Chapter 9

12. Since the center of mass of the two-skater system does not move, both skaters will end up at the center of mass of the system. Let the center of mass be a distance x from the 40-kg skater, then

$$(65 \text{ kg})(10 \text{ m}-x)=(40 \text{ kg})x \Rightarrow x=6.2 \text{ m}.$$

Thus the 40-kg skater will move by 6.2 m.

18. The magnitude of the ball's momentum change is

$$\Delta p = m\left|v_i - v_f\right| = (0.70 \text{ kg})\left|(5.0 \text{ m/s})-(-2.0 \text{ m/s})\right| = 4.9 \text{ kg}\cdot\text{m/s}.$$

20. We infer from the graph that the horizontal component of momentum p_x is 4.0 $\text{kg}\cdot\text{m/s}$. Also, its initial magnitude of momentum p_o is 6.0 $\text{kg}\cdot\text{m/s}$. Thus,

$$\cos\theta_o = \frac{p_x}{p_o} \quad \Rightarrow \quad \theta_o = 48°.$$

26. (a) By energy conservation, the speed of the victim when he falls to the floor is

$$\frac{1}{2}mv^2 = mgh \quad \Rightarrow \quad v = \sqrt{2gh} = \sqrt{2(9.8 \text{ m/s}^2)(0.50 \text{ m})} = 3.1 \text{ m/s}.$$

Thus, the magnitude of the impulse is

$$J = |\Delta p| = m|\Delta v| = mv = (70 \text{ kg})(3.1 \text{ m/s}) \approx 2.2\times10^2 \text{ N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 0.082$ s for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{2.2\times10^2 \text{ N}\cdot\text{s}}{0.082 \text{ s}} \approx 2.7\times10^3 \text{ N}.$$

29. We choose the positive direction in the direction of rebound so that $\vec{v}_f > 0$ and $\vec{v}_i < 0$. Since they have the same speed v, we write this as $\vec{v}_f = v$ and $\vec{v}_i = -v$. Therefore, the change in momentum for each bullet of mass m is $\Delta\vec{p} = m\Delta v = 2mv$. Consequently, the total change in momentum for the 100 bullets (each minute) $\Delta\vec{P} = 100\Delta\vec{p} = 200mv$. The average force is then

$$\vec{F}_{avg} = \frac{\Delta \vec{P}}{\Delta t} = \frac{(200)(3 \times 10^{-3} \text{kg})(500 \text{ m/s})}{(1 \text{min})(60 \text{ s/min})} \approx 5 \text{ N}.$$

39. **THINK** This problem deals with momentum conservation. Since no external forces with horizontal components act on the man-stone system and the vertical forces sum to zero, the total momentum of the system is conserved.

EXPRESS Since the man and the stone are initially at rest, the total momentum is zero both before and after the stone is kicked. Let m_s be the mass of the stone and v_s be its velocity after it is kicked. Also, let m_m be the mass of the man and v_m be his velocity after he kicks the stone. Then, by momentum conservation,

$$m_s v_s + m_m v_m = 0 \implies v_m = -\frac{m_s}{m_m} v_s.$$

ANALYZE We take the axis to be positive in the direction of motion of the stone. Then

$$v_m = -\frac{m_s}{m_m} v_s = -\frac{0.068 \text{ kg}}{91 \text{ kg}}(4.0 \text{ m/s}) = -3.0 \times 10^{-3} \text{ m/s}$$

or $|v_m| = 3.0 \times 10^{-3}$ m/s.

LEARN The negative sign in v_m indicates that the man moves in the direction opposite to the motion of the stone. Note that his speed is much smaller (by a factor of m_s / m_m) compared to the speed of the stone.

