$ its velocity vector, and *m* its mass. The cross product of $\vec{r}$ and $\vec{v}$ is (using Eq. 3-30)

$$\vec{r} \times \vec{v} = \;\; yv_z - zv_y \;\; \hat{\mathrm{i}} + \;\; zv_x - xv_z \;\; \hat{\mathrm{j}} + \;\; xv_y - yv_x \;\; \hat{\mathrm{k}}.$$

Since only the *x* and *z* components of the position and velocity vectors are nonzero (i.e., $y = 0$ and $v_y = 0$), the above expression becomes $\vec{r} \times \vec{v} = \left(-xv_z + zv_z\right)\hat{\mathrm{j}}.$ As for the torque, writing $\vec{F} = F_x\hat{\mathrm{i}} + F_y\hat{\mathrm{j}} + F_z\hat{\mathrm{k}},$ then we find $\vec{r} \times \vec{F}$ to be

$$\vec{\tau} = \vec{r} \times \vec{F} = \;\; yF_z - zF_y \;\; \hat{\mathrm{i}} + \;\; zF_x - xF_z \;\; \hat{\mathrm{j}} + \;\; xF_y - yF_x \;\; \hat{\mathrm{k}}.$$

(a) With $\vec{r} = (2.0 \text{ m})\hat{\mathrm{i}} - (2.0 \text{ m})\hat{\mathrm{k}}$ and $\vec{v} = (-5.0 \text{ m/s})\hat{\mathrm{i}} + (5.0 \text{ m/s})\hat{\mathrm{k}}$, in unit-vector notation, the angular momentum of the object is

$$\vec{\ell} = m \;\; -xv_z + zv_x \;\; \hat{\mathrm{j}} = \;\; 0.25 \text{ kg} \;\;\; - \;\; 2.0 \text{ m} \;\;\; 5.0 \text{ m/s} \;\; + \;\; -2.0 \text{ m} \;\;\; -5.0 \text{ m/s} \;\;\; \hat{\mathrm{j}} = 0.$$

(b) With $x$ = 2.0 m, $z$ = –2.0 m, $F_y$ = 4.0 N, and all other components zero, the expression above yields

$$\vec{\tau} = \vec{r} \times \vec{F} = (8.0 \text{ N}\cdot\text{m})\hat{\mathrm{i}} + (8.0 \text{ N}\cdot\text{m})\hat{\mathrm{k}}.$$

Note: The fact that $\vec{\ell} = 0$ implies that $\vec{r}$ and $\vec{v}$ are parallel to each other ($\vec{r} \times \vec{v} = 0$). Using $\tau = |\vec{r} \times \vec{F}| = rF\sin\phi$, we find the angle between $\vec{r}$ and $\vec{F}$ to be

$$\sin\phi = \frac{\tau}{rF} = \frac{8\sqrt{2} \text{ N}\cdot\text{m}}{(2\sqrt{2} \text{ m})(4.0 \text{ N})} = 1 \;\; \Rightarrow \;\; \phi = 90°$$

That is, $\vec{r}$ and $\vec{F}$ are perpendicular to each other.

34. We use a right-handed coordinate system with $\hat{\mathrm{k}}$ directed out of the *xy* plane so as to be consistent with counterclockwise rotation (and the right-hand rule). Thus, all the angular momenta being considered are along the $-\hat{\mathrm{k}}$ direction; for example, in part (b) $\vec{\ell} = -4.0t^2\,\hat{\mathrm{k}}$ in SI units. We use Eq. 11-23.

(a) The angular momentum is constant so its derivative is zero. There is no torque in this instance.

(b) Taking the derivative with respect to time, we obtain the torque:

$$\vec{\tau} = \frac{d\vec{\ell}}{dt} = \left(-4.0\hat{\mathrm{k}}\right)\frac{dt^2}{dt} = (-8.0t\ \mathrm{N\cdot m})\hat{\mathrm{k}}\,.$$

This vector points in the $-\hat{\mathrm{k}}$ direction (causing the clockwise motion to speed up) for all $t > 0$.

