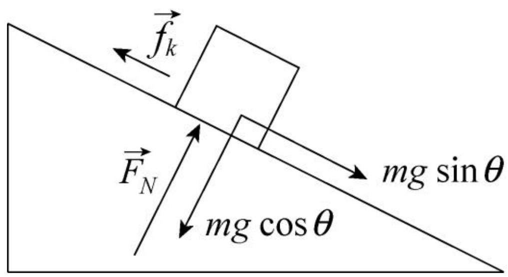
Chapter 6

1. The greatest deceleration (of magnitude a) is provided by the maximum friction force (Eq. 6-1, with $F_N = mg$ in this case). Using Newton's second law, we find

$$a = f_{s,\max}/m = \mu_s g.$$

Eq. 2-16 then gives the shortest distance to stop: $|\Delta x| = v^2/2a = 36$ m. In this calculation, it is important to first convert v to 13 m/s.

4. We first analyze the forces on the pig of mass m. The incline angle is θ.



The $+x$ direction is "downhill.'' Application of Newton's second law to the x- and y-axes leads to

$$mg\sin\theta - f_k = ma$$
$$F_N - mg\cos\theta = 0.$$

Solving these along with Eq. 6-2 ($f_k = \mu_k F_N$) produces the following result for the pig's downhill acceleration:

$$a = g\left(\sin\theta - \mu_k\cos\theta\right).$$

To compute the time to slide from rest through a downhill distance ℓ, we use Eq. 2-15:

$$\ell = v_0 t + \frac{1}{2}at^2 \;\Rightarrow\; t = \sqrt{\frac{2\ell}{a}}.$$

We denote the frictionless ($\mu_k = 0$) case with a prime and set up a ratio:

$$\frac{t}{t'} = \frac{\sqrt{2\ell/a}}{\sqrt{2\ell/a'}} = \sqrt{\frac{a'}{a}}$$

which leads us to conclude that if $t/t' = 2$ then $a' = 4a$. Putting in what we found out above about the accelerations, we have

$$g\sin\theta = 4g\left(\sin\theta - \mu_k\cos\theta\right).$$

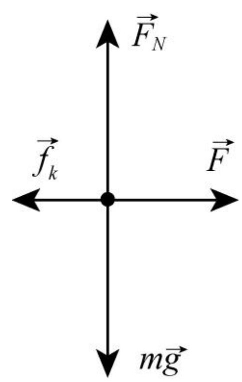
Using $\theta = 35°$, we obtain $\mu_k = 0.53$.

6. The free-body diagram for the player is shown to the right. $\vec{F}_N$ is the normal force of the ground on the player, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. The force of friction is related to the normal force by $f = \mu_k F_N$. We use Newton's second law applied to the vertical axis to find the normal force. The vertical component of the acceleration is zero, so we obtain $F_N - mg = 0$; thus, $F_N = mg$. Consequently,

$$\mu_k = \frac{f}{F_N} = \frac{470\ \text{N}}{(79\ \text{kg})(9.8\ \text{m/s}^2)} = 0.61.$$

7. **THINK** A force is being applied to accelerate a crate in the presence of friction. We apply Newton's second law to solve for the acceleration.

EXPRESS The free-body diagram for the crate is shown to the right. We denote $\vec{F}$ as the horizontal force of the person exerted on the crate (in the $+x$ direction), $\vec{f}_k$ is the force of kinetic friction (in the $-x$ direction), F_N is the vertical normal force exerted by the floor (in the $+y$ direction), and $m\vec{g}$ is the force of gravity. The magnitude of the force of friction is given by Eq. 6-2: $f_k = \mu_k F_N$. Applying Newton's second law to the x and y axes, we obtain



$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively.

ANALYZE (a) The second equation above yields the normal force $F_N = mg$, so that the friction is

$$f_k = \mu_k F_N = \mu_k mg = (0.35)(55\ \text{kg})(9.8\ \text{m/s}^2) = 1.9\times10^2\ \text{N}.$$

(b) The first equation becomes

$$F - \mu_k mg = ma$$

which (with $F = 220$ N) we solve to find

$$a = \frac{F}{m} - \mu_k g = \frac{220\ \text{N}}{55\ \text{kg}} - (0.35)(9.8\ \text{m/s}^2) = 0.56\ \text{m/s}^2.$$

LEARN For the crate to accelerate, the condition $F > f_k = \mu_k mg$ must be met. As can be seen from the equation above, the greater the value of μ_k, the smaller the acceleration under the same applied force.

18. (a) We apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma,$$

where, using Eq. 6-11,

$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta.$$

Thus, with μ_k = 0.600, we have

$$a = g\sin\theta - \mu_k\cos\theta = -3.72\ \text{m/s}^2$$

which means, since we have chosen the positive direction in the direction of motion (down the slope) then the acceleration vector points "uphill"; it is decelerating. With $v_0 = 18.0\ \text{m/s}$ and $\Delta x = d = 24.0$ m, Eq. 2-16 leads to

$$v = \sqrt{v_0^2 + 2ad} = 12.1\ \text{m/s}.$$

(b) In this case, we find $a = +1.1\ \text{m/s}^2$, and the speed (when impact occurs) is 19.4 m/s.