46. Our $+x$ direction is east and $+y$ direction is north. The linear momenta for the two $m = 2.0$ kg parts are then

$$\vec{p}_1 = m\vec{v}_1 = mv_1\,\hat{j}$$

where $v_1 = 3.0$ m/s, and

$$\vec{p}_2 = m\vec{v}_2 = m\left(v_{2x}\,\hat{i} + v_{2y}\,\hat{j}\right) = mv_2\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

where $v_2 = 5.0$ m/s and $\theta = 30°$. The combined linear momentum of both parts is then

$$\vec{P} = \vec{p}_1 + \vec{p}_2 = mv_1\,\hat{j} + mv_2\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right) = \left(mv_2\cos\theta\right)\hat{i} + \left(mv_1 + mv_2\sin\theta\right)\hat{j}$$

$$= (2.0 \text{ kg})(5.0 \text{ m/s})(\cos 30°)\hat{i} + (2.0 \text{ kg})\left(3.0 \text{ m/s} + (5.0 \text{ m/s})(\sin 30°)\right)\hat{j}$$

$$= \left(8.66\hat{i} + 11\hat{j}\right) \text{ kg·m/s.}$$

From conservation of linear momentum we know that this is also the linear momentum of the whole kit before it splits. Thus the speed of the 4.0-kg kit is

$$v = \frac{P}{M} = \frac{\sqrt{P_x^2 + P_y^2}}{M} = \frac{\sqrt{\left(8.66 \text{ kg} \cdot \text{m/s}\right)^2 + \left(11 \text{ kg} \cdot \text{m/s}\right)^2}}{4.0 \text{ kg}} = 3.5 \text{ m/s.}$$

50. (a) We choose $+x$ along the initial direction of motion and apply momentum conservation:

$$m_{\text{bullet}} \vec{v}_i = m_{\text{bullet}} \vec{v}_1 + m_{\text{block}} \vec{v}_2$$
$$(5.2 \text{ g})(672 \text{ m/s}) = (5.2 \text{ g})(428 \text{ m/s}) + (700 \text{ g})\vec{v}_2$$

which yields $v_2 = 1.81$ m/s.

(b) It is a consequence of momentum conservation that the velocity of the center of mass is unchanged by the collision. We choose to evaluate it before the collision:

$$\vec{v}_{\text{com}} = \frac{m_{\text{bullet}} \vec{v}_i}{m_{\text{bullet}} + m_{\text{block}}} = \frac{(5.2 \text{ g})(672 \text{ m/s})}{5.2 \text{ g} + 700 \text{ g}} = 4.96 \text{ m/s.}$$

91. No external forces with horizontal components act on the cart-man system and the vertical forces sum to zero, so the total momentum of the system is conserved. Let m_c be the mass of the cart, v be its initial velocity, and v_c be its final velocity (after the man jumps off). Let m_m be the mass of the man. His initial velocity is the same as that of the cart and his final velocity is zero. Conservation of momentum yields $(m_m + m_c)v = m_c v_c$. Consequently, the final speed of the cart is

$$v_c = \frac{v(m_m + m_c)}{m_c} = \frac{(2.3 \text{ m/s})(75 \text{ kg} + 39 \text{ kg})}{39 \text{ kg}} = 6.7 \text{ m/s.}$$

The cart speeds up by 6.7 m/s − 2.3 m/s = + 4.4 m/s. In order to slow himself, the man gets the cart to push backward on him by pushing forward on it, so the cart speeds up.

102. (a) Since the center of mass of the man-balloon system does not move, the balloon will move downward with a certain speed u relative to the ground as the man climbs up the ladder.

(b) The speed of the man relative to the ground is $v_g = v − u$. Thus, the speed of the center of mass of the system is

$$v_{\text{com}} = \frac{mv_g - Mu}{M + m} = \frac{m(v - u) - Mu}{M + m} = 0.$$

This yields

$$u = \frac{mv}{M + m} = \frac{(80 \text{ kg})(2.5 \text{ m/s})}{320 \text{ kg} + 80 \text{ kg}} = 0.50 \text{ m/s.}$$

(c) Now that there is no relative motion within the system, the speed of both the balloon and the man is equal to v_{com}, which is zero. So the balloon will again be stationary.