(c) With $\vec{\ell} = (-4.0\sqrt{t})\hat{\mathrm{k}}$ in SI units, the torque is

$$\vec{\tau} = \left(-4.0\hat{\mathrm{k}}\right)\frac{d\sqrt{t}}{dt} = \left(-4.0\hat{\mathrm{k}}\right)\left(\frac{1}{2\sqrt{t}}\right) = \left(-\frac{2.0}{\sqrt{t}}\hat{\mathrm{k}}\right)\mathrm{N\cdot m}\,.$$

This vector points in the $-\hat{\mathrm{k}}$ direction (causing the clockwise motion to speed up) for all $t > 0$ (and it is undefined for $t < 0$).

(d) Finally, we have

$$\vec{\tau} = \left(-4.0\hat{\mathrm{k}}\right)\frac{dt^{-2}}{dt} = \left(-4.0\hat{\mathrm{k}}\right)\left(\frac{-2}{t^3}\right) = \left(\frac{8.0}{t^3}\hat{\mathrm{k}}\right)\mathrm{N\cdot m}.$$

This vector points in the $+\hat{\mathrm{k}}$ direction (causing the initially clockwise motion to slow down) for all $t > 0$.

38. (a) Equation 10-34 gives $\alpha = \tau/I$ and Eq. 10-12 leads to $\omega = \alpha t = \tau t/I$. Therefore, the angular momentum at $t = 0.033$ s is

$$I\omega = \tau t = \left(16\,\mathrm{N\cdot m}\right)\left(0.033\mathrm{s}\right) = 0.53\,\mathrm{kg\cdot m^2/s}$$

where this is essentially a derivation of the angular version of the impulse-momentum theorem.

(b) We find

$$\omega = \frac{\tau t}{I} = \frac{\left(16\ \mathrm{N\cdot m}\right)\left(0.033\,\mathrm{s}\right)}{1.2\times10^{-3}\ \mathrm{kg\cdot m^2}} = 440\,\mathrm{rad/s}$$

which we convert as follows:

$$\omega = (440\ \mathrm{rad/s})(60\ \mathrm{s/min})(1\ \mathrm{rev}/2\pi\ \mathrm{rad}) \approx 4.2\times10^3\ \mathrm{rev/min}.$$

40. Torque is the time derivative of the angular momentum. Thus, the change in the angular momentum is equal to the time integral of the torque. With $\tau = (5.00 + 2.00t)\ \text{N}\cdot\text{m}$, the angular momentum (in units $\text{kg}\cdot\text{m}^2/\text{s}$) as a function of time is

$$L(t) = \int \tau\, dt = \int (5.00 + 2.00t)dt = L_0 + 5.00t + 1.00t^2 .$$

Since $L = 5.00\ \text{kg}\cdot\text{m}^2/\text{s}$ when $t = 1.00\ \text{s}$, the integration constant is $L_0 = -1$. Thus, the complete expression of the angular momentum is

$$L(t) = -1 + 5.00t + 1.00t^2 .$$

At $t = 3.00\ \text{s}$, we have $L(t = 3.00) = -1 + 5.00(3.00) + 1.00(3.00)^2 = 23.0\ \text{kg}\cdot\text{m}^2/\text{s}.$

45. (a) No external torques act on the system consisting of the man, bricks, and platform, so the total angular momentum of the system is conserved. Let $I_i$ be the initial rotational inertia of the system and let $I_f$ be the final rotational inertia. Then $I_i\omega_i = I_f\omega_f$ and

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{6.0\ \text{kg}\cdot\text{m}^2}{2.0\ \text{kg}\cdot\text{m}^2}\right)\ 1.2\ \text{rev/s} \ = 3.6\ \text{rev/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2} I_i\omega_i^2$, the final kinetic energy is $K_f = \frac{1}{2} I_f\omega_f^2$, and their ratio is

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2/2}{I_i\omega_i^2/2} = \frac{2.0\ \text{kg}\cdot\text{m}^2 \quad 3.6\ \text{rev/s}\ ^2/2}{6.0\ \text{kg}\cdot\text{m}^2 \quad 1.2\ \text{rev/s}\ ^2/2} = 3.0.$$

(c) The man did work in decreasing the rotational inertia by pulling the bricks closer to his body. This energy came from the man's store of internal energy.