30. We use the familiar horizontal and vertical axes for x and y directions, with rightward and upward positive, respectively. The rope is assumed massless so that the force exerted by the child $\vec{F}$ is identical to the tension uniformly through the rope. The x and y components of $\vec{F}$ are $F\cos\theta$ and $F\sin\theta$, respectively. The static friction force points leftward.

(a) Newton's Law applied to the y-axis, where there is presumed to be no acceleration, leads to

$$F_N + F\sin\theta - mg = 0$$

which implies that the maximum static friction is $\mu_s(mg - F\sin\theta)$. If $f_s = f_{s,\max}$ is assumed, then Newton's second law applied to the x axis (which also has a = 0 even though it is "verging" on moving) yields

$$F\cos\theta - f_s = ma \quad \Rightarrow \quad F\cos\theta - \mu_s(mg - F\sin\theta) = 0$$

which we solve, for θ = 42° and μ_s = 0.42, to obtain F = 74 N.

(b) Solving the above equation algebraically for F, with W denoting the weight, we obtain

$$F = \frac{\mu_s W}{\cos\theta + \mu_s \sin\theta} = \frac{(0.42)(180\text{ N})}{\cos\theta + (0.42)\sin\theta} = \frac{76\text{ N}}{\cos\theta + (0.42)\sin\theta}.$$

(c) We minimize the above expression for F by working through the condition:

$$\frac{dF}{d\theta} = \frac{\mu_s W(\sin\theta - \mu_s \cos\theta)}{(\cos\theta + \mu_s \sin\theta)^2} = 0$$

which leads to the result $\theta = \tan^{-1}\mu_s = 23°$.

(d) Plugging $\theta = 23°$ into the above result for F, with $\mu_s = 0.42$ and $W = 180$ N, yields $F = 70\text{ N}$.

33. **THINK** In this problem, the frictional force is not a constant, but instead proportional to the speed of the boat. Integration is required to solve for the speed.

EXPRESS We denote the magnitude of the frictional force as αv, where $\alpha = 70\text{ N}\cdot\text{s/m}$. We take the direction of the boat's motion to be positive. Newton's second law gives

$$-\alpha v = m\frac{dv}{dt} \;\Rightarrow\; \frac{dv}{v} = -\frac{\alpha}{m}dt.$$

Integrating the equation gives

$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{\alpha}{m}\int_0^t dt$$

where v_0 is the velocity at time zero and v is the velocity at time t. Solving the integral allows us to calculate the time it takes for the boat to slow down to 45 km/h, or $v = v_0/2$, where $v_0 = 90\text{ km/h}$.

ANALYZE The integrals are evaluated with the result

$$\ln\left(\frac{v}{v_0}\right) = -\frac{\alpha t}{m}$$

With $v = v_0/2$ and $m = 1000$ kg, we find the time to be

$$t = -\frac{m}{\alpha}\ln\left(\frac{v}{v_0}\right) = -\frac{m}{\alpha}\ln\left(\frac{1}{2}\right) = -\frac{1000\text{ kg}}{70\text{ N}\cdot\text{s/m}}\ln\left(\frac{1}{2}\right) = 9.9\text{ s}.$$

LEARN The speed of the boat is given by $v(t) = v_0 e^{-\alpha t/m}$, showing exponential decay with time. The greater the value of α, the more rapidly the boat slows down.

41. Perhaps surprisingly, the equations pertaining to this situation are exactly those in Sample Problem – "Car in flat circular turn," although the logic is a little different. In the Sample Problem, the car moves along a (stationary) road, whereas in this problem the cat is stationary relative to the merry-go-around platform. But the static friction plays the same role in both cases since the bottom-most point of the car tire is instantaneously at rest with respect to the race track, just as static friction applies to the contact surface between cat and platform. Using Eq. 6-23 with Eq. 4-35, we find

$$\mu_s = (2\pi R/T)^2/gR = 4\pi^2 R/gT^2.$$

With T = 6.0 s and R = 5.4 m, we obtain μ_s = 0.60.

42. The magnitude of the acceleration of the car as it rounds the curve is given by v^2/R, where v is the speed of the car and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If F_N is the normal force of the road on the car and m is the mass of the car, the vertical component of Newton's second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is

$$f_{s,\max} = \mu_s F_N = \mu_s mg.$$

If the car does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \quad \Rightarrow \quad v \le \sqrt{\mu_s R g}.$$

Consequently, the maximum speed with which the car can round the curve without slipping is

$$v_{\max} = \sqrt{\mu_s R g} = \sqrt{(0.60)(30.5\text{ m})(9.8\text{ m/s}^2)} = 13\text{ m/s} \approx 48\text{ km/h}.